

RESEARCH ARTICLE



Some results on commutativity of MA-semirings

Liaquat Ali^{1*}, Muhammad Aslam², Yaqoub Ahmed Khan³

¹ Assistant Professor of Mathematics, Government M.A.O College Lahore, Pakistan.
Tel.: +923004557189

² Professor of Mathematics, Government College University, Lahore, Pakistan

³ Assistant Professor of Mathematics, GIC, Lahore, Pakistan



Received: 27-06-2020

Accepted: 25-07-2020

Published: 29-08-2020

Editor: Dr. Natarajan Gajendran

Citation: Ali L, Aslam M, Ahmed Khan Y (2020) Some results on commutativity of MA-semirings. Indian Journal of Science and Technology 13(31): 3198-3203. <https://doi.org/10.17485/IJST/v13i31.1022>

* **Corresponding author.**

Tel: +923004557189
rehmani.pk786@gmail.com

Funding: None

Competing Interests: None

Copyright: © 2020 Ali et al. This is an open access article distributed under the terms of the [Creative Commons Attribution License](https://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

Published By Indian Society for Education and Environment ([iSee](http://www.isee.org))

ISSN

Print: 0974-6846

Electronic: 0974-5645

Abstract

Objective: The main aim of this article is to extend the concept of involution for a certain class of semirings known as MA-semirings. Now a days the commutativity conditions in the theory of rings and semirings becomes crucial for researchers. This motivates us to discuss some conditions on MA-semirings with involution which enforces commutativity. **Method:** We use the tools of derivations and involutions of second kind on MA-semirings. **Findings:** We are able to find the conditions of commutativity in semirings through these particular mappings. **Novelty:** To define the concept of Hermitian elements in MA-semirings with involution and to establish some commutativity results through different conditions involving Hermitian elements is the novel idea.

Keywords: MA-semirings; prime rings; involution; derivations

1 Introduction and Preliminaries

Involution plays a vital role in defining the important structures like B*-algebra and C*-algebra. The rings with involution are also investigated by many algebraists (see⁽¹⁻⁵⁾). The concept of involution was also discussed for some other types inverse semirings (see⁽⁶⁻⁹⁾). The notion of MA-semiring was introduced by M.A Javed, M. Aslam and M. Hussain⁽¹⁰⁾. The theory of commutators along with derivations and certain additive mappings was further investigated in⁽¹⁰⁻¹⁵⁾. For more on MA-semirings and MA-semirings with involution one can see⁽¹²⁻²⁴⁾. Main objective of this paper is to introduce the notion of involution of First and Second kind for MA-semirings and generalize some results of Ring theory published in⁽²⁵⁾.

Now we include some necessary preliminaries for completion. By a semiring R , we mean a semiring with absorbing zero '0' in which addition is commutative. A semiring R is said to be additive inverse semiring if for each $x \in R$ there is a unique $x' \in R$ such that $x + x' + x = x$ and $x' + x + x' = x'$ and x' is called the Pseudo inverse of x . An additive inverse semiring R is said to be an MA-semiring if it satisfies $x + x' \in Z(R), \forall x \in R$, where $Z(R)$ is the center of R .

Following example shows that every ring with absorbing zero '0' is an MA-semiring but converse may not be true in general.

Example 1.1.⁽¹⁰⁾ The set $R = \{0, 1, 2, 3, \dots\}$ with addition \oplus and multiplication \otimes respectively defined by $a \oplus b = \sup\{a, b\}$ and $a \otimes b = \inf\{a, b\}$ is an MA-semiring

which is not a ring. In fact R is a commutative prime MA-semiring.

