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Some properties of multivalued positive Boolean dependencies in the database model of block form

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Abstract

Objectives: The article proposed a new type of dependency on blocks and slices. Then found and proved the properties of this new dependency. **Method:** Logical inference methods were used. **Findings:** A new type of data relationship has been proposed: Multivalued positive Boolean dependencies on block and slice in the database model of block form. From this new concept, the article stated and demonstrated the equivalence of the three types of deduction, namely: m-deduction by logic, m-deduction by block, m-deduction by block has no more than two elements. Next are the necessary and sufficient criteria of the tight m-expression for the set of multivalued positive Boolean dependencies on block and slice, the sufficient properties for a set of functions $\{I, \wedge, \vee\}$. The properties related to this new concept when the block degenerated into relation. **Novelty:** The proposed new dependency with their properties on the block and on the slice are completely new.

Keywords: Multivalued positive Boolean dependencies; block; Boolean dependencies; block schemes

1 Introduction

1.1 The block, slice of the block

Definition 1 .1 [1]

Let R = (id; $A_1, A_2, \ldots, A_n$) is a finite set of elements, where id is non-empty finite index set, A_i (i=1.. n) is the attribute. Each attribute A_i (i=1.. n) there is a corresponding value domain dom (A_i). A block r on R, denoted r(R) consists of a finite number of elements that each element is a family of mappings from the index set id to the value domain of the attributes A_i (i = 1.. n).

We have:

$$t \in r(R) \Leftrightarrow t = \left\{ t^i : \text{ id } \to \text{dom}(A_i) \right\}_{i=1.n}$$

Then, block is denoted r(R) or r (id; $A_1, A_2, \ldots, A_n$), if without fear of confusion we simply denoted r.

Definition 1.2 [1]

Let R = (id; A_1, A_2,..., A_n), r(R) is a block over R. For each $x \in$ id we denoted r(R_x) is a block with R_x = ({x}; A_1, A_2,..., A_n) such that:

$$t_x \in r(R_x) \Leftrightarrow t_x = \left\{ t_x^i = t_{|x}^i \right\}_{i=1..n}, \text{ where } t \in r(R),\ t = \left\{ t^i : \text{ id } \rightarrow \text{dom}(A_i) \right\}_{i=1..n}$$

Then $r(R_x)$ *is called a slice of the block* r(R) *at point* x.

1.2 Functional dependencies

Here, for simplicity we use the notation:

$$\mathbf{x}^{(i)} = (x;\ A_i);\ id^{(i)} = \left\{ x^{(i)} \mid x \in id \right\}$$

and $x^{(i)} (x \in id, i = 1 \ldots n)$ is called an index attribute of block scheme R = (id; A_1,A_2,...,A_n).

Definition 1.3 [2]

Let R = (id; A_1,A_2 ,..., A_n), r(R) is a block over R and $X, Y \subseteq \bigcup_{i=1}^{n} id^{(i)}, X \rightarrow Y$ *is a notation of functional dependency. A block r satisfies* $X \rightarrow Y$ *if:*

$$\forall t_1, t_2 \in R \text{ such that } t_1(X) = t_2(X) \text{ then } t_1(Y) = t_2(Y)$$

Definition 1.4 [2]

Let block scheme $\alpha = (R, F)$, $R = (id; A_1, A_2, \ldots, A_n)$ F *is the set of functional dependencies over* R. *Then, the closure of* F *denoted* F^+ *is defined as follows*:

$$F^+ = \{X \rightarrow Y \mid F \Rightarrow X \rightarrow Y\}$$

If $X = \left\{ x^{(m)} \right\} \subseteq id^{(m)}, Y = \left\{ y^{(k)} \right\} \subseteq id^{(k)}$ then we denoted functional dependency $X \rightarrow Y$ is simply $x^{(m)} \rightarrow y^{(k)}$.

The block r satisfies $\mathbf{x}^{(m)} \rightarrow \mathbf{y}^{(k)}$ if $\forall \mathbf{t}_1, \mathbf{t}_2 \in \mathbf{r}$ such that $\mathbf{t}_1\left(\mathbf{x}^{(m)}\right) = \mathbf{t}_2\left(\mathbf{x}^{(m)}\right)$ then $t_1\left(\mathbf{y}^{(k)}\right) = t_2\left(\mathbf{y}^{(k)}\right)$,

where: $t_1\left(x^{(m)}\right) = t_1(x; A_m), t_2\left(x^{(m)}\right) = t_2(x; A_m)$, $t_1\left(y^{(k)}\right) = t_1(y; A_k)$, $t_2\left(y^{(k)}\right) = t_2(y; A_k)$.

