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Periodic orbits in the neighborhood of the triangular equilibrium points in the photogravitational restricted three body problem – Part 1

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Abstract

Objective: To find the periodic orbits using Fourier series expansions around the libration points L_4 and L_5 in the frame work of planar restricted three-body by considering the more massive primary a source of radiation. **Methods/Statistical analysis:** Period of the periodic orbits around the libration point L_4 is found using the variational equations and function of the two finite masses. The period is independent on the size the orbit. When terms of higher order are retained in the analysis the period depends on the size of the orbit. **Findings**: The value of the critical mass is found in the photo-gravitational restricted three–body problem and is shown that the critical mass value corresponding to the small mass increases with the size of the orbit. It is shown that the classes of periodic orbits with infinitesimal limiting orbits L_4 exist for values of the small mass greater than the critical mass value $\mu_{0.}$ A comparison between these orbits with and without radiation pressure is made. **Applications**: Periodic orbits can be used to explore small solar system bodies, including asteroids and comets. Perturbation due to solar radiation pressure has to be understood and should be taken care of during human exploration mission.

Keywords: Photogravitational restricted problem; triangular liberation point; periodic orbits; solar radiation pressure; Fourier series expansions

1 Introduction

Brown [1] considered the long period orbits around the triangular libration points by supposing finite amplitudes of libration and discussed in some detail the dependence of period and orbit shape on amplitude. Gylden [2] had begun the study of the infinitesimal periodic orbits around the libration points in the restricted body problem of three bodies. After that a number of investigations of the problem have appeared in the literature. E. Strö mgren in 1928 has pointed out that the variational equations are suitable as a basis for numerical integrations. The period of the periodic orbits is found using the variational equations around a libration point and a function of two finite masses, while it is independent of the size of the orbit. The study in [3] showed that the period of the periodic orbits around the libration points L_4 and L_5 appears as a function of the size of the orbits, when terms of second and third order are retained in the series expression, however the investigation to the case that the masses have values in the neighbourhood of the critical values μ_0 and 1- μ_0. In this paper we have utilized the approach of [3] in the planar RTBP when the more massive primary is a source of radiation.

Restricted three-body problem (RTBP) describes the motion of a massless body which moves under the gravitational effect of two finite masses called primaries. The primaries are supposed to move in circular orbits around their center of mass on account of their mutual attraction. [4] is a fundamental book on the RTBP. During the last few decades many perturbing forces, such as oblateness and radiation forces of the primaries, variation in Coriolis and centrifugal forces, variation of the masses of the primaries have been included in the RTBP to study different aspects. In [5] Radzievskii formulated the photo-gravitational restricted three-body problem (PRTBP) and studied the location of the equilibrium points. This arises from the classical problem when one of the masses is an intense emitter of radiation. Some of the papers in [6–16] have discussed the effect of the radiation pressure in the RTBP. [17] have studied the existence and stability of equilibrium points in the plane of motion of the primaries, where the primaries are sources of radiation. In [18,19] the effect of radiation pressure is discussed in the four-body problem.

The effect of radiation pressure of a source can be expressed by a mass reduction factor q =1 - ε, where the radiation coefficient ε is the ratio of the force F_p which is caused by radiation to the force F_g which results from gravitation, i.e., $\varepsilon = \frac{F_p}{F_g}$ .q is expressed in terms of particle radius 'a', density 'δ' and radiation pressure efficiency χ(in CGS system) as

$$q = 1 - \frac{5.6\ \times 10^{-5}}{a\delta}\chi$$

Knowing the mass and the luminosity of the radiating body, ε can be found for any given radius and density. Solar radiation pressure force F_p changes with distance by the same law of gravitational attraction force F_g and acts opposite to it. Thus, Sun's resulting force acting on the particle is [6,9]

$$\mathrm{F} = \mathrm{F_g} - \mathrm{F_p} = \left(1 - \frac{\mathrm{F_p}}{\mathrm{F_g}}\right)\mathrm{F_g} = (1-\varepsilon)\mathrm{F_g} = (\mathrm{q})\mathrm{F_g}$$

If q =1, radiation has no effect. If q < 0, then radiation surpasses gravity and if 0 < q $\leq$ 1, gravitational force exceeds radiation.

2 Equations of Motion

In the dimensionless synodic coordinate system with origin of the system positioned at the center of mass of the primaries, considering the more massive primary at the location (-μ,0) and the smaller primary at (1-μ,0), the equations of motion for the circular photogravitational planar restricted three–body problem in the dimensionless barycentric synodic coordinates (x, y) are :

$$\begin{aligned} \ddot{x} - 2\dot{y} &= \Omega_x \\ \ddot{y} + 2\dot{x} &= \Omega_y, \end{aligned} \tag{1}$$

where

$$\Omega = \frac{1}{2}\left[(1-\mu)\mathrm{r}_1^2 + \mu r_2^2\right] + \frac{q(1-\mu)}{r_1} + \frac{\mu}{r_2} \tag{2}$$

and r_1 and r_2 are determined by the equations

$$\begin{aligned} r_1^2 &= (x+\mu)^2 + y^2 \\ r_2^2 &= (x-1+\mu)^2 + y^2 \end{aligned} \tag{3}$$

The position of the libration triangular points L_4 and L_5 in terms of ε = 1 - q are:

$$\left(\frac{1}{2} - \frac{\varepsilon}{3} - \mu,\ \frac{\sqrt{3}}{2} - \frac{\sqrt{3}\,\varepsilon}{9}\right), \left(\frac{1}{2} - \frac{\varepsilon}{3} - \mu, -\frac{\sqrt{3}}{2} + \frac{\sqrt{3}\,\varepsilon}{9}\right)$$

To investigate the motion of the infinitesimal mass in the neighborhood of the libration point L_4, we introduce a new coordinate system with its origin at L_4.

$$\begin{aligned} x &= \frac{1}{2} + \xi - \frac{\varepsilon}{3} - \mu, \\ y &= \frac{\sqrt{3}}{2} + \eta - \frac{\sqrt{3}\varepsilon}{2}. \end{aligned} \tag{4}$$

The differential equations of motion of the infinitesimal mass in the new co-ordinate system are

$$\begin{aligned} \ddot{\xi} - 2\dot{\eta} &= \frac{\partial \Omega}{\partial \xi} \\ \ddot{\eta} + 2\dot{\xi} &= \frac{\partial \Omega}{\partial \eta} \end{aligned} \tag{5}$$

where

$$\Omega = \frac{1}{2}\left[\left(\frac{1}{2} + \xi - \frac{\varepsilon}{3} - \mu\right)^2 + \left(\frac{\sqrt{3}}{2} + \eta - \frac{\sqrt{3}\varepsilon}{9}\right)^2\right] + \frac{q(1-\mu)}{\rho_1} + \frac{\mu}{\rho_2}, \tag{6}$$

and where ρ_1 and ρ_2 are determined by the equations

$$\begin{aligned} \rho_1^2 &= \left(\frac{\sqrt{3}}{2} - \frac{\varepsilon}{3\sqrt{3}} + \eta\right)^2 + \left(\frac{1}{2} - \frac{\varepsilon}{3} + \xi\right)^2 \\ \rho_2^2 &= \left(\frac{\sqrt{3}}{2} - \frac{\varepsilon}{3\sqrt{3}} + \eta\right)^2 + \left(-\frac{1}{2} - \frac{\varepsilon}{3} + \xi\right)^2 \end{aligned} \tag{7}$$

The Jacobi integral is

$$\dot{\xi}^2 + \dot{\eta}^2 = 2\,\Omega - C. \tag{8}$$

With the help of series expansions, we get

$$\begin{aligned} \frac{1}{\rho_1} = \Bigg(\Bigg(&\frac{3\sqrt{3}}{\sqrt{27-18\varepsilon+4\varepsilon^2}} + \frac{27(-9+2\varepsilon)\eta}{2\,(27-18\varepsilon+4\varepsilon^2)^{3/2}} - \frac{81\left(\sqrt{3}\left(-135+36\varepsilon+4\varepsilon^2\right)\right)\eta^2}{8\,(27-18\varepsilon+4\varepsilon^2)^{5/2}} - \frac{729\left(729+162\varepsilon-324\varepsilon^2+56\varepsilon^3\right)\eta^3}{16\,(27-18\varepsilon+4\varepsilon^2)^{7/2}}\Bigg) + \\ &\Bigg(\frac{27\sqrt{3}(-3+2\varepsilon)}{2\,(27-18\varepsilon+4\varepsilon^2)^{3/2}} + \frac{729(-9+2\varepsilon)(-3+2\varepsilon)\eta}{4\,(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{2187\sqrt{3}(-3+2\varepsilon)\left(297-108\varepsilon+4\varepsilon^2\right)\eta^2}{16\,(27-18\varepsilon+4\varepsilon^2)^{7/2}} - \\ &\frac{32805\left((-3+2\varepsilon)\left(2187-810\varepsilon-108\varepsilon^2+40\varepsilon^3\right)\right)\eta^3}{32\,(27-18\varepsilon+4\varepsilon^2)^{9/2}}\Bigg)\xi + \Bigg(\frac{81\sqrt{3}\left(-27-36\varepsilon+20\varepsilon^2\right)}{8\,(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{2187\left(-243+1026\varepsilon-612\varepsilon^2+88\varepsilon^3\right)\eta}{16\,(27-18\varepsilon+4\varepsilon^2)^{7/2}} + \\ &\frac{6561\sqrt{3}\left(29889-73872\varepsilon+48600\varepsilon^2-10944\varepsilon^3+656\varepsilon^4\right)\eta^2}{64\,(27-18\varepsilon+4\varepsilon^2)^{9/2}} - \\ &\frac{98415\left(452709-1010394\varepsilon+694008\varepsilon^2-171504\varepsilon^2+7632\varepsilon^4+1376\varepsilon^5\right)\eta^3}{128\,(27-18\varepsilon+4\varepsilon^2)^{11/2}}\Bigg)\xi^2 + \Bigg(\frac{2187\sqrt{3}(-3+2\varepsilon)\left(-63+12\varepsilon+4\varepsilon^2\right)}{16\,(27-18\varepsilon+4\varepsilon^2)^{7/2}} + \\ &\frac{295245(-3+2\varepsilon)\left(135+6\varepsilon-44\varepsilon^2+8\varepsilon^3\right)\eta}{32\,(27-18\varepsilon+4\varepsilon^2)^{9/2}} + \frac{295245\sqrt{3}(-3+2\varepsilon)\left(-10449-15552\varepsilon+18216\varepsilon^2-4992\varepsilon^3+368\varepsilon^4\right)\eta^2}{128\,(27-18\varepsilon+4\varepsilon^2)^{11/2}} + \\ &\frac{18600435(-3+2\varepsilon)\left(-6561+106434\varepsilon-103032\varepsilon^2+33264\varepsilon^2-3600\varepsilon^4+32\varepsilon^5\right)\eta^3}{256\,(27-18\varepsilon+4\varepsilon^2)^{13/2}}\Bigg)\xi^3\Bigg), \end{aligned}$$