Such examples motivate us to generalize the results of ring theory in MA-semirings. Throughout the paper by a semiring R we mean a MA-semiring unless mentioned otherwise. R is prime if $aRb = \{0\}$ implies that $a = 0$ or $b = 0$ and semiprime if $aRa = \{0\}$ implies that $a = 0$. R is 2-torsion free if for $x \in R$, $2x = 0$ implies $x = 0$. An additive mapping $*$: $R \rightarrow R$ is said to be involution if $\forall x, y \in R, (x^*)^* = x$ and $(xy)^* = y^*x^*$. The set $H = \{x \in R : x^* = x\}$ is the set of all Hermitian elements and $S = \{x \in R : x^* = x'\}$ is the set of all Skew Hermitian elements. An additive mapping d : $R \rightarrow R$ is derivation if $d(xy) = d(x)y + xd(y)$. We define commutator as $[x, y] = xy - yx$. By Jordan product we mean $x \circ y = xy + yx$. If I is an ideal of R , then $V_R(I) = \{x \in R : [x, y] = 0, \forall y \in I\}$. We include some useful identities $[x, xy] = x[x, y], [xy, z] = x[y, z] + [x, z]y, [x, yz] = [x, y]z + y[x, z], [x, y] + [y, x] = y(x + x') = x(y + y'), (xy)' = x'y = xy', [x, y]' = [x', y'] = [x', y], x \circ (y + z) = x \circ y + x \circ z$, for all $x, y, z \in R$ (For more see^(10,14,15)).

2 Main Results

First we introduce the notions of skew hermitian elements, involution of first and second kind for MA-semirings which are the generalization of the relevant concepts of rings.

Definition 2.1. An element $x \in R$ is skew hermitian if $x^* = x'$ and we denote the set of all skew hermitian elements by S .

Definition 2.2. An involution $*$ is said to be of First kind if $Z \subseteq H$ otherwise it is of second kind.

Lemma 2.3. Let R be a semiprime inverse semiring with involution $*$ of second kind. Then $S \cap Z \neq \{0\}$ and hence $H \cap Z \neq \{0\}$

Proof. As $*$ is of second kind, therefore $Z \not\subseteq H$. Let $z_0 \in Z$ such that $z_0 \notin H$. Then $z_0r = rz_0, \forall r \in R$ which further gives $(z_0r)^* = (rz_0)^*, \forall r \in R$ and hence $r^*z_0^* = z_0^*r^*, \forall r \in R$. Replacing r by r^* , we get $rz_0^* = z_0^*r, \forall r \in R$. and hence $z_0^* \in Z$. On the other hand, as $z_0 \in Z, z_0' \in Z$ and hence $z_0' + z_0^* \in Z$. We can easily see that $z_0' + z_0^* \in S$. This means $z_0' + z_0^* \in S \cap Z$. Suppose that $S \cap Z = \{0\}$. Therefore $z_0' + z_0^* = 0$. This means $z_0^* = -z_0'$ and hence $z_0 \in H$, a contradiction. Hence $S \cap Z \neq \{0\}$. Next let $s \in S \cap Z \neq \{0\}$. Then $s \in S$ and $s \in Z$ which implies $s^2 \in H$ and $s^2 \in Z$. If $s^2 = 0$, then $sRs = \{0\}$. By the semiprimeness of R , $s = 0$, which is not possible. Therefore $0 \neq s^2 \in H \cap Z$ and hence $H \cap Z \neq \{0\}$.

Lemma 2.4. Let R be a 2-torsion free prime semiring with involution $*$ of second kind. If

$$[x, x^*] = 0, \forall x \in R \tag{1}$$

then R is commutative.

Proof. Linearizing (1), we get $[x, y^*] + [y, x^*] = 0, \forall x, y \in R$ and replacing y by y^* , we get

$$[x, y] + [y^*, x^*] = 0, \forall x, y \in R \tag{2}$$

In (2) replacing y by $ys, s \in S \cap Z - \{0\}$, we get $([x, y]' + [y^*, x^*])s = 0, \forall x, y \in R, s \in S \cap Z - \{0\}$ and hence $([x, y]' + [y^*, x^*])Rs = \{0\}, \forall x, y \in R, s \in S \cap Z - \{0\}$. By the primeness of R , we have $[x, y]' + [y^*, x^*] = 0, \forall x, y \in R$ which implies that

$$[x, y] = [y^*, x^*], \forall x, y \in R \tag{3}$$

Using (3) into (2) and hence by the 2-torsion freeness of R , we get $[x, y] = 0, \forall x, y \in R$ which clearly shows that R is commutative.

Lemma 2.5. Let R be a 2-torsion free prime semiring with involution $*$ of second kind and d be a nonzero derivation satisfying $d[x, x^*] = 0, \forall x \in R$. Then R is commutative.