Let block scheme R= (id; A_1, A_2,..., A_n), we denoted the subsets of functional dependencies over R:

$$F_h = \left\{ X \rightarrow Y \mid X = \bigcup_{i \in A} x^{(i)}, Y = \bigcup_{j \in B} x^{(j)} A, B \subseteq \{1, 2, \ldots, n\}, x \in id \right\}$$

$$F_{hx} = F_{h|_{\bigcup_{i=1}^{n} x^{(i)}}} = \left\{ X \rightarrow Y \in F_h \mid X, Y \subseteq \bigcup_{i=1}^{n} x^{(i)} \right\}$$

Definition 1.5 [3]

Let block scheme $\alpha = (R, F_h)$, $R = (id; A_1, A_2, \ldots, A_n)$, *then* F_h *is called the complete set of functional dependencies if:*

$$F_{hx} = F_{h|_{\bigcup_{i=1}^{n} x^{(i)}}} \quad \text{is the same with every} \quad x \in id$$

A more specific way:

F_{hx} is the same with every $x \in$ id mean: $\forall x, y \in id : M \rightarrow N \in F_{hx} \Leftrightarrow M' \rightarrow N' \in F_{hy}$ with M', N' respectively, formed from M, N by replacing x by y.

1.3 Closure of the index attributes sets:

Definition 1.6 [3]

Let block scheme$\alpha = (R, F), R = (id; A_1, A_2, \ldots, A_n)$, F *is the set of functional dependencies on R.*

With each $X \subseteq \bigcup_{i=1}^{n} id^{(i)}$*,we define closure of* X *for* F *denoted* X^+ *as follows:*

$$X^+ = \left\{ x^{(i)} \mid X \rightarrow x^{(i)} \in F^+,\ x \in id,\ i = 1..n \right\}$$

We denote the set of all subsets of a set $\bigcup_{i=1}^{n} id^{(i)}$ as set SubSet ($\bigcup_{i=1}^{n} id^{(i)}$).

1.4 Key of the block scheme α = (R, F)

Definition 1.7[4]

Let block scheme α *= (R, F), R = (id;* A_1*,* A_2*,...,* A_n *), Fis the set of functional dependencies on R,* $K \subseteq \bigcup_{i=1}^{n} \mathrm{id}^{(i)}$*. K called a key of block schema* α *if it satisfies two conditions:*

i) $\mathrm{K} \to \mathrm{x}^{(\mathrm{i})} \in \mathrm{F}^{+}, \forall\, \mathrm{x} \in \mathrm{id},\ \mathrm{i} = 1..\mathrm{n}$.

ii) $\forall \mathrm{K}' \subset \mathrm{K}$ then K' has no properties i).

If K is a key and $\mathrm{K} \subseteq \mathrm{K}''$ *then* K" called a super key of the block scheme R for F.

2 Multivalued Boolean formulas

2.1 Multivalued boolean formulas

Definition 2.1[5]

For the set of Boolean values $\mathrm{B} = \{b_1, b_2, \ldots, b_k\}$ *including k values in [0;1],* $k \geq 2$ *are in ascending order and satisfy the following conditions:*

(i) $0 \in B$.

(ii) $\forall\, b \in B \quad \Rightarrow 1 - b \in B$.

We choose the operations and basic multivalued logical function: $\forall\, a,\ b \in B$

- $a \wedge b = \min(a, b)$,
- $a \vee b = \max(a, b)$,
- $\neg a = 1 - a$
- With each value $b \in B$, we define the function I_b:

$$\forall \mathrm{x} \in \mathrm{B}: \quad \mathrm{I_b(x)} = 1 \text{ if } \mathrm{x} = \mathrm{b} \text{ and } \mathrm{I_b(x)} = 0 \text{ if } \mathrm{x} \neq \mathrm{b}$$

The functions $\mathrm{I}_b,\ b \in B$ called generalized negative functions.