$$\frac{1}{\rho_2}=\left(\left(\frac{3\sqrt{3}}{\sqrt{27+4\varepsilon^2}}+\frac{27(-9+2\varepsilon)\eta}{2\left(27+4\varepsilon^2\right)^{3/2}}-\frac{81\left(\sqrt{3}\left(-135+108\varepsilon+4\varepsilon^2\right)\right)\eta^2}{8\left(27+4\varepsilon^2\right)^{5/2}}-\frac{729\left(729-1782\varepsilon+108\varepsilon^2+56\varepsilon^2\right)\eta^2}{16\left(27+4\varepsilon^2\right)^{7/2}}\right)+\right.$$
$$\left(\frac{27\sqrt{3}(3+2\varepsilon)}{2\left(27+4\varepsilon^2\right)^{3/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\eta}{4\left(27+4\varepsilon^2\right)^{5/2}}+\frac{2187\sqrt{3}(3+2\varepsilon)\left(297-180\varepsilon+4\varepsilon^2\right)\eta^2}{16\left(27+4\varepsilon^2\right)^{7/2}}-\right.$$
$$\left.\frac{32805\left((3+2\varepsilon)\left(2187-2754\varepsilon+324\varepsilon^2+40\varepsilon^3\right)\right)\eta^3}{32\left(27+4\varepsilon^2\right)^{9/2}}\right)\xi+\left(\frac{81\sqrt{3}\left(-27+108\varepsilon+20\varepsilon^2\right)}{8\left(27+4\varepsilon^2\right)^{5/2}}+\frac{2187\left(-243-1566\varepsilon-36\varepsilon^2+88\varepsilon^3\right)\eta}{16\left(27+4\varepsilon^2\right)^{7/2}}+\right.$$
$$\frac{6561\sqrt{3}\left(29889+68040\varepsilon-21384\varepsilon^2-10080\varepsilon^3+656\varepsilon^4\right)\eta^2}{64\left(27+4\varepsilon^2\right)^{9/2}}-$$
$$\left.\frac{98415\left(452709+511758\varepsilon-600696\varepsilon^2-132624\varepsilon^3+46224\varepsilon^4+1376\varepsilon^5\right)\eta^3}{128\left(27+4\varepsilon^2\right)^{11/2}}\right)\xi^2+\left(\frac{2187\sqrt{3}(3+2\varepsilon)\left(-63+60\varepsilon+4\varepsilon^2\right)}{16\left(27+4\varepsilon^2\right)^{7/2}}+\right.$$
$$\frac{295245(3+2\varepsilon)\left(135-282\varepsilon+20\varepsilon^2+8\varepsilon^2\right)\eta}{32\left(27+4\varepsilon^2\right)^{9/2}}+\frac{295245\sqrt{3}(3+2\varepsilon)\left(-10449+58968\varepsilon-17208\varepsilon^2-3360\varepsilon^3+368\varepsilon^4\right)\eta^2}{128\left(27+4\varepsilon^2\right)^{11/2}}+$$
$$\left.\left.\frac{18600435(3+2\varepsilon)\left(-6561-190998\varepsilon+126360\varepsilon^2+9936\varepsilon^3-6480\varepsilon^4+32\varepsilon^5\right)\eta^3}{256\left(27+4\varepsilon^2\right)^{13/2}}\right)\xi^3\right)$$

Inserting (9) in (6), we find the expression of Ω $after$ simplifications as:

$$\Omega=\frac{1}{2}\left(\frac{\sqrt{3}}{2}-\frac{\varepsilon}{3\sqrt{3}}+\eta\right)^2+\frac{1}{2}\left(\frac{1}{2}-\frac{\varepsilon}{3}-\mu+\xi\right)^2+\mu\left(\frac{3\sqrt{3}}{\sqrt{27+4\varepsilon^2}}+\frac{27(-9+2\varepsilon)\eta}{2(27+4\varepsilon^2)^{3/2}}-\right.$$
$$\frac{81\sqrt{3}(-135+108\varepsilon+4\varepsilon^2)\eta^2}{8(27+4\varepsilon^2)^{5/2}}-\frac{729(729-1782\varepsilon+108\varepsilon^2+56\varepsilon^3)\eta^3}{16(27+4\varepsilon^2)^{7/2}}+\left(\frac{27\sqrt{3}(3+2\varepsilon)}{2(27+4\varepsilon^2)^{3/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\eta}{4(27+4\varepsilon^2)^{5/2}}+\right.$$
$$\left.\frac{2187\sqrt{3}(3+2\varepsilon)(297-180\varepsilon+4\varepsilon^2)\eta^2}{16(27+4\varepsilon^2)^{7/2}}-\frac{32805(3+2\varepsilon)(2187-2754\varepsilon+324\varepsilon^2+40\varepsilon^3)\eta^3}{32(27+4\varepsilon^2)^{9/2}}\right)\xi+\left(\frac{81\sqrt{3}(-27+108\varepsilon+20\varepsilon^2)}{8(27+4\varepsilon^2)^{5/2}}+\right.$$
$$\frac{2187(-243-1566\varepsilon-36\varepsilon^2+88\varepsilon^3)\eta}{16(27+4\varepsilon^2)^{7/2}}+\frac{6561\sqrt{3}(29889+68040\varepsilon-21384\varepsilon^2-10080\varepsilon^3+656\varepsilon^4)\eta^2}{64(27+4\varepsilon^2)^{9/2}}-$$
$$\left.\frac{98415(452709+511758\varepsilon-600696\varepsilon^2-132624\varepsilon^3+46224\varepsilon^4+1376\varepsilon^5)\eta^3}{128(27+4\varepsilon^2)^{11/2}}\right)\xi^2+\left(\frac{2187\sqrt{3}(3+2\varepsilon)(-63+60\varepsilon+4\varepsilon^2)}{16(27+4\varepsilon^2)^{7/2}}+\right.$$
$$\frac{295245(3+2\varepsilon)(135-282\varepsilon+20\varepsilon^2+8\varepsilon^3)\eta}{32(27+4\varepsilon^2)^{9/2}}+\frac{295245\sqrt{3}(3+2\varepsilon)(-10449+58968\varepsilon-17208\varepsilon^2-3360\varepsilon^3+368\varepsilon^4)\eta^2}{128(27+4\varepsilon^2)^{11/2}}+$$
$$\left.\left.\frac{18600435(3+2\varepsilon)(-6561-190998\varepsilon+126360\varepsilon^2+9936\varepsilon^3-6480\varepsilon^4+32\varepsilon^5)\eta^3}{256(27+4\varepsilon^2)^{13/2}}\right)\xi^3\right)+(1-\varepsilon)(1-\mu)\left(\frac{3\sqrt{3}}{\sqrt{27-18\varepsilon+4\varepsilon^2}}+\right.$$
$$\frac{27(-9+2\varepsilon)\eta}{2(27-18\varepsilon+4\varepsilon^2)^{3/2}}-\frac{81\sqrt{3}(-135+36\varepsilon+4\varepsilon^2)\eta^2}{8(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{729(729+162\varepsilon-324\varepsilon^2+56\varepsilon^3)\eta^3}{16(27-18\varepsilon+4\varepsilon^2)^{7/2}}+\left(\frac{27\sqrt{3}(-3+2\varepsilon)}{2(27-18\varepsilon+4\varepsilon^2)^{3/2}}+\right.$$
$$\left.\frac{729(-9+2\varepsilon)(-3+2\varepsilon)\eta}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{2187\sqrt{3}(-3+2\varepsilon)(297-108\varepsilon+4\varepsilon^2)\eta^2}{16(27-18\varepsilon+4\varepsilon^2)^{7/2}}-\frac{32805(-3+2\varepsilon)(2187-810\varepsilon-108\varepsilon^2+40\varepsilon^3)\eta^3}{32(27-18\varepsilon+4\varepsilon^2)^{9/2}}\right)\xi+$$
$$\left(\frac{81\sqrt{3}(-27-36\varepsilon+20\varepsilon^2)}{8(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{2187(-243+1026\varepsilon-612\varepsilon^2+88\varepsilon^3)\eta}{16(27-18\varepsilon+4\varepsilon^2)^{7/2}}+\frac{6561\sqrt{3}(29889-73872\varepsilon+48600\varepsilon^2-10944\varepsilon^3+656\varepsilon^4)\eta^2}{64(27-18\varepsilon+4\varepsilon^2)^{9/2}}-\right.$$
$$\left.\frac{98415(452709-1010394\varepsilon+694008\varepsilon^2-171504\varepsilon^3+7632\varepsilon^4+1376\varepsilon^5)\eta^3}{128(27-18\varepsilon+4\varepsilon^2)^{11/2}}\right)\xi^2+\left(\frac{2187\sqrt{3}(-3+2\varepsilon)(-63+12\varepsilon+4\varepsilon^2)}{16(27-18\varepsilon+4\varepsilon^2)^{7/2}}+\right.$$
$$\frac{295245(-3+2\varepsilon)(135+6\varepsilon-44\varepsilon^2+8\varepsilon^3)\eta}{32(27-18\varepsilon+4\varepsilon^2)^{9/2}}+\frac{295245\sqrt{3}(-3+2\varepsilon)(-10449-15552\varepsilon+18216\varepsilon^2-4992\varepsilon^3+368\varepsilon^4)\eta^2}{128(27-18\varepsilon+4\varepsilon^2)^{11/2}}+$$
$$\left.\left.\frac{18600435(-3+2\varepsilon)(-6561+106434\varepsilon-103032\varepsilon^2+33264\varepsilon^3-3600\varepsilon^4+32\varepsilon^5)\eta^3}{256(27-18\varepsilon+4\varepsilon^2)^{13/2}}\right)\xi^3\right)$$