Proof. We have

$$d[x, x^*] = 0, \forall x \in R \tag{4}$$

Linearizing (4) and using it again, we have

$$d[x, y^*] + d[y, x^*] = 0, \forall x, y \in R \tag{5}$$

Replacing y by y^* in (5), we get

$$d[x, y] + d[y^*, x^*] = 0, \forall x, y \in R \tag{6}$$

Replacing y by $yh, h \in H \cap Z - \{0\}$ in (6), we get $(d([x, y] + [y^*, x^*])h) = 0, \forall x, y \in R$ and hence $d([x, y] + d[y^*, x^*])h + ([x, y] + [y^*, x^*])d(h) = 0, \forall x, y \in R$. Using (6) again, we get

$$([x, y] + [y^*, x^*])d(h) = 0, \forall x, y \in R \tag{7}$$

Replacing y by $ys, s \in S \cap Z - \{0\}$ in (6) and using it again, we get

$$([x, y] + [y^*, x^*]') d(s) = 0, \forall x, y \in R \tag{8}$$

As $hs \in S \cap Z - \{0\}, \forall h \in H \cap Z - \{0\}, s \in S \cap Z - \{0\}$, therefore in (8) replacing s by hs , we get $([x, y] + [y^*, x^*]') d(hs) = 0, \forall x, y \in R$ and hence $([x, y] + [y^*, x^*]') (hd(s) + d(h)s) = 0, \forall x, y \in R$ which on simplification gives $([x, y] + [y^*, x^*]') d(s)h + ([x, y] + [y^*, x^*]') d(h)s = 0, \forall x, y \in R$. Using (5) again, we get $([x, y] + [y^*, x^*]') d(h)s = 0, \forall x, y \in R$ and hence $([x, y] + [y^*, x^*]') d(h)Rs = \{0\}, \forall x, y \in R$. By the primeness of R , we get $([x, y] + [y^*, x^*]') d(h) = 0, \forall x, y \in R$ and hence

$$[x, y] d(h) = [y^*, x^*] d(h), \forall x, y \in R \tag{9}$$

Using (9) into (7), we get $2[x, y]d(h) = 0, \forall x, y \in R$ and by the 2-torsion freeness of R , we have

$$[x, y]d(h) = 0, \forall x, y \in R \tag{10}$$

In (10), replacing y by yr and using (10) again, we get $[x, y]Rd(h) = 0, \forall x, y \in R$ and by the primeness of R , we have $[x, y] = 0, \forall x, y \in R$ or $d(h) = 0, \forall h \in H \cap Z - \{0\}$. If $[x, y] = 0, \forall x, y \in R$, then R is commutative. If

$$d(h) = 0, \forall h \in H \cap Z - \{0\} \tag{11}$$

In (11), replacing h by $s^2, s \in S \cap Z - \{0\}$, we get $2sd(s) = 0, \forall s \in S \cap Z - \{0\}$, and by the 2-torsion freeness of R , we get $sd(s) = 0, \forall s \in S \cap Z - \{0\}$ and hence $Rd(s) = \{0\}, \forall s \in S \cap Z - \{0\}$. By the primeness of R , we have

$$d(s) = 0, \forall s \in S \cap Z - \{0\} \tag{12}$$

Since for each $z \in Z, z + z^* \in H \cap Z$, therefore from (11), we have

$$d(z) + d(z^*) = 0, \forall z \in Z \tag{13}$$

Also since for each $z \in Z, z' + z^* \in H \cap Z$, therefore from (12), we have $d(z') + d(z^*) = 0, \forall z \in Z$, which further gives

$$d(z) = d(z^*), \forall z \in Z \tag{14}$$

Using (14) into (13) and 2-torsion freeness of R , we get

$$d(z) = 0, \forall z \in Z \tag{15}$$

In (6), replacing y by $yz, z \in Z$, we get $d[x, yz] + d[z^*y^*, x] = 0, \forall x, y \in R, z \in Z$. This means $d([x, y]z) + d([y^*, x^*]z^*) = 0, \forall x, y \in R, z \in Z$ and so $d[x, y]z + [x, y]d(z) + d[y^*, x^*]z^* + [y^*, x^*]d(z^*) = 0, \forall x, y \in R, z \in Z$. Using (15) again, we get