Definition 2.2[5]

Let P = {x_1, x_2, ..., x_n} is a finite set of Boolean variables, B is the set of Boolean values. Then the multivalued boolean formulas (CTBĐT) also known as multivalued logic formulas are constructed as follows:

1. Each value in B is a CTBĐT.
2. Each variable in P is a CTBĐT.
3. Each function $\mathrm{I}_b,\ b \in B$ is a CTBĐT.
4. If a is a multivalued Boolean formula then (a) is a CTBĐT.
5. If a and b are CTBĐT then $a \wedge b,\ a \vee b$ and $\neg a$ is a CTBĐT.
6. Only formulas created by rules from (1) –(5) are CTBĐT.

We denote MVL(P) as a set of CTBĐT building on the set of variables P = {x_1, x_2, ..., x_n} and set of values B = {b_1, b_2,..., b_k} including k values in [0;1], $k \geq 2$.

Definition 2.3[5]

We define $a \to b$ *equivalent to CTBĐT* $(\neg a) \vee b$ *and then:* $a \to b$ *= max(1- a, b).*

Definition 2.4[5]

Each vector of elements v = {v_1*,* v_2*, ...,* v_n*} in space* $B^n = B\ X\ B\ X \ldots X\ B$ *is called a value assignment. Thus, with each CTBĐT* $f \in$ *MVL(P) we have f(v) = f(*v_1*,* v_2*, ...,* v_n*) is the value of formula f for v value assignments.*

In the case where there is no confusion, we understand the symbol $\mathrm{X} \subseteq \mathrm{P}$ at the same time performing for the following subjects:

- An attribute set in P.
- A set of logical variables in P.

- A multivalued Boolean formula is the logical union of variables in X.

On the other hand, if X = {B_1, B_2, ..., B_n} $\subseteq$ P , we denoted:

$\wedge X = B_1 \wedge B_2 \wedge \ldots \wedge B_n$ called the associational form.

$\vee X = \mathrm{B_1} \vee \mathrm{B_2} \vee \cdots \vee \mathrm{B_n}$ called the recruitmental form.

We call formula f: Z $\rightarrow$ V is :

• Multivalued derivative formula if Z and V has the associational form, mean:

$$\mathrm{f}: \wedge \mathrm{Z} \rightarrow \wedge \mathrm{V}$$

• Strong multivalued derivative formula if Z has the ecruitmental form and V has the associational form, mean:

$$f: \vee Z \rightarrow \wedge V$$

• Weak multivalued derivative formula if Z has the associational form and V has the ecruitmental form, mean:

$$\mathrm{f}: \wedge \mathrm{Z} \rightarrow \vee V$$

• Duality multivalued derivative formula if Z and V are in recruitment form, mean:

$$f: \vee Z \rightarrow \vee V$$

For each finite set CTBĐT F = {f_1, f_2, ..., f_m} in MVL(P), we consider F as a formatted formula $\mathrm{F} = \mathrm{f_1} \wedge \mathrm{f_2} \wedge \ldots \wedge \mathrm{f_m}$. Then we have:

$$\mathrm{F(v)} = \mathrm{f_1(v)} \wedge \mathrm{f_2(v)} \wedge \ldots \wedge \mathrm{f_m(v)}$$

2.2 Table of values and truth tables

With each formula f on P, table of values for f, denote that V_f contains n+1 columns, with the first n columns containing the values of the variables in U, and the last column contains the value of f for each values signment of the corresponding row. Thus, the value table contains k^n row, n is the element number of P, k is the element number of B.

Definition 2.5[5]

Let m ∈ [0;1], truth table with m threshold of f or the m-truth table of f, denoted $T_{f,m}$ is the set of assignments v such that f(v) receive value not less than m:

$$\mathrm{T}_{f,m} = \{\mathrm{v} \in B^{\mathrm{n}} \mid \mathrm{f(v)} \geq \mathrm{m}\}$$

Then, the m-truth table $T_{F,m}$ of finite sets of formulas F on P, is the intersection of the m-truth tables of each member formula in F.

$$T_{F,m} = \bigcap_{f \in F} T_{f,m}$$

We have: $v \in T_{F,m}$ necessary and sufficient are $\forall f \in F \ : \ f(v) \geq m$.