From above equation, we obtain the expression of $\frac{\partial\Omega}{\partial\xi}$ and $\frac{\partial\Omega}{\partial\eta}$ and substitute in equation (5):

$$\ddot{\xi}-2\dot{\eta}=\frac{1}{2}-\frac{\varepsilon}{3}-\mu+\xi+\mu(\frac{27\sqrt{3}(3+2\varepsilon)}{2(27+4\varepsilon^2)^{3/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\eta}{4(27+4\varepsilon^2)^{5/2}}+\frac{2187\sqrt{3}(3+2\varepsilon)(297-180\varepsilon+4\varepsilon^2)\eta^2}{16(27+4\varepsilon^2)^{7/2}}-$$
$$\frac{32805(3+2\varepsilon)(2187-2754\varepsilon+324\varepsilon^2+40\varepsilon^3)\eta^3}{32(27+4\varepsilon^2)^{9/2}}+2(\frac{81\sqrt{3}(-27+108\varepsilon+20\varepsilon^2)}{8(27+4\varepsilon^2)^{5/2}}+\frac{2187(-243-1566\varepsilon-36\varepsilon^2+88\varepsilon^3)\eta}{16(27+4\varepsilon^2)^{7/2}}+$$
$$\frac{6561\sqrt{3}(29889+68040\varepsilon-21384\varepsilon^2-10080\varepsilon^3+656\varepsilon^4)\eta^2}{64(27+4\varepsilon^2)^{9/2}}-$$
$$\frac{98415(452709+511758\varepsilon-600696\varepsilon^2-132624\varepsilon^3+46224\varepsilon^4+1376\varepsilon^5)\eta^3}{128(27+4\varepsilon^2)^{11/2}})\xi+3(\frac{2187\sqrt{3}(3+2\varepsilon)(-63+60\varepsilon+4\varepsilon^2)}{16(27+4\varepsilon^2)^{7/2}}+$$
$$\frac{295245(3+2\varepsilon)(135-282\varepsilon+20\varepsilon^2+8\varepsilon^3)\eta}{32(27+4\varepsilon^2)^{9/2}}+\frac{295245\sqrt{3}(3+2\varepsilon)(-10449+58968\varepsilon-17208\varepsilon^2-3360\varepsilon^3+368\varepsilon^4)\eta^2}{128(27+4\varepsilon^2)^{11/2}}+$$
$$\frac{18600435(3+2\varepsilon)(-6561-190998\varepsilon+126360\varepsilon^2+9936\varepsilon^3-6480\varepsilon^4+32\varepsilon^5)\eta^3}{256(27+4\varepsilon^2)^{13/2}})\xi^2)+(1-\varepsilon)(1-\mu)(\frac{27\sqrt{3}(-3+2\varepsilon)}{2(27-18\varepsilon+4\varepsilon^2)^{3/2}}+$$
$$\frac{729(-9+2\varepsilon)(-3+2\varepsilon)\eta}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{2187\sqrt{3}(-3+2\varepsilon)(297-108\varepsilon+4\varepsilon^2)\eta^2}{16(27-18\varepsilon+4\varepsilon^2)^{7/2}}-\frac{32805(-3+2\varepsilon)(2187-810\varepsilon-108\varepsilon^2+40\varepsilon^3)\eta^3}{32(27-18\varepsilon+4\varepsilon^2)^{9/2}}+$$
$$2(\frac{81\sqrt{3}(-27-36\varepsilon+20\varepsilon^2)}{8(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{2187(-243+1026\varepsilon-612\varepsilon^2+88\varepsilon^3)\eta}{16(27-18\varepsilon+4\varepsilon^2)^{7/2}}+\frac{6561\sqrt{3}(29889-73872\varepsilon+48600\varepsilon^2-10944\varepsilon^3+656\varepsilon^4)\eta^2}{64(27-18\varepsilon+4\varepsilon^2)^{9/2}}-$$
$$\frac{98415(452709-1010394\varepsilon+694008\varepsilon^2-171504\varepsilon^3+7632\varepsilon^4+1376\varepsilon^5)\eta^3}{128(27-18\varepsilon+4\varepsilon^2)^{11/2}})\xi+3(\frac{2187\sqrt{3}(-3+2\varepsilon)(-63+12\varepsilon+4\varepsilon^2)}{16(27-18\varepsilon+4\varepsilon^2)^{7/2}}+$$
$$\frac{295245(-3+2\varepsilon)(135+6\varepsilon-44\varepsilon^2+8\varepsilon^3)\eta}{32(27-18\varepsilon+4\varepsilon^2)^{9/2}}+\frac{295245\sqrt{3}(-3+2\varepsilon)(-10449-15552\varepsilon+18216\varepsilon^2-4992\varepsilon^3+368\varepsilon^4)\eta^2}{128(27-18\varepsilon+4\varepsilon^2)^{11/2}}+$$
$$\frac{18600435(-3+2\varepsilon)(-6561+106434\varepsilon-103032\varepsilon^2+33264\varepsilon^3-3600\varepsilon^4+32\varepsilon^5)\eta^3}{256(27-18\varepsilon+4\varepsilon^2)^{13/2}})\xi^2)$$

$$
\begin{aligned}
&\dot{\xi}+2\dot{\xi}=\frac{\sqrt{3}}{2}-\frac{\varepsilon}{3\sqrt{3}}+\eta+\mu\Big(\frac{27(-9+2\varepsilon)}{2(27+4\varepsilon^2)^{3/2}}-\frac{81\sqrt{3}(-135+108\varepsilon+4\varepsilon^2)\eta}{4(27+4\varepsilon^2)^{5/2}}-\frac{2187(729-1782\varepsilon+108\varepsilon^2+56\varepsilon^3)\eta^2}{16(27+4\varepsilon^2)^{7/2}}+\\
&\Big(\frac{729(-9+2\varepsilon)(3+2\varepsilon)}{4(27+4\varepsilon^2)^{5/2}}+\frac{2187\sqrt{3}(3+2\varepsilon)(297-180\varepsilon+4\varepsilon^2)\eta}{8(27+4\varepsilon^2)^{7/2}}-\frac{98415(3+2\varepsilon)(2187-2754\varepsilon+324\varepsilon^2+40\varepsilon^3)\eta^2}{32(27+4\varepsilon^2)^{9/2}}\Big)\xi+\\
&\Big(\frac{2187(-243-1566\varepsilon-36\varepsilon^2+88\varepsilon^3)}{16(27+4\varepsilon^2)^{7/2}}+\frac{6561\sqrt{3}(29889+68040\varepsilon-21384\varepsilon^2-10080\varepsilon^3+656\varepsilon^4)\eta}{32(27+4\varepsilon^2)^{9/2}}-\\
&\frac{295245(452709+511758\varepsilon-600696\varepsilon^2-132624\varepsilon^3+46224\varepsilon^4+1376\varepsilon^5)\eta^2}{128(27+4\varepsilon^2)^{11/2}}\Big)\xi^2+\\
&\Big(\frac{295245(3+2\varepsilon)(135-282\varepsilon+20\varepsilon^2+8\varepsilon^3)}{32(27+4\varepsilon^2)^{9/2}}+\frac{295245\sqrt{3}(3+2\varepsilon)(-10449+58968\varepsilon-17208\varepsilon^2-3360\varepsilon^3+368\varepsilon^4)\eta}{64(27+4\varepsilon^2)^{11/2}}+\\
&\frac{55801305(3+2\varepsilon)(-6561-190998\varepsilon+126360\varepsilon^2+9936\varepsilon^3-6480\varepsilon^4+32\varepsilon^5)\eta^2}{256(27+4\varepsilon^2)^{13/2}}\Big)\xi^3\Big)+(1-\varepsilon)(1-\mu)\\
&\Big(\frac{27(-9+2\varepsilon)}{2(27-18\varepsilon+4\varepsilon^2)^{3/2}}-\frac{81\sqrt{3}(-135+36\varepsilon+4\varepsilon^2)\eta}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{2187(729+162\varepsilon-324\varepsilon^2+56\varepsilon^3)\eta^2}{16(27-18\varepsilon+4\varepsilon^2)^{7/2}}+\Big(\frac{729(-9+2\varepsilon)(-3+2\varepsilon)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\\
&\frac{2187\sqrt{3}(-3+2\varepsilon)(297-108\varepsilon+4\varepsilon^2)\eta}{8(27-18\varepsilon+4\varepsilon^2)^{7/2}}-\frac{98415(-3+2\varepsilon)(2187-810\varepsilon-108\varepsilon^2+40\varepsilon^3)\eta^2}{32(27-18\varepsilon+4\varepsilon^2)^{9/2}}\Big)\xi+\\
&\Big(\frac{2187(-243+1026\varepsilon-612\varepsilon^2+88\varepsilon^3)}{16(27-18\varepsilon+4\varepsilon^2)^{7/2}}+\frac{6561\sqrt{3}(29889-73872\varepsilon+48600\varepsilon^2-10944\varepsilon^3+656\varepsilon^4)\eta}{32(27-18\varepsilon+4\varepsilon^2)^{9/2}}-\\
&\frac{295245(452709-1010394\varepsilon+694008\varepsilon^2-171504\varepsilon^3+7632\varepsilon^4+1376\varepsilon^5)\eta^2}{128(27-18\varepsilon+4\varepsilon^2)^{11/2}}\Big)\xi^2+\\
&\Big(\frac{295245(-3+2\varepsilon)(135+6\varepsilon-44\varepsilon^2+8\varepsilon^3)}{32(27-18\varepsilon+4\varepsilon^2)^{9/2}}+\frac{295245\sqrt{3}(-3+2\varepsilon)(-10449-15552\varepsilon+18216\varepsilon^2-4992\varepsilon^3+368\varepsilon^4)\eta}{64(27-18\varepsilon+4\varepsilon^2)^{11/2}}+\\
&\frac{55801305(-3+2\varepsilon)(-6561+106434\varepsilon-103032\varepsilon^2+33264\varepsilon^3-3600\varepsilon^4+32\varepsilon^5)\eta^2}{256(27-18\varepsilon+4\varepsilon^2)^{13/2}}\Big)\xi^3\Big)
\end{aligned}
\tag{12}
$$

If ε= 0, the equations of motion become the same as in Pedersen[3]. The first equation of motion is:

$$
\begin{aligned}
\ddot{\xi}-2\eta=\frac{3\xi}{4}+\frac{3\sqrt{3}}{4}(1-2\mu)\eta+\frac{21}{16}(1-2\mu)\xi^2-\frac{3}{8}\sqrt{3}\eta\xi-\frac{33}{16}(1-2\mu)\eta^2-\\
\frac{75}{32}\sqrt{3}(1-2\mu)\eta\xi^2+\frac{123\eta^2\xi}{32}+\frac{45\sqrt{3}}{32}(1-2\mu)\eta^3.
\end{aligned}
\tag{13}
$$