$$d[x, y]z + d[y^*, x^*]z^* = 0, \forall x, y \in R, z \in Z \tag{16}$$

From (6), we have

$$(d[x, y])' = d[y^*, x^*], \forall x, y \in R \tag{17}$$

Using (17) into (16), we get $d[x, y] (z' + z^*) = 0, \forall x, y \in R, z \in Z$ and hence $d[x, y]R(z' + z^*) = \{0\}, \forall x, y \in R, z \in Z$. By the primeness of R , we have $d[x, y] = 0$ or $z' + z^* = 0, \forall x, y \in R, z \in Z$. If $z' + z^* = 0, \forall z \in Z$, then $z^* = z, \forall z \in Z$ and hence $Z \subset H$ which contradicts the fact that the involution $*$ is of second kind. Hence the only possibility is

$$d[x, y] = 0, \forall x, y \in R \tag{18}$$

In (18), replacing x by rs and using it again, we get

$$d(r)[s, y] + [r, s]d(s) = 0, \forall r, s, y \in R \tag{19}$$

In (19) replacing s by $[s, t]$ and using it again, we get

$$d(r)[[s, t], y] = 0, \forall r, s, t, y \in R \tag{20}$$

In (20), replacing y by yx and using it again, we get $d(r) y[[s, t], x] = 0, \forall r, s, t, x, y \in R$ and hence $d(r) R[[s, t], x] = \{0\}, \forall r, s, t, x \in R$. Since $d \neq 0$, therefore $d(r_0) \neq 0$, for some $r_0 \in R$ and hence $d(r_0) R[[s, t], x] = \{0\}, \forall s, t, x \in R$. By the primeness of R , we have

$$[[s, t], x] = 0, \forall s, t, x \in R \tag{21}$$

Replacing t by st in (21) and using it again, we get

$$[s, x] [s, t] = 0, \forall s, t, x \in R \tag{22}$$

Replacing t by rt in (22) and using it again, we get $[s, x] R[s, t] = \{0\}, \forall s, t, x \in R$. By the primeness of R , we conclude that $[s, t] = 0, \forall s, t \in R$ and hence R is commutative.

Theorem 2.6. Let R be a 2-torsion free prime semiring with involution $'$ of second kind. If d is a derivation such that

$$d(x \circ x^*) = 0, \forall x \in R \tag{23}$$

then R is commutative.

Proof. In (23), replacing x by $h, h \in H \cap Z - \{0\}$, we get $d(h \circ h^*) = 0, \forall h \in H \cap Z - \{0\}$ and hence $4d(h)h = 0, \forall h \in H \cap Z - \{0\}$. By the 2-torsion freeness of R we have $d(h)h = 0, \forall h \in H \cap Z - \{0\}$ which further gives $d(h)Rh = 0, \forall h \in H \cap Z - \{0\}$. By the primeness of R , we have

$$d(h) = 0, \forall h \in H \cap Z - \{0\} \tag{24}$$

Since for each $s \in S \cap Z - \{0\}, s^2 \in H \cap Z - \{0\}$ and so replacing h by s^2 in (24), we get

$$d(s) = 0, \forall s \in S \cap Z - \{0\} \tag{25}$$

Since for each $z \in Z, z + z^* \in H \cap Z$ and $z' + z^* \in S \cap Z$, therefore replacing h by $z + z^*$ and s by $z' + z^*$ in (24) and (25) respectively, we get

$$d(z) + d(z^*) = 0 \tag{26}$$

and $d(z') + d(z^*) = 0$ which further implies

$$d(z) = d(z^*) \tag{27}$$

Using (27) into (26), we get $2d(z) = 0, \forall z \in Z$ and by the 2-torsion freeness of R , we have

$$d(z) = 0, \forall z \in Z \tag{28}$$

Linearizing (23) and using it again, we get

$$d(x \circ y^* + y \circ x^*) = 0, \forall x, y \in R \tag{29}$$

In (29) replacing y by $z \in Z$, we get $d(xz^* + z^*x + zx^* + x^*z) = 0, \forall x \in R, z \in Z$. since $z^* \in Z$, therefore $2d(xz^* + z^*x) = 0, \forall x \in R, z \in Z$ and by the 2-torsionfreeness of R , we have $d(xz^* + z^*x) = 0, \forall x \in R, z \in Z$. Using (28), we get