2.3 Logical deduction

Definition 2.6[5]

Let f, g is two CTBĐT and value m ∈ B. We say formula f derives formula g from threshold m and denoted $f \models_m g$ if $T_{f,m} \subseteq T_{g,m}$. We say f and g are two m-equivalent formulas, denoted $f \equiv_x g$ if $T_{f,m} = T_{g,m}$.

With F, G in MVL(P) and value m ∈ [0;1], we have F derives G from threshold m, denoted $F \models_m G$ if $T_{F,m} \subseteq T_{G,m}$. Moreover, we say F and G are m-equivalents, denoted $F \equiv_n G$ if $T_{F,m} = T_{G,m}$.

2.4 Multivalued positive Boolean formula

Definition 2.7[5]

Formula f ∈ MVL(P) is called a multivalued positive Boolean formula (CTBDĐT) if f(e) = 1 with e is the unit value assignment: e = (1, 1, ..., 1), we denoted MVP(P) is the set of all multivalued positive Boolean formulas on P.

3 Research results

3.1 The m-truth block of the data block

Definition 3.1

Let R = (id; A_1,A_2,...,A_n), r(R) is a block on R , $U = \bigcup_{i=1}^{n} i\, d^{(i)}$, |id|=s, We call each vector of elements v = {v_{i1}, v_{i2}, ..., v_{in} } $_{i=1..s}$ in space B^{nxs} is a value assignment. Thus, with each CTBĐT f ∈ MVL(U) we have f(v) = f (v_{i1}, v_{i2}, ..., v_{in}) $_{i=1..s}$ is the value of formula f for v value assignments.

Example 3.1: Let $R = (\{1,2\}, A_1, A_2, A_3)$ then $U = \left\{1^{(1)}, 1^{(2)}, 1^{(3)}, 2^{(1)}, 2^{(2)}, 2^{(3)}\right\}$, *B*={0, 0.5, 1}.

Let $v = \begin{pmatrix} 0.5 & 1 & 0.5 \\ 1 & 0.5 & 0.5 \end{pmatrix}$, $f = 1^{(1)}1^{(2)}2^{(1)}2^{(2)} \to 1^{(3)}2^{(3)}$, then we have f(v) = max(1 - min(0.5, 1, 1, 0.5), min(0.5, 0.5)).

Inferred: f (v) = 0.5.

We have two special assignment:

Unit assignment: $e = \begin{pmatrix} 1 & 1 & 1 \\ . & . & . \\ 1 & 1 & 1 \end{pmatrix}$ and the assignment value 0: $Z = \begin{pmatrix} 0 & 0 & 0 \\ \cdot & \cdot & \cdot \\ 0 & 0 & 0 \end{pmatrix}$

Definition 3.2

Let m ∈ [0;1], truth block threshold m of f or m-truth block of f, denoted $T_{f,\,m}$ is the set of assignments v such that f(v) receive value not less than m:

$$T_{f,m} = \{v \in B^{nxs} \mid f(v) \geq m\}$$

Then, the m-truth block $T_{F,m}$ of a finite set of formulas F on U, is the intersection of the m-truth blocks of each formula of member f in F.

$$T_{F,m} = \bigcap_{f \in F} T_{f,m}$$

We have: $v \in T_{F,m}$ if and only if $\forall f \in F : f(v) \geq m$.

With | *B*| = k then | B^{nxs} | = k ^{nxs}, we have the following theorem:

Theorem 3.1

Let block $T = \{t_1, t_2, \ldots, t_d\} \subseteq B^{nxs}$ and values m_1, m_2,..., m_d trong B, $I \leq d \leq k^{n \times s}$. Then:

a) There exists one CTBĐT f satisfies the following two properties:

(i) $\forall t_i \in T : f(t_i) = m_i,$

(ii) $\forall t \in B^{nxs} \mid T : f(t) = 0$

b) With every slice : T_x , $x \in id$, $|id| = s$, CTBĐ*T* $f_x = f_{|T_x}$ *also satisfies the following two properties:*