Finding the periodic orbits around L_4 is equivalent to determining the coefficients a and b in the Fourier expansions of ξ and η :

$$
\begin{aligned}
\xi &= a_0+a_1cos\omega t+a_2sin\omega t+a_3cos2\omega t+a_4sin2\omega t\\
\eta &= b_0+b_1cos\omega t+b_2sin\omega t+b_3cos2\omega t+b_4sin2\omega t
\end{aligned}
\tag{14}
$$

a_1, a_2, b_1 and b_2 are of first order, while the other coefficients are of second order. Following the method of Pedersen[3], thefirst order terms are being retained.

$$
\begin{aligned}
\xi &= a_1cos\omega t+a_2sin\omega t,\\
\eta &= b_1cos\omega t+b_2sin\omega t
\end{aligned}
\tag{15}
$$

Insert the values of ξ and η in equation (11) and (12), we get

$$\begin{aligned}
&-a_1\omega^2 cos\omega t - a_2\omega^2 sin\omega t - 2\left(-b_1\omega sin\omega t + b_2\omega cos\omega t\right) = \frac{1}{2} + a_2 sin\omega t - \frac{\varepsilon}{3} + \\
&\frac{729b2sin\omega t(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{27\sqrt{3}(1-\varepsilon)(-3+2\varepsilon)(1-\mu)}{2(27-18\varepsilon+4\varepsilon^2)^{3/2}} + \\
&\frac{81\sqrt{3}a2sin\omega t(1-\varepsilon)(-27-36\varepsilon+20\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} - \mu + \frac{729b2sin\omega t(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}} + \\
&\frac{27\sqrt{3}(3+2\varepsilon)\mu}{2(27+4\varepsilon^2)^{3/2}} + \frac{81\sqrt{3}a2sin\omega t(-27+108\varepsilon+20\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}} + cos\omega t(a_1 + \\
&\frac{729b1(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{81\sqrt{3}a1(1-\varepsilon)(-27-36\varepsilon+20\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \\
&\frac{729b1(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}} + \frac{81\sqrt{3}a1(-27+108\varepsilon+20\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}),
\end{aligned} \tag{16}$$

$$\begin{aligned}
&-b_1\omega^2 cos\omega t - b_2\omega^2 sin\omega t + 2\left(-a_1\omega sin\omega t + a_2\omega cos\omega t\right) = \frac{\sqrt{3}}{2} + b_2 sin\omega t - \frac{\varepsilon}{3\sqrt{3}} + \\
&\frac{729a2sin\omega t(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{27(1-\varepsilon)(-9+2\varepsilon)(1-\mu)}{2(27-18\varepsilon+4\varepsilon^2)^{3/2}} - \\
&\frac{81\sqrt{3}b_2sin\omega t(1-\varepsilon)(-135+36\varepsilon+4\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{729a2sin\omega t(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}} + \\
&\frac{27(-9+2\varepsilon)\mu}{2(27+4\varepsilon^2)^{3/2}} - \frac{81\sqrt{3}b2sin\omega t(-135+108\varepsilon+4\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}} + \\
&cos\omega t(b_1 + \frac{729a1(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} - \frac{81\sqrt{3}b1(1-\varepsilon)(-135+36\varepsilon+4\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \\
&\frac{729a1(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}} - \frac{81\sqrt{3}b1(-135+108\varepsilon+4\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}})
\end{aligned} \tag{17}$$

Equating the coefficient of $cos\omega t$ and $sin\omega t$ in the equations (16) and (17) to zero, we get following system of four equation

$$\begin{aligned}
&a_2\left(\frac{729\,(1-\varepsilon)\,(-9+2\varepsilon)\,(-3+2\varepsilon)\,(1-\mu)}{4\,(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{729\,(-9+2\varepsilon)\,(3+2\varepsilon)\,\mu}{4\,(27+4\varepsilon^2)^{5/2}}\right) + 2a_1\omega + b_2 \\
&(1 - \frac{81\sqrt{3}\,(1-\varepsilon)\,\left(-135+36\varepsilon+4\varepsilon^2\right)(1-\mu)}{4\,(27-18\varepsilon+4\varepsilon^2)^{5/2}} - \frac{81\sqrt{3}\left(-135+108\varepsilon+4\varepsilon^2\right)\mu}{4\,(27+4\varepsilon^2)^{5/2}} + \omega^2) = 0
\end{aligned} \tag{18}$$

$$\begin{aligned}
&a_1\left(\frac{729\,(1-\varepsilon)\,(-9+2\varepsilon)\,(-3+2\varepsilon)\,(1-\mu)}{4\,(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{729\,(-9+2\varepsilon)\,(3+2\varepsilon)\,\mu}{4\,(27+4\varepsilon^2)^{5/2}}\right) - 2a_2\omega + \\
&b_1(1 - \frac{81\sqrt{3}\,(1-\varepsilon)\,\left(-135+36\varepsilon+4\varepsilon^2\right)(1-\mu)}{4\,(27-18\varepsilon+4\varepsilon^2)^{5/2}} - \frac{81\sqrt{3}\left(-135+108\varepsilon+4\varepsilon^2\right)\mu}{4\,(27+4\varepsilon^2)^{5/2}} + \omega^2) = 0
\end{aligned} \tag{19}$$

$$\begin{aligned}
&b_2\left(\frac{729\,(1-\varepsilon)\,(-9+2\varepsilon)\,(-3+2\varepsilon)\,(1-\mu)}{4\,(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{729\,(-9+2\varepsilon)\,(3+2\varepsilon)\,\mu}{4\,(27+4\varepsilon^2)^{5/2}}\right) - 2b_1\omega + \\
&a_2(1 + \frac{81\sqrt{3}\,(1-\varepsilon)\,\left(-27-36\varepsilon+20\varepsilon^2\right)(1-\mu)}{4\,(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{81\sqrt{3}\left(-27+108\varepsilon+20\varepsilon^2\right)\mu}{4\,(27+4\varepsilon^2)^{5/2}} + \omega^2) = 0
\end{aligned} \tag{20}$$

$$b_1\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}}\right)+2b_2\omega+ \\ a_1(1+\frac{81\sqrt{3}(1-\varepsilon)\left(-27-36\varepsilon+20\varepsilon^2\right)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{81\sqrt{3}\left(-27+108\varepsilon+20\varepsilon^2\right)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2)=0 \tag{21}$$

From the above equations (19) and (21), we express a_2 and b_2 in terms of a_1 and b_1:

$$(a_1(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}})+b_1(1-\frac{81\sqrt{3}(1-\varepsilon)(-135+36\varepsilon+4\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}- \\ \frac{81\sqrt{3}(-135+108\varepsilon+4\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2))(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}})+4a_1\omega^2+ \\ (-b_1(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}})-a_1(1+\frac{81\sqrt{3}(1-\varepsilon)(-27-36\varepsilon+20\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+ \\ \frac{81\sqrt{3}(-27+108\varepsilon+20\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2))(1-\frac{81\sqrt{3}(1-\varepsilon)(-135+36\varepsilon+4\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{81\sqrt{3}(-135+108\varepsilon+4\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2)=0$$

$$(-b_1(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}})-a_1(1+\frac{81\sqrt{3}(1-\varepsilon)(-27-36\varepsilon+20\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+ \\ \frac{81\sqrt{3}(-27+108\varepsilon+20\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2))(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}})-4b_1\omega^2+ \\ (a_1(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}})+b_1(1-\frac{81\sqrt{3}(1-\varepsilon)(-135+36\varepsilon+4\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}- \\ \frac{81\sqrt{3}(-135+108\varepsilon+4\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2))(1+\frac{81\sqrt{3}(1-\varepsilon)(-27-36\varepsilon+20\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{81\sqrt{3}(-27+108\varepsilon+20\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2)=0$$

We get the following equations which determine a_1 and b_1.

$$(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}})2-(1-\frac{81\sqrt{3}(1-\varepsilon)\left(-135+36\varepsilon+4\varepsilon^2\right)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}- \\ \frac{81\sqrt{3}\left(-135+108\varepsilon+4\varepsilon^2\right)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2)(1+\frac{81\sqrt{3}(1-\varepsilon)\left(-27-36\varepsilon+20\varepsilon^2\right)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+ \\ \frac{81\sqrt{3}\left(-27+108\varepsilon+20\varepsilon^2\right)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2)+4\omega^2)a_1=0 \tag{22}$$

$$\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}}\right)^2- \\ \left(1-\frac{81\sqrt{3}(1-\varepsilon)\left(-135+36\varepsilon+4\varepsilon^2\right)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{81\sqrt{3}\left(-135+108\varepsilon+4\varepsilon^2\right)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2\right) \\ \left(1+\frac{81\sqrt{3}(1-\varepsilon)\left(-27-36\varepsilon+20\varepsilon^2\right)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{81\sqrt{3}\left(-27+108\varepsilon+20\varepsilon^2\right)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2\right)+4\omega^2)b1=0 \tag{23}$$

If now

$$\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}}\right)^2- \\ \left(1-\frac{81\sqrt{3}(1-\varepsilon)\left(-135+36\varepsilon+4\varepsilon^2\right)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{81\sqrt{3}\left(-135+108\varepsilon+4\varepsilon^2\right)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2\right) \\ \left(1+\frac{81\sqrt{3}(1-\varepsilon)\left(-27-36\varepsilon+20\varepsilon^2\right)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{81\sqrt{3}\left(-27+108\varepsilon+20\varepsilon^2\right)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2\right)+4\omega^2 \gtrless 0 \tag{24}$$

Then we have $a_1 = b1 = a_2 = b_2 = 0$.