$$d(x)z^* + d(x^*)z = 0, \forall x \in R, z \in Z \tag{30}$$

Replacing z by $h \in H \cap Z - \{0\}$, we have $(d(x) + d(x^*))h = 0, \forall x \in R, h \in H \cap Z - \{0\}$ and so $(d(x) + d(x^*))Rh = 0, \forall x \in R, h \in H \cap Z - \{0\}$ and by the primeness of R , we have

$$d(x) + d(x^*) = 0, \forall x \in R \tag{31}$$

which further implies that

$$d(x^*) = (d(x))', \forall x \in R \tag{32}$$

In (31) replacing x by $h + s, h \in H, s \in S$, we get $d(h + s) + d(h + s') = 0, \forall h \in H, s \in S$ and therefore $2d(h) + d(s + s') = 0, \forall h \in H, s \in S$. since $s + s' \in Z$ and R is 2-torsion free, therefore using (28), we get

$$d(h) = 0, \forall h \in H \tag{33}$$

In (31), replacing x by $xh, h \in H$ and using (33), we get

$$d(x)h + hd(x^*) = 0, \forall x \in R, h \in H \tag{34}$$

Using (32) into (34), we get

$$[d(x), h] = 0, \forall x \in R, h \in H \tag{35}$$

In (35) replacing x by xy , we get $[d(x)y, h] + [xd(y), h] = 0, \forall x, y \in R, h \in H$ and hence $d(x)[y, h] + [d(x), h]y + [x, h]d(y) + x[d(y), h] = 0, \forall x, y \in R, h \in H$. Using (35) again, we get

$$d(x)[y, h] + [x, h]d(y) = 0, \forall x, y \in R, h \in H \tag{36}$$

In (36), replacing y by $d(y)$, we get $d(x)[d(y), h] + [x, h]d^2(y) = 0, \forall x, y \in R, h \in H$ and using (35) again, we obtain

$$[x, h]d^2(y) = 0, \forall x, y \in R, h \in H \tag{37}$$

Replacing x by xr in (35) and using it again, we get $[x, h]rd^2(y) = 0, \forall x, y, r \in R, h \in H$ and hence $[x, h]Rd^2(y) = \{0\}, \forall x, y \in R, h \in H$. By the primeness of R , we obtain $[x, h] = 0$ or $d^2(y) = 0, \forall x, y \in R, h \in H$. When

$$d^2(y) = 0, \forall y \in R \tag{38}$$

In (38) replacing y by xy and using it again, we get $2d(x)d(y) = 0, \forall x, y \in R$ and by the 2-torsion freeness of R , we have

$$d(x)d(y) = 0, \forall x, y \in R \tag{39}$$

In (39), replacing x by xr , we get $d(x)rd(y) + xd(r)d(y) = 0, \forall x, y, r \in R$ and using (39) again, we get $d(x)rd(y) = 0, \forall x, y, r \in R$ and therefore $d(x)Rd(y) = \{0\}, \forall x, y \in R$. By the primeness of R , we have $d(x) = 0, \forall x \in R$ and therefore $d=0$, which is not possible. On the other hand, when

$$[x, h] = 0, \forall x \in R, h \in H \tag{40}$$

Since $\forall x \in R, x + x^* \in H$, so replacing h by $x + x^*$ in (40), we get $[x, x + x^*] = 0, \forall x \in R$ which implies

$$[x, x] + [x, x^*] = 0, \forall x \in R \tag{41}$$

Also $\forall x \in R$, we have $[x, x' + x^*] = [x, x'] + [x, x^*] = [x, x'] + [x, x^*] = [x, x] + [x, x^*] = [x, x + x^*] = 0$ and hence

$$[x, x] = [x, x^*], \forall x \in R \tag{42}$$

Using (42) into (41) and the 2-torsion freeness of R , we get $[x, x^*] = 0, \forall x \in R$. By Lemma 2.4, we conclude that R is commutative.

3 Concluding Remarks

The word commutativity carries much importance for every noncommutative algebraic structure. This article discusses some conditions on MA-semirings with involution especially of second kind which guarantee commutativity. The commutativity of algebraic structures brings convenience in calculations. Therefore this research is useful and opens the door of further research in this area by using different types of additive mappings.