$(iii) \forall t_{xi} \in T_x : f_x(t_{xi}) = m_i, T_x$

$(iv) \forall t_x \in B^n \mid T_x : f_x(t_x) = 0$

Proof:

a) With each $t_i \in T$: $t_i = \{t_{ij1}, t_{ij2}, \ldots, t_{ijn}\}_{j=1..s}$, $1 \leq i \leq d$, we built formula:

$$h_i\left(x^{(j1)}, x^{(j2)}, \ldots, x^{(in)}\right)_{j=1..s} = \wedge\left(I_{tij1}\left(x^{(j1)}\right), I_{tj2}\left(x^{(j2)}\right), \ldots, I_{tjin}\left(x^{(in)}\right), m_i\right)_{j=1..S}$$

and we have: *if* $\left(x^{(j1)}, x^{(j2)}, \ldots, x^{(jn)}\right)_{j=1..s} = t_i = \{t_{ij1}, t_{ij2}, \ldots, t_{ijn}\}_{j=1..s}$ then:

$$h_i(t_i) = m_i,\ h_i(t) = 0 \text{ with } t \neq t_i,\ 1 \leq i \leq d$$

Therefore, if we set:

$$f\left(x^{(j1)}, x^{(j2)}, \ldots, x^{(jn)}\right)_{j=1..s} = (h_1 \vee h_2 \vee \cdots \vee h_d)\left(x^{(j1)}, x^{(j2)}, \ldots, x^{(jn)}\right)_{j=1..s}$$

then f is the formula to look for.

Indeed, we have:

$$\mathrm{f}(\mathrm{t_i}) = (\mathrm{h_1} \vee \mathrm{h_2} \vee \cdots \vee \mathrm{h_d})(\mathrm{t_i}) = \mathrm{h_1}(\mathrm{t_i}) \vee \mathrm{h_2}(\mathrm{t_i}) \vee \cdots \vee \mathrm{h_i}(\mathrm{t_i}) \vee \cdots \vee \mathrm{h_d}(\mathrm{t_i})$$

.Which according to the properties of

$$\mathrm{h_{i:}} \quad \mathrm{h_i}(\mathrm{t_i}) = \mathrm{h_i}\left(\{\mathrm{t_{ij1}}, \mathrm{t_{ij2}}, \ldots, \mathrm{t_{ijn}}\}_{\mathrm{j=1..s}}\right) = \wedge\left(\mathrm{I_{tij1}}(\mathrm{t_{ij1}}), \mathrm{I_{tij2}}(\mathrm{t_{ij2}}), \ldots, \mathrm{I_{tjn}}(\mathrm{t_{ijn}}), \mathrm{m_i}\right)_{\mathrm{j=1..s}} = \mathrm{m_i}$$

$$\mathrm{h_i}(\mathrm{t}) = 0 \text{ vi } \mathrm{t} \neq \mathrm{t_i},\ 1 \le \mathrm{i} \le \mathrm{d}$$

So infer: $f(t_i) = m_i, 1 \le i \le d$ and $\forall t \in B^{\mathrm{nxs}} \setminus T : f(t) = 0 \Rightarrow f$ is CTBĐT to look for.

b) From CTBĐT f we have: $f_x = (h_{x1} \vee h_{x2} \vee \ldots vh_{xd})$ Then:

$\forall \mathrm{t_{xi}} \in \mathrm{T_x} : \mathrm{f_x}(\mathrm{t_{xi}}) = (\mathrm{h_{x1}} \vee \mathrm{h_{x2}} \vee \ldots \mathrm{v_{hxd}})(\mathrm{t_{xi}}) = \mathrm{h_{x1}}(\mathrm{t_{xi}}) \vee \mathrm{h_{x2}}(\mathrm{t_{xi}}) \vee \ldots \mathrm{vh_{xi}}(\mathrm{txi}) \vee \ldots \mathrm{vh_{xd}}$

Which we have: $\mathrm{h_{xi}}(\mathrm{t_{xi}}) = \mathrm{h_{xi}}(\{\mathrm{t_{xi1}}, \mathrm{t_{xi2_2......,x}t_{xin}}\}) = \wedge(\mathrm{I_{xxi1}}(\mathrm{t_{txi1}}), \mathrm{I_{txi2}}(\mathrm{t_{xi2}}), \ldots, \mathrm{I_{txin}}(\mathrm{t_{xin}}), \mathrm{m_i}) = \mathrm{m_i}$

$\mathrm{h_{xi}}(\mathrm{t_x}) = 0$ với $\mathrm{t_x} \neq \mathrm{t_{xi}}, 1 \le \mathrm{i} \le \mathrm{d}$.

we infer: $f_x(t_{xi}) = m_i, 1 \le i \le d$

and $\forall \mathrm{t_x} \in B^{\mathrm{n}} \setminus \mathrm{T} : \mathrm{f_x}(\mathrm{t_x}) = 0 \Rightarrow \mathrm{f_x}$ satisfies 2 conditions required.