The mathematical condition for infinitesimal periodic orbit, therefore is

$$\begin{aligned}&\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}}\right)^2-\\&\left(1-\frac{81\sqrt{3}(1-\varepsilon)\left(-135+36\varepsilon+4\varepsilon^2\right)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{81\sqrt{3}\left(-135+108\varepsilon+4\varepsilon^2\right)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2\right)\\&\left(1+\frac{81\sqrt{3}(1-\varepsilon)\left(-27-36\varepsilon+20\varepsilon^2\right)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{81\sqrt{3}\left(-27+108\varepsilon+20\varepsilon^2\right)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2\right)+4\omega^2=0\end{aligned}\tag{25}$$

If $\varepsilon = 0$, the equation (25) is same as in Pedersen [2].

$$\begin{aligned}&\left(\frac{3}{4}\sqrt{3}(1-\mu)-\frac{3\sqrt{3}\mu}{4}\right)^2+4\omega^2-\left(1+\frac{1}{4}(-1+\mu)-\frac{\mu}{4}+\omega^2\right)\left(1+\frac{5(1-\mu)}{4}+\frac{5\mu}{4}+\omega^2\right)=0\\&-\omega^2+\frac{27}{16}+\frac{27(1-2\mu)2}{16}+\omega^4=0\end{aligned}\tag{26}$$

Now expanding equation (25)

$$\begin{aligned}&-1+\frac{28697814}{(27-18\varepsilon+4\varepsilon^2)^5}-\frac{95659380\varepsilon}{(27-18\varepsilon+4\varepsilon^2)^5}+\frac{126482958\varepsilon^2}{(27-18\varepsilon+4\varepsilon^2)^5}-\frac{86447736\varepsilon^3}{(27-18\varepsilon+4\varepsilon^2)^5}+\frac{33224904\varepsilon^4}{(27-18\varepsilon+4\varepsilon^2)^5}-\frac{6928416\varepsilon^5}{(27-18\varepsilon+4\varepsilon^2)^5}+\frac{629856\varepsilon^6}{(27-18\varepsilon+4\varepsilon^2)^5}-\\&\frac{2187\sqrt{3}}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{3645\sqrt{3}\varepsilon}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{1782\sqrt{3}\varepsilon^2}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{324\sqrt{3}\varepsilon^3}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{2187\sqrt{3}\mu}{(27+4\varepsilon^2)^{5/2}}-\frac{324\sqrt{3}\varepsilon^2\mu}{(27+4\varepsilon^2)^{5/2}}-\frac{57395628\mu}{(27-18\varepsilon+4\varepsilon^2)^5}+\\&\frac{191318760\varepsilon\mu}{(27-18\varepsilon+4\varepsilon^2)^5}-\frac{252965916\varepsilon^2\mu}{(27-18\varepsilon+4\varepsilon^2)^5}+\frac{172895472\varepsilon^3\mu}{(27-18\varepsilon+4\varepsilon^2)^5}-\frac{66449808\varepsilon^4\mu}{(27-18\varepsilon+4\varepsilon^2)^5}+\frac{13856832\varepsilon^5\mu}{(27-18\varepsilon+4\varepsilon^2)^5}-\frac{1259712\varepsilon^6\mu}{(27-18\varepsilon+4\varepsilon^2)^5}+\frac{2187\sqrt{3}\mu}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\\&\frac{3645\sqrt{3}\varepsilon\mu}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{1782\sqrt{3}\varepsilon^2\mu}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{324\sqrt{3}\varepsilon^3\mu}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{157837977\mu}{4(27+4\varepsilon^2)^{5/2}(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{176969853\varepsilon\mu}{4(27+4\varepsilon^2)^{5/2}(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\\&\frac{7440174\varepsilon^2\mu}{(27+4\varepsilon^2)^{5/2}(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{17891847\varepsilon^3\mu}{(27+4\varepsilon^2)^{5/2}(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{6928416\varepsilon^4\mu}{(27+4\varepsilon^2)^{5/2}(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{1259712\varepsilon^5\mu}{(27+4\varepsilon^2)^{5/2}(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{28697814\mu^2}{(27+4\varepsilon^2)^5}+\\&\frac{8503056\varepsilon^2\mu^2}{(27+4\varepsilon^2)^5}+\frac{629856\varepsilon^4\mu^2}{(27+4\varepsilon^2)^5}+\frac{28697814\mu^2}{(27-18\varepsilon+4\varepsilon^2)^5}-\frac{95659380\varepsilon\mu^2}{(27-18\varepsilon+4\varepsilon^2)^5}+\frac{126482958\varepsilon^2\mu^2}{(27-18\varepsilon+4\varepsilon^2)^5}-\frac{86447736\varepsilon^3\mu^2}{(27-18\varepsilon+4\varepsilon^2)^5}+\frac{33224904\varepsilon^4\mu^2}{(27-18\varepsilon+4\varepsilon^2)^5}-\frac{6928416\varepsilon^5\mu^2}{(27-18\varepsilon+4\varepsilon^2)^5}+\\&\frac{629856\varepsilon^6\mu^2}{(27-18\varepsilon+4\varepsilon^2)^5}+\frac{157837977\mu^2}{4(27+4\varepsilon^2)^{5/2}(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{176969853\varepsilon\mu^2}{4(27+4\varepsilon^2)^{5/2}(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{7440174\varepsilon^2\mu^2}{(27+4\varepsilon^2)^{5/2}(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{17891847\varepsilon^3\mu^2}{(27+4\varepsilon^2)^{5/2}(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\\&\frac{6928416\varepsilon^4\mu^2}{(27+4\varepsilon^2)^{5/2}(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{1259712\varepsilon^5\mu^2}{(27+4\varepsilon^2)^{5/2}(27-18\varepsilon+4\varepsilon^2)^{5/2}}+2\omega^2-\frac{2187\sqrt{3}\omega^2}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{3645\sqrt{3}\varepsilon\omega^2}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{1782\sqrt{3}\varepsilon^2\omega^2}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\\&\frac{324\sqrt{3}\varepsilon^3\omega^2}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{2187\sqrt{3}\mu\omega^2}{(27+4\varepsilon^2)^{5/2}}-\frac{324\sqrt{3}\varepsilon^2\mu\omega^2}{(27+4\varepsilon^2)^{5/2}}+\frac{2187\sqrt{3}\mu\omega^2}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{3645\sqrt{3}\varepsilon\mu\omega^2}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{1782\sqrt{3}\varepsilon^2\mu\omega^2}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{324\sqrt{3}\varepsilon^3\mu\omega^2}{(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\omega^4=0.\end{aligned}$$

For $\varepsilon = 0.01$, equation (27) becomes

$$\omega^4-\omega^2+0.000017\mu\omega^2+6.76\mu-6.76\mu^2+0.000117=0\tag{28}$$

Further condition is that ω^2 should be real and positive

From (28), we get

$$\omega^2=\frac{1}{2}\left(1-0.000017\mu+\sqrt{(1-0.000017\mu)^2-4(0.000117+6.764\mu-6.7645\mu^2)}\right)$$

The condition for ω^2 being real is

$$\mu(1-\mu)\leq 0.0367307\tag{29}$$

or the product of the two finite masses should be less than or equal to 0.0367307.

Now we have

$$\mu_0(1-\mu_0)=0.0367307\tag{30}$$

The two equations:

$$\begin{aligned}\xi&=a_1 cos\omega(t-t_0)+a_2 sin\omega(t-t_0),\\\eta&=b_1 cos\omega(t-t_0)+b_2 sin\omega(t-t_0)\end{aligned}$$

correspond to the same motion as the two equations (15), but rearranging the expressions on the right-hand side of the equations, we immediately see that other coefficients of cosϖt and sinϖt are obtained. The arbitrary constant t_0 , which may vary without changing the orbital motion, we shall call the phase constant. It is possible to choose the phase –constant in such a way that the coefficients of cosϖt in the equation $\eta = b_1 cos\omega(t-t_0) + b_2 sin\omega(t-t_0)$ vanishes; t_0 then has to satisfy the equation $b_1 cos\omega(t_0) - b_2 sin\omega(t_0) = 0$. Therefore, we can choose the phase constant in such a way that no terms with cosϖt appear in the expansion of η.

Let us assume that the phase-constant has been fixed that b_1=0.

Now we shall denote that $a_1 = \lambda$,

$$\begin{aligned} 2a_2\omega &= \lambda\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}}\right) \\ 2b_2\omega &= -\lambda\left(1+\frac{81\sqrt{3}(1-\varepsilon)\left(-27-36\varepsilon+20\varepsilon^2\right)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{81\sqrt{3}\left(-27+108\varepsilon+20\varepsilon^2\right)\mu}{4(27+4\varepsilon^2)^{5/2}} + \omega^2\right) \end{aligned} \tag{31}$$

We summarize the above results as follows:

When μ (1- μ) $\leq$ 0.0367307, infinitesimal period orbits around L_4 exist, corresponding to the following equations:

$$\begin{aligned} \xi &= \lambda cos\omega t + \frac{\lambda}{2w}\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}}\right) sin\omega t \\ \eta &= -\frac{\lambda}{2\omega}\left(1+\frac{81\sqrt{3}(1-\varepsilon)\left(-27-36\varepsilon+20\varepsilon^2\right)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{81\sqrt{3}\left(-27+108\varepsilon+20\varepsilon^2\right)\mu}{4(27+4\varepsilon^2)^{5/2}} + \omega^2\right) sin\omega t \end{aligned} \tag{32}$$

If ε = 0, equation (32) becomes the same as in Pedersen [3] i.e.

$$\begin{aligned} \xi &= \lambda\cos\omega t + \frac{3\lambda}{8w}(\sqrt{3}(1-2\mu)\sin\omega t \\ \eta &= \frac{-\lambda}{2\omega}\left(\frac{3}{4}+\omega^2\right)\sin\omega t \end{aligned} \tag{33}$$

In order to find the Fourier coefficients in the Fourier expressions for ξ and η, first we find the coefficients of the second order (coefficient scheme, Pedersen [3]) corresponding to ξ^2, $\xi\eta$ andη^2. Introducing the values found, together with a_0 and b_0 , we can find the constant terms in the Fourier expressions (11) and (12). Putting these terms equal to zero two equations of the first order are found for the determination of a_0 and b_0. Similarly we compute the coefficient of the second order corresponding to $cos2\omega t$ and $sin2\omega t$ and ξ^2 $\xi\eta$ and η^2.

We get for more exact determination of a_1, a_2, b_1 and b_2 four equations of the form:

$$\begin{aligned} &-b_1\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}}\right) - 2b_2\omega - \\ &a_1\left(1+\frac{81\sqrt{3}(1-\varepsilon)\left(-27-36\varepsilon+20\varepsilon^2\right)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{81\sqrt{3}\left(-27+108\varepsilon+20\varepsilon^2\right)\mu}{4(27+4\varepsilon^2)^{5/2}} + \omega^2\right) + \tau_1\lambda = 0 \end{aligned} \tag{34}$$

$$\begin{aligned} &-a_1\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}}\right) + 2a_2\omega - \\ &b_1\left(1-\frac{81\sqrt{3}(1-\varepsilon)\left(-135+36\varepsilon+4\varepsilon^2\right)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} - \frac{81\sqrt{3}\left(-135+108\varepsilon+4\varepsilon^2\right)\mu}{4(27+4\varepsilon^2)^{5/2}} + \omega^2\right) + \tau_2\lambda = 0 \end{aligned} \tag{35}$$

$$-b_2\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}}\right)+2b_1\omega- \\ a_2\left(1+\frac{81\sqrt{3}(1-\varepsilon)(-27-36\varepsilon+20\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{81\sqrt{3}(-27+108\varepsilon+20\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2\right)+\tau_3\lambda=0 \tag{36}$$

$$-a_2\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}}\right)-2a_1\omega- \\ b_2\left(1-\frac{81\sqrt{3}(1-\varepsilon)(-135+36\varepsilon+4\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{81\sqrt{3}(-135+108\varepsilon+4\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2\right)+\tau_4\lambda=0 \tag{37}$$

where $\tau_1, \tau_2, \tau_3,$ *and* τ_4 are known function of λ, which are small quantities of the second order.