References

- 1) Beidar KI, Martindale WS. On Functional Identities in Prime Rings with Involution. *Journal of Algebra*. 1998;203(2):491–532. Available from: <https://dx.doi.org/10.1006/jabr.1997.7285>.
- 2) Herstein N. Rings with involution. and others, editor;University of Chicago. 1976.
- 3) Lanski C. Commutation with skew elements in rings with involution. *Pacific Journal of Mathematics*. 1979;83(2):393–399. Available from: <https://dx.doi.org/10.2140/pjm.1979.83.393>.
- 4) Lee T. On Derivations of prime rings with involution. *Chin J of Math*. 1992;20(2):191–203.
- 5) Lee P. On subrings of rings with involution. *Pacific J Math*. 1975;60(2):131–147.
- 6) Chadja I, Langer H. Near semirings and semirings with involution. *Misk Math Notes*. 2017;17(2):801–810.
- 7) Dolinka I. Idempotent distributive semirings with involution. *International Journal of Algebra and Computation*. 2003;13(05):597–625. Available from: <https://dx.doi.org/10.1142/s0218196703001614>.
- 8) Markov RV. Pierce sheaf for semirings with involution. *Russian Mathematics*. 2014;58(4):14–19. Available from: <https://dx.doi.org/10.3103/s1066369x14040033>.
- 9) Salli VN, Polukolec KTI. *Izv, Vyssh UcZav Mat*. 1969;3:52–60.
- 10) Javed M, Aslam M, Hussain M. On condition (A2) of Bandlet and Petrich for inverse semirings. *Int Math Forum*. 2012;7(59):2903–2914.
- 11) Javed M, Aslam M. Commuting mappings of semiprime MA-semirings. *WASJ*.
- 12) Nadeem M, Aslam M. On the additive maps satisfying skew Engel conditions. *J of Alg and Rel Top*. 2017;5(2):47–58.
- 13) Nadeem M, Aslam M, Javed MA. On the generalization of Bresar theorems. *Quasi and Rel Sys*. 2016;24:123–128.
- 14) Aslam M, Javed MA, Sara S. On centralizer of semiprime inverse semiring. *Discussiones Mathematicae - General Algebra and Applications*. 2016;36(1):71–71. Available from: <https://dx.doi.org/10.7151/dmgaa.1252>.
- 15) Shafiq S, Aslam M. On Jordan mappings of inverse semirings. *Open Mathematics*. 2017;15(1):1123–1131. Available from: <https://dx.doi.org/10.1515/math-2017-0088>.
- 16) Ali L, Aslam M, Khan Y. On additive maps of MA-semirings with involution;vol. 39. *Proyecciones (Antofagasta, On)*. 2020.
- 17) Ali L, Aslam M, Khan Y. Commutativity of semirings with involution. *Asian-European J of Math*. 2019. Available from: <https://doi.org/10.1142/S1793557120501533>.
- 18) Ali L, Aslam M, Khan Y. On Jordan ideals of inverse semirings with involution. *Indian J of Sci and Tech*. 2020;13(04):430–438.
- 19) Ali L, Aslam M, Khan YA. SOME COMMUTATIVITY CONDITIONS ON *-PRIME SEMIRINGS. *JP Journal of Algebra, Number Theory and Applications*. 2020;46(2):109–121. Available from: <https://dx.doi.org/10.17654/nt046020109>.
- 20) Ali L, Aslam M, Khan Y, Farid G. On generalized derivations of semirings with involution. *J of Mech Cont and Math Sci*. 2020;15(4):138–152.
- 21) Atteya M. Derivations on MA-semirings. *Math Sci Lett*. 2015;4(3):1–5.
- 22) Ahmed Y, Dudek AW. Stronger Lie derivations on MA-semirings. *Afrika Matematika*. 2020;31(5-6):891–901. Available from: <https://dx.doi.org/10.1007/s13370-020-00768-3>.
- 23) Khan Y, Nadeem M, Aslam M. On centralizers Of MA-semirings. *Jof Mech Cont and Math Sci*. 2020;15(4):47–57.
- 24) Khan YA, Aslam M, Ali L. Commutativity of additive inverse semirings through $f(xy) = [x, f(y)]$. *Thai J of Math, Special Issue*. 2018;p. 288–300.
- 25) Ali S, Dar NA, Ascii M. On derivations and commutativity of prime rings with involution. *Georgian Mathematical Journal*. 2016;23(1). Available from: <https://dx.doi.org/10.1515/gmj-2015-0016>.