Consequence 3.1*: With each block $T \subseteq B^{nxs, T \neq \varnothing}$ and each value m>0 in B, exists one CTBĐT f take T as the m-truth block, and f_x get T_x as the m-truth block.*

Proof:

Use the result of the theorem 3.1 with special cases: $m_1 = m_2 = \ldots = m_d = m$ we obtained CTBĐT f satisfies two conditions:

(i) $\forall t_i \in T : f(t_i) = m$

(ii) $\forall t \in \mathrm{B}^{nxs} \mid T : f(t) = 0$

and then CTBĐT f_x also satisfies two conditions:

(iii) $\forall t_{xi} \in T_x : f(t_{xi}) = m$

(iv) $\forall t_{xi} \in \mathrm{B}^{\mathrm{n}} \mid T_x : f(t_{xi}) = 0$. Thence inferred: $\mathrm{T}_{f,m} = \mathrm{T}$ và $\mathrm{T}_{fx,m} = \mathrm{T}_x$.

Definition 3.3

Let R = (id; A_1, A_2,...,A_n), r(R) is a block on R, $U = \bigcup_{i=1}^{n} i\, d^{(i)}$, each CTBĐT $f \in MVL(U)$ is called a multivalued positive Boolean formula (CTBDĐT) if f(e) = 1, with i e is the unit value assignment. Here: $e = \begin{pmatrix} 1 & 1 & 1 \\ . & . & . \\ 1 & 1 & 1 \end{pmatrix}$

Example 3.2:

Let $\mathrm{R} = (\{1,2\}, \mathrm{A_1}, \mathrm{A_2}, \mathrm{A_3}), \mathrm{U} = \left\{1^{(1)}, 1^{(2)}, 1^{(3)}, 2^{(1)}, 2^{(2)}, 2^{(3)}\right\}$ $B = \{0, 0.5, 1\}$. Then:

- The formulars: $1^{(1)} \wedge 1^{(2)} \wedge 2^{(1)} \wedge 2^{(2)}$, $\quad 1^{(1)} \wedge 1^{(2)} \wedge 2^{(1)} \rightarrow 2^{(2)}$ are the CTBDĐT.
- The formulars: $1^{(2)} \wedge \left(\neg 2^{(3)}\right)$, $\quad \left(\neg 1^{(3)}\right) \wedge \left(\neg 2^{(1)}\right)$ are not the CTBDĐT.

We denoted MVP(U) is the set of all multivalued positive Boolean formulas on U.

Definition 3.4

Let R = (id; A_1, A_2,...,A_n), r(R) is a block on R, we denote d_i is the value domain of the attribute A_i (is also of index attribute $x^{(i)}, x \in id\big)$, $1 \le i \le n$. Then, for each value domain we consider mapping: $\alpha_i : d_i \, x \, d_i \rightarrow B$ satisfies the following conditions:

(i) *Reflectivity*: $\forall a \in d_i : \alpha_i(a,a) = 1$,

(ii) Symmetry: $\forall\, a, b \in d_i : \alpha_i(a,b) = \alpha_i(b,a)$,

(iii) *Sufficiency*: $\forall\, m \in B,\ \exists\, a, b \in d_i : \alpha_i(a,b) = m$.

Thus, we see the mapping $\alpha_i$are the relationships above d_i satisfies the reflective, symmetrical and sufficiency properties. Equality relationships with logic of two values B ={0,1} is the separate case of the above relationship.