From (34) and (35), we find a_2 and b_2 in terms of a_1 andb_1:

$$2a_2\omega+a_1\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}}\right)+ \\ b_1\left(1-\frac{81\sqrt{3}(1-\varepsilon)(-135+36\varepsilon+4\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{81\sqrt{3}(-135+108\varepsilon+4\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2\right)-\tau_2\lambda \tag{38}$$

$$2b_2\omega-b_1\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}}\right)- \\ a_1\left(1+\frac{81\sqrt{3}(1-\varepsilon)(-27-36\varepsilon+20\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{81\sqrt{3}(-27+108\varepsilon+20\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2\right)+\tau_1\lambda \tag{39}$$

Inserting these values of a_2 and b_2 in the (36) and (37), we get the two equations:

$$-\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}}\right)\tau_1\lambda-\tau_2\lambda(-1 \\ -\frac{81\sqrt{3}(1-\varepsilon)(-27-36\varepsilon+20\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{81\sqrt{3}(-27+108\varepsilon+20\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}-\omega^2)+ \\ b1(\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}}\right)^2+4\omega^2+ \\ (-1-\frac{81\sqrt{3}(1-\varepsilon)(-27-36\varepsilon+20\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{81\sqrt{3}(-27+108\varepsilon+20\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}-\omega^2) \\ (1-\frac{81\sqrt{3}(1-\varepsilon)(-135+36\varepsilon+4\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{81\sqrt{3}(-135+108\varepsilon+4\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2))+2\omega\tau_3\lambda=0. \tag{40}$$

If ε = 0, equation (40) becomes same as in Pedersen [2] i.e.

$$(-\tfrac{3}{4}\sqrt{3}(1-\mu)+\tfrac{3\sqrt{3}\mu}{4})\tau_1\lambda-\tau_2\lambda(-1+\tfrac{1-\mu}{4}+\tfrac{\mu}{4}-\omega^2)+ \\ b_1((\tfrac{3}{4}\sqrt{3}(1-\mu)-\tfrac{3\sqrt{3}\mu}{4})^2+4\omega^2+(-1+\tfrac{1-\mu}{4}+\tfrac{\mu}{4}-\omega^2) \\ (1+\tfrac{5(1-\mu)}{4}+\tfrac{5\mu}{4}+\omega^2))+2\omega\tau_3\lambda=0$$

$$
\begin{aligned}
&b_1(-\frac{27}{16}(1-2\mu)^2-\omega^2+\frac{27}{16}+\omega^4)=\tau_2\lambda(\frac{3}{4}+\omega^2)+2\omega\tau_3\lambda-\frac{3}{4}\sqrt{3}(1-2\mu)\tau_1\lambda\\
&b_1(-\frac{27}{16}(1-2\mu)^2-\omega^2+\frac{27}{16}+\omega^4)=\tau_2\lambda(\frac{3}{4}+\omega^2)+2\omega\tau_3\lambda-\frac{3}{4}\sqrt{3}(1-2\mu)\tau_1\lambda
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
&(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}})\tau_2\lambda+\\
&\tau_1\lambda(-1+\frac{81\sqrt{3}(1-\varepsilon)(-135+36\varepsilon+4\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{81\sqrt{3}(-135+108\varepsilon+4\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}-\omega^2)+\\
&b_1((-\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}})(-1+\frac{81\sqrt{3}(1-\varepsilon)(-135+36\varepsilon+4\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\\
&\frac{81\sqrt{3}(-135+108\varepsilon+4\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}-\omega^2)+(-\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}})\\
&(1-\frac{81\sqrt{3}(1-\varepsilon)(-135+36\varepsilon+4\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{81\sqrt{3}(-135+108\varepsilon+4\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2))+\\
&a1(-(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}})^2-4\omega^2+\\
&(1-\frac{81\sqrt{3}(1-\varepsilon)(-135+36\varepsilon+4\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{81\sqrt{3}(-135+108\varepsilon+4\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2)\\
&(1+\frac{81\sqrt{3}(1-\varepsilon)(-27-36\varepsilon+20\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{81\sqrt{3}(-27+108\varepsilon+20\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2))+2\omega\tau_4\lambda=0
\end{aligned}
\tag{42}
$$

One important characteristics of the solution of the nonlinear equation is that the orbit constant or the value of an orbital parameter is related to the orbital period

$$\mathrm{T}(\lambda)=\mathrm{T}_0+\mathrm{P}(\lambda)$$

where $\mathrm{P}(\lambda)$ goes to zero as the orbital parameter, λ approaches zero. Here $\mathrm{T}(\lambda)$ is the period (considering the non linear system) and T_0 is the period of the orbit in the linear framework that is, when λ = 0. The quantity λ controls the size of the orbit and its related to the initial conditions.

Introducing now in equations (40) and (42), the values for the orbit constant a_1 and the phase constantb_1:

$$a_1=\lambda \text{ and } b_1=0$$

we get the two equations:

$$
\begin{aligned}
&(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}})\tau_2+\\
&\tau_1(-1+\frac{81\sqrt{3}(1-\varepsilon)(-135+36\varepsilon+4\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{81\sqrt{3}(-135+108\varepsilon+4\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}-\omega^2)+\\
&(-(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}})^2-4\omega^2+\\
&(1-\frac{81\sqrt{3}(1-\varepsilon)(-135+36\varepsilon+4\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{81\sqrt{3}(-135+108\varepsilon+4\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2)\\
&(1+\frac{81\sqrt{3}(1-\varepsilon)(-27-36\varepsilon+20\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{81\sqrt{3}(-27+108\varepsilon+20\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}+\omega^2))+2\omega\tau_4=0
\end{aligned}
\tag{43}
$$

$$\begin{aligned}&-(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}+\frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4(27+4\varepsilon^2)^{5/2}})\tau_1-\\&\tau_2(-1-\frac{81\sqrt{3}(1-\varepsilon)(-27-36\varepsilon+20\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}}-\frac{81\sqrt{3}(-27+108\varepsilon+20\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}}-\omega^2)+2\omega\tau_3=0\end{aligned} \tag{44}$$

If $\varepsilon = 0$ equations (43) and (44) becomes

$$\begin{aligned}&-\frac{27}{16}(1-2\mu)^2-\omega^2+\frac{27}{16}+\omega^4-\tau_1\left(\frac{9}{4}+\omega^2\right)+2\omega\tau_4+\frac{3}{4}\sqrt{3}(1-2\mu)\tau_2=0\\&(-\frac{3}{4}\sqrt{3}(1-2\mu))\tau_1+\tau_2(\frac{3}{4}+\omega^2)+2\omega\tau_3=0\end{aligned} \tag{45}$$

For ε = 0.01, equation (43) becomes

$$\begin{gathered}0.0001176+6.764\mu-6.764\mu^2+(1.29-2.60\mu)\tau_2\lambda-\omega^2+\\0.000017\mu\omega^2+\omega^4+\tau_1\lambda\left(-2.2549+0.0150\mu-\omega^2\right)+2\omega\tau_4\lambda=0\end{gathered} \tag{46}$$

Now considering the values of μ in the neighborhood of the critical value of μ_0, the smaller root of the equation :

$$\mu_0\,(1-\mu_0)\ =0.0367307$$

From the above equation we find

$$\mu_0\ =0.0381891. \tag{47}$$

The corresponding values of ω, determined with the aid of the variational equations, is found from the following equation

$$\omega^2=\tfrac{1}{2}(1-0.000017\mu+\sqrt{(1-0.000017\mu)^2-4(0.000117+6.764\mu-6.7645\mu^2)})$$

$$\omega_0^2=\tfrac{1}{2}$$

Now we have assumed that μ lies in the neighborhood of μ_0 , we can put

$$\mu(1-\mu)=0.0367307+\delta \tag{48}$$

where δ is infinitely small.

Introducing (48) in (43), we get

$$(-\frac{1}{2}+\omega^2)^2+\frac{27\delta}{4}-\left(\frac{9}{4}+\omega^2-0.015\mu\right)\tau_1+\frac{3}{4}\sqrt{3}(1-2\mu)\tau_2+2\omega\tau_4=0. \tag{49}$$

The last three terms in equation (49) are small quantities of the second order. As only terms of the second order are retained in this equation, we can substitute ω_0 and μ_0 for ω and μ in these terms. The condition for real values of ω and therefore the condition for infinitesimal periodic orbits around L_4 is

$$27\delta\leq 9\tau_1-3\sqrt{3}(k)\tau_2-4\sqrt{2\tau_4} \tag{50}$$

and we have found the upper limit of the mass product

$$\frac{1}{27}(1+9\tau1-3,\overline{3}(k)\tau2-4\sqrt{2}\tau4) \tag{51}$$

Now we turn to the computation of the quantitiesτ_1, $\tau_2\tau_3$ and τ_4, which are functions of μ and λ of the following form

$$\tau_n=f_n(\mu)\lambda,^2\quad(\mathrm{n}=1,2,3,4)$$

As in this equation only terms of the second order are retained, we can in the computation of $f_n(\mu)$ substitute μ_0 and ω_0 for μ and ω. The computation is based on following equation

$$a_1 = \lambda_2 \quad b_1 = 0$$

$$2a_2\omega = \lambda\left(\frac{729(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)(1-\mu)}{4\left(27-18\varepsilon+4\varepsilon^2\right)^{5/2}} + \frac{729(-9+2\varepsilon)(3+2\varepsilon)\mu}{4\left(27+4\varepsilon^2\right)^{5/2}}\right),$$

$$2b_2\omega = -\lambda(1+\frac{81\sqrt{3}(1-\varepsilon)\left(-27-36\varepsilon+20\varepsilon^2\right)(1-\mu)}{4\left(27-18\varepsilon+4\varepsilon^2\right)^{5/2}} + \frac{81\sqrt{3}\left(-27+108\varepsilon+20\varepsilon^2\right)\mu}{4\left(27+4\varepsilon^2\right)^{5/2}} + \omega^2)$$

Substitute the values of $\mu = \mu_0$ and $\omega = \omega_0$ inequations(31)become
$a_1 = \lambda$, $b_1 = 0$,

$$\sqrt{2}a_2 = (\frac{6.959(-9+2\varepsilon)(3+2\varepsilon)}{(27+4\varepsilon^2)^{5/2}} + \frac{175.29(1-\varepsilon)(-9+2\varepsilon)(-3+2\varepsilon)}{(27-18\varepsilon+4\varepsilon^2)^{5/2}})\lambda \tag{52}$$

$$\sqrt{2}b_2 = -\lambda(1+\frac{81\sqrt{3}(1-\varepsilon)(-27-36\varepsilon+20\varepsilon^2)(1-\mu)}{4(27-18\varepsilon+4\varepsilon^2)^{5/2}} + \frac{81\sqrt{3}(-27+108\varepsilon+20\varepsilon^2)\mu}{4(27+4\varepsilon^2)^{5/2}} + \omega^2) \tag{53}$$

For $\varepsilon = 0.01$, we get

$$a_2 = 0.848\lambda, b_2 = -0.880\lambda \tag{54}$$

With the values given by (54), we find the coefficients of the second order of the coefficient scheme(Pedersen [2]) corresponding to ξ^2, η^2 and $\xi\eta$,

ξ^2	$0.859\lambda^2$
$\xi\eta$	$-0.374\lambda^2$
η^2	$0.390\lambda^2$

Introducing these values in the differential equation (11) and (12), we get

$$\begin{aligned} &0.0000184672 + 0.7454a_0 + 1.196b_0 + 0.537\lambda^2 = 0 \\ &0.000033331 + 1.196a_0 + 2.254b_0 + 0.773\lambda^2 = 0 \end{aligned} \tag{55}$$

Which have the following roots

$$\begin{aligned} a_0 &= -0.000007026 - 1.148\lambda^2 \\ b_0 &= -0.0000110557 + 0.266\lambda^2 \end{aligned} \tag{56}$$

We compute the coefficients of the second order in the coefficient scheme corresponding to $cos\omega t$ and $sin\omega t$ and ξ^2, η^2 and $\xi\eta$

	Cos2ωt	**Sin2ωt**
ξ^2	$0.140\lambda^2$	$0.848\lambda^2$
$\xi\eta$	$0.373\lambda^2$	$-0.440\lambda^2$
η^2	$-0.387\lambda^2$	0

Introducing these values in the differential equations (11) and (12), we get for the determination of a_3, a_4, b_3 and b_4 the four equations :

$$\begin{aligned}
&-2.82b_4-2.74a_3-1.19b_3=0.67\lambda^2\\
&2.82b_3-2.74a_4-1.19b_4=1.31\lambda^2\\
&2.82a_4-4.25b_3-1.19a_3=-1.85\lambda^2\\
&-2.82a_3-4.25b_4-1.19a_4=1.41\lambda^2
\end{aligned} \tag{57}$$

which have the roots:

$$\begin{aligned}
&a_3=-0.43\lambda^2, \quad a_4=0.57\lambda^2\\
&b_3=0.94\lambda^2, \quad b_4=-0.21\lambda^2
\end{aligned} \tag{58}$$

With these values of a_0, b_0, a_3, a_4, b_3 and b_4 we can compute the coefficients in the coefficients scheme corresponding to *cosωt and sinωt*.

	cosωt	**sinωt**
ξ^2	-2.24λ^3	-0.99λ^3
$\xi\eta$	0.40λ^3	0.54λ^3
η^2	0.18λ^3	0.36λ^3

Introducing these values in the differential equations (11) and (12), we get for the more exact determination of a_1, a_2, b_1 and b_2 of four equations (35),(36),(37) and (38). Comparing two systems of equations we can determine the values of τ_1, τ_2, τ_3 and τ_4.

$$\begin{aligned}
&\tau_1=3.18\lambda^2, \quad \tau_2=0.96\lambda^2\\
&\tau_3=1.84\lambda^2, \quad \tau_4=5.37\lambda^2
\end{aligned} \tag{59}$$

With the values of τ_1, τ_2, τ_3 and τ_4 found we can show that equation (44) is satisfied(We have obtained accuracy in the range of 10^{-2}).

Now we insert the values of $a_1=\lambda$, $b_1=0$, $\tau_1 = 3.18\ \lambda^2$ and $\tau_2 = 0.96\lambda^2$ in equations (38) and (39) and get the following values:

$$\begin{aligned}
a_1&=\lambda, b_1=0\\
2a_2\omega&=0.96\lambda^3+1.3\lambda(1-2\mu)\\
2b_2\omega&=3.18\lambda^3-\lambda\left(0.75+\omega^2\right)
\end{aligned} \tag{60}$$

Inserting the values of τ_1, τ_2 and τ_4 in (49), we get the equation

$$\left(-0.5+\omega^2\right)^2+6.75\delta-0.032\lambda^2=0. \tag{61}$$

The condition for real values of ω and hence for infinitesimal periodic orbits around L_4 is

$$27\delta\leq 0.128\lambda^2 \tag{62}$$

and the upper limit of the mass product is

$$0.03703\left[1+0.128\lambda^2\right] \tag{63}$$

We can determine the orbit constant λ_0 by the equation

$$27\delta=0.128{\lambda_0}^2. \tag{64}$$

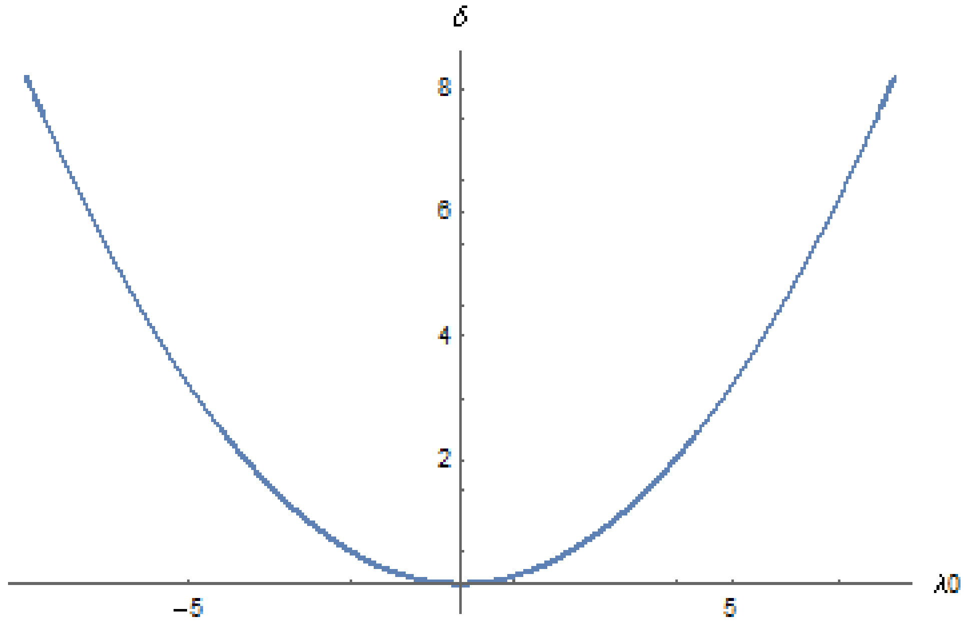

Fig 1. Periodic orbits around L_4 exists for values of the mass-product greater than 0.0381891

Brown [1] found that for mass ratio $\mu > \mu_0$, two families of periodic orbits exist at L_4 and both families depend analytically on a real parameter λ. This orbital parameter admits a strictly positive lower bound δ the same for both families. δ is an analytical function of mass ratio μ, when λ goes to δ, both families close to a common periodic orbit, which is called the limiting orbit

We did not carry out the computation of the terms of the third order due to the computation of the expression. Therefore we carry out the determination of the Fourier coefficients of ξ and η for limiting orbits. For the limiting orbits the equation of the condition (61) yields only one value of $\omega^2(\omega^2 = \frac{1}{2}$).The orbit constant of the limiting orbit is given by equation (64)

$$27\delta = 0.128\lambda_0^2 \tag{65}$$

and δ is given by equation (48):

$$\mu(1-\mu) = 0.0367307 + \delta \tag{66}$$

From (64) and (66) we get the following equation

$$27\mu(1-\mu) = 0.991 + 0.128\lambda_0^2 \tag{67}$$

and from (67) we find,

$$1-2\mu = 0.92\left(1-0.0103\lambda_0^2\right) \tag{68}$$

Introducing ω = 0.707 and values of 1-2 μ given by (68) in equations (60), we get the following values of the coefficients of the first order for the limiting orbits:

$$\begin{gathered} a_1 = \lambda, \mathrm{b}_1 = 0 \\ a_2 = 0.636\lambda^3 + 0.85\lambda \\ b_2 = 2.24\lambda^3 - 0.88\lambda \end{gathered} \tag{69}$$

Now we can find the coefficients of the second order by substituting λ_0 for λ

$$\begin{gathered} a_0 = -0.0000070 - 1.1484976\lambda_0^2, \\ b_0 = -0.000011 + 0.266356\lambda_0^2 \end{gathered}$$

$$\begin{array}{c} a_3 = -0.43\lambda_0^2, \quad a_4 = 0.57\lambda_0^2 \\ b_3 = 0.94\lambda_0^2, b_4 = -0.21\lambda_0^2 \end{array} \tag{70}$$

Introducing (69) and (70) in the following equations

$$\begin{aligned} \xi = \; & a_0 + a_1 cos\omega t + a_2 sin\omega t + a_3 cos2\omega t + a_4 sin2\omega t, \quad \xi = \; a_0 + a_1 cos\omega t + a_2 sin\omega t + a_3 cos2\omega t + a_4 sin2\omega t, \\ \eta = \; & b_0 + b_1 cos\omega t + b_2 sin\omega t + b_3 cos2\omega t + b_4 sin2\omega t \quad \eta = \; b_0 + b_1 cos\omega t + b_2 sin\omega t + b_3 cos2\omega t + b_4 sin2\omega t \end{aligned} \tag{71}$$

we get the Fourier expressions for the limiting orbits

$$\begin{aligned} \xi &= -1.15\lambda_0^2 + \lambda_0 cos\omega_0 t + \left(0.85\lambda_0 + 0.636\lambda_0^3\right) sin\omega_0 t - 0.43\lambda_0^2 cos2\omega_0 t + 0.57\lambda_0^2 sin2\omega_0 t \\ \eta &= 0.266\lambda_0^2 + \left(-0.88\lambda_0 + 2.24\lambda_0^3\right) sin\omega_0 t + 0.94\lambda_0^2 cos2\omega_0 t - 0.21\lambda_0^2 sin2\omega_0 t. \end{aligned} \tag{72}$$

where $\omega_0 = 0.707$

Now we introduce the values of 0.1 and 0.2 for λ_0. With λ_0 = 0.1, we get from (72) the Fourier expressions:

$$\begin{aligned} 10^5.\xi &= -1150 - 430 cos2\omega_0 t + 10000\ cos\omega_0 t + 570\ sin2\omega_0 t + 8563 sin\omega_0 t, \\ 10^5.\eta &= 266.35 + 940 cos2\omega_0 t - 210\ sin2\omega_0 t - 8576\ sin\omega_0 t \end{aligned} \tag{73}$$

and with λ_0 = 0.2, we get

$$\begin{aligned} 10^5.\xi &= -4600 - 1720 cos2\omega_0 t + 20000\ cos\omega_0 t + 2280\ sin2\omega_0 t + 17508\ sin\omega_0 t, \\ 10^5.\eta &= 1065.4 + 3760 cos2\omega_0 t - 840\ sin2\omega_0 t - 15808\ sin\omega_0 t. \end{aligned} \tag{74}$$

The corresponding values of μ are found with the aid of equation (68).Denoting the value of μ corresponding to λ_0 = 0.1 and λ_0 = 0.2 by μ_1 and μ_2, respectively, we get

$$\begin{array}{c} \mu_1 = 0.040047 = \mu_0 + 0.00185 \\ \mu_2 = 0.040108 = \mu_0 + 0.00200 \end{array} \tag{75}$$

Figure 2 correspond to the equations (73) and (74).

3 Results

When $\mu > \mu_0$ and orbital parameter reach a critical value λ_0, at which the value of the two periodic orbits coincide and their period becomes equal. Figure 2 shows two limiting orbits belonging to various values of the mass parameter for ε = 0.01. First corresponding to the short period limiting orbit (inner orbit) with $\mu_1 = 0.04007$, the second belonging to the long period limiting orbit (outer orbit) with $\mu_2 = 0.040108$. At $\mu = \mu_0 = 0.03852$ the limiting orbits shrinks to the libration points. At $\mu = \mu_0$, $\lambda_0 = 0$. Figure 3 shows the effect of radiation pressure (ε = 0.01) of more massive primary on the limiting orbit (Inner orbit) around L_4 , when the small mass has the value $\mu_1 = \mu_0$ + 0.00185. A comparison between these orbits with and without radiation pressure is made and presented in Figure 4 . The semi major axis a and eccentricity e of the periodic orbits are calculated and presented in Table 1. It can be noticed that with the inclusion of radiation pressure, the semi-major axis and the eccentricity of the limiting orbit increases.

Table 1. Semi major axis and eccentricity of limiting orbits

	Inner Orbit		Outer Orbit	
	Semi Major Axis	Eccentricity	Semi Major Axis	Eccentricity
Pedersen	0.143	0.845	0.247	0.907
With perturbation	0.145	0.883	0.287	0.926

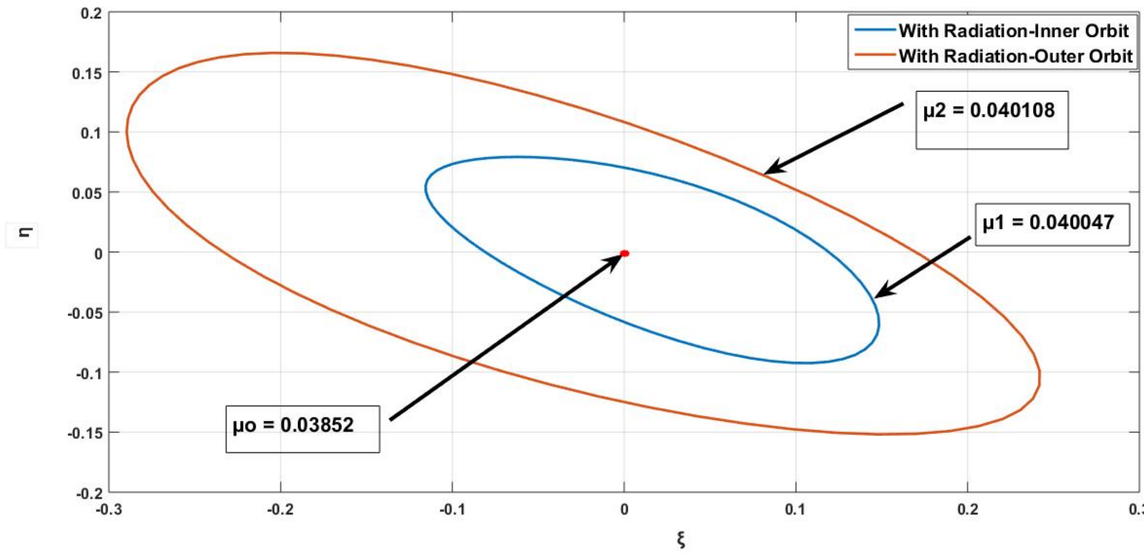

Fig 2. The limiting orbits around L_4 for $\varepsilon = 0.01$, when the small mass has the value $\mu_1 = \mu_0 + 0.00185$ (Inner orbit) and $\mu_2 = \mu_0 + 0.00200$ (outer orbit)

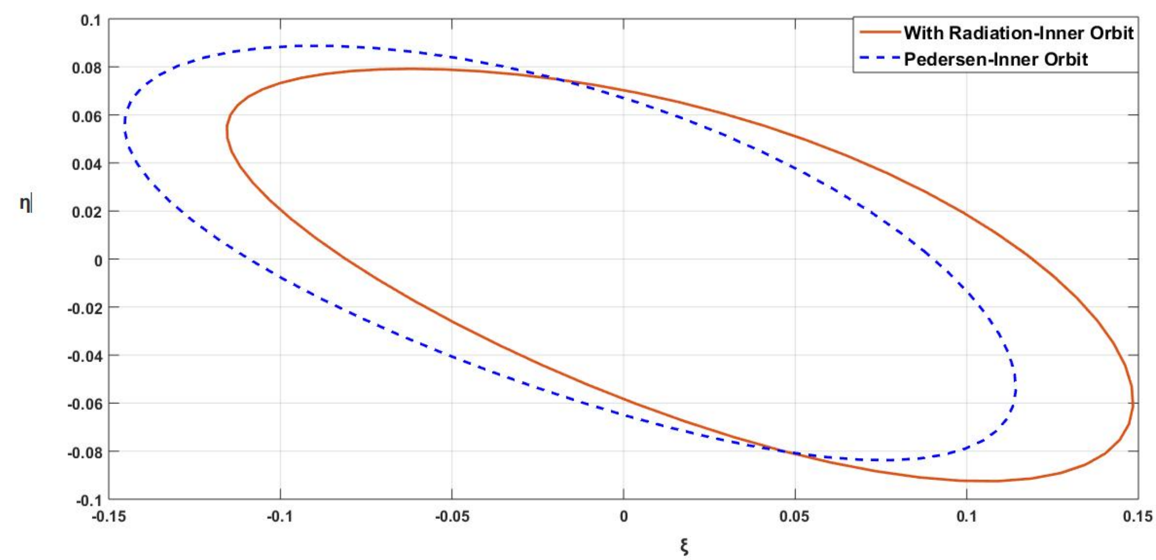

Fig 3. Comparison of the limiting orbit (inner orbit) with and without perturbation around L_4, when the small mass has the value $\mu_1 = \mu_0 + 0.00185$.

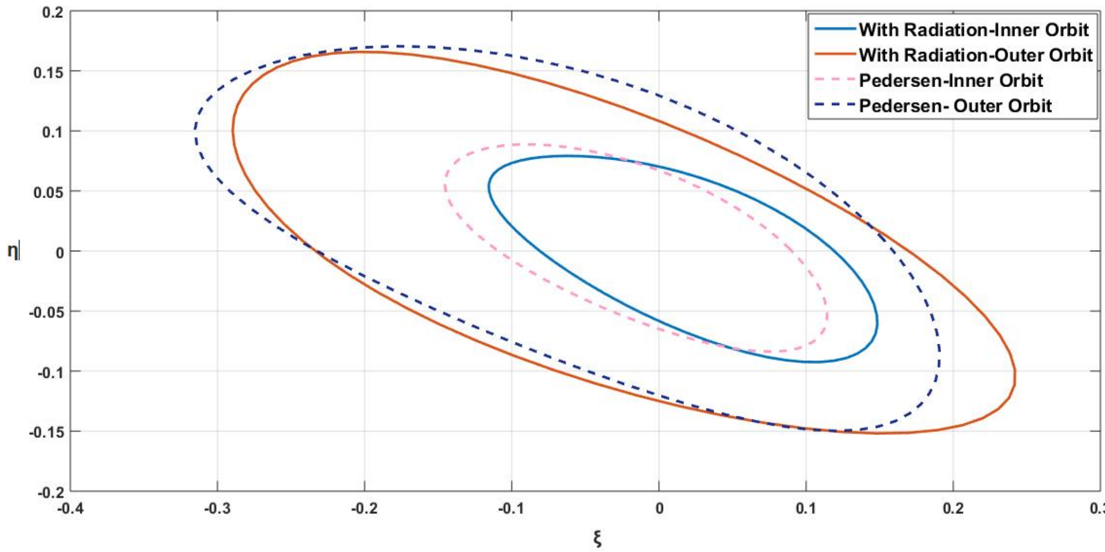

Fig 4. Comparison of the limiting orbits with and without perturbation L_4, when the small Mass has the value $\mu_1 = \mu_0 + 0.00185$ (inner orbit) and $\mu_2 = \mu_0 + 0.00200$ (outer Orbit)

4 Conclusion

Fourier expansion of the periodic orbits in the immediate vicinity of the triangular libration points in the photogravitational restricted three–body problem are obtained. Periodic orbits are generated around the libration points L_4 and L_5 by considering the more massive primary as a source of radiation. We observe that the shape of periodic orbits changed with the inclusion of radiation pressure. A comparison between these orbits with and without radiation pressure is made. The value of the critical mass is found and is shown that the critical mass value corresponding to the small mass increases with the size of the orbit. Calculations were carried out using Wolfram Mathematica 11 and MATLAB 2015B software.

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