Definition 3.5

Let R = (id; A_1,A_2,...,A_n), r(R) is a block on R, $u,v \in r$, mappings α_i define on each value domain d_i, $1 \le i \le n$. we call $\alpha(u,v)$ is the value assignment:

$$\alpha(u,v) = \left(\alpha_1\left(u.x^{(1)},v.x^{(1)}\right),\alpha_2\left(u.x^{(2)},v.x^{(2)}\right),\ldots,\alpha_n\left(u.x^{(n)},v.x^{(n)}\right)\right)_{x\in id}$$

Then, for each block r, we denote the truth block of block r as T_r:

$$T_r = \{\alpha(u,v) \mid u,v \in r\}$$

If block r contains at least a certain element k then: $\alpha(u,u) = 1 \Rightarrow e \in T_r$.

In the case id = {x}, then the block degenerates into a relation and the concept of the truth block of the block becomes the concept of truth table of relation in the relational data model. In other words, the truth block of a block is to expand the concept of the truth table of relation in the relational data model.

3.2 The multivalued positive Boolean dependencies on block

Definition 3.6

Let $R = (id;A_1,A_2,\ldots,A_n)$, $r(R)$is a block on R, $U = \bigcup_{i=1}^{n} i\,d^{(i)}$, we call each multivalued positive Boolean formula in MVP(U) is a multivalued positive Boolean dependency (PTBDĐT) on block.

We say block r is m-satisfying the multivalued positive Boolean dependency f and denoted r(f,m) if $T_r \subseteq T_{f,m}$.

The block r is m-satisfying set of multivalued positive Boolean dependency F and denoted r(F,m) if r satisfies all PTBDĐT f in F:

$$r(F,m) \Leftrightarrow \forall f \in F : r(f,m) \Leftrightarrow T_r \subseteq T_{F,m}$$

If r(f,m) then we say PTBDĐT f is m-right in the block r.

Proposition 3.1

Let R = (id; A_1,A_2,...,A_n), r(R) is a block on R, $U = \bigcup_{i=1}^{n} i\,d^{(i)}$. Then:

i) If r is m-satisfying the multivalued positive Boolean dependency f : $r(f,m)$ then $r_x(f_x,m)$, $\forall x \in id$

ii) If r is m-satisfying set of multivalued positive Boolean dependency F : $r(F,m)$ then $r_x(F_x,m)$, $\forall x \in id$

Proof

i) Under the assumption we have $r(f,m) \Rightarrow T_r \subseteq T_{f,m} \Rightarrow T_{rx} = (T_r)_x \subseteq (T_{f,m})_x = T_{f_{x,m}}, \forall x \in id$

So we have $T_{rx} \subseteq T_{fx,m}, \forall x \in id \Rightarrow r_x(f_x,m), \forall x \in id$

ii) Under the assumption $r(F,m) \Rightarrow T_r \subseteq T_{F,m} \Rightarrow T_{rx} = (T_r)_x \subseteq (T_{F,m})_x = T_{Fx,m}, \forall x \in id$

Therefore: $T_{rx} \subseteq T_{Fx,m}, \forall x \in id \Rightarrow r_x(F_x,m), \forall x \in id$

Proposition 3.2

Let R = (id; A_1,A_2,...,A_n), r(R) is a block on R, $U = \bigcup_{i=1}^{n} id^{(i)}$, $f = \bigcup_{x\in id} f_x$. Then:

i) $If\ r_x(f_x,m), \forall x \in id$ then r is m-satisfying the multivalued positive Boolean dependency f: r(f,m).

ii) $If\ r_x(F_x,m), \forall x \in id$ then r m-satisfying set of multivalued positive Boolean dependency F: r(F,m).

Proof

i) Under the assumption we have: $r_x(f_x,m), \forall x \in id \Rightarrow T_{rx} \subseteq T_{fx,m}, \forall x \in id \Rightarrow (T_r)_x \subseteq (T_{f,m})_x, \forall x \in id$

So we have: $T_r \subseteq T_{f,m} \Rightarrow r(f,m)$.

$\Rightarrow$ *r* is m-satisfying the multivalued positive Boolean dependency f.

ii) Under the assumption $r_x(F_x,m), \forall x \in id \Rightarrow T_{rx} \subseteq T_{Fx,m}, \forall x \in id \Rightarrow (T_r)_x \subseteq (T_{F,m})_x, \forall x \in id$

So we have: $T_r \subseteq T_{F,m} \Rightarrow r(F,m)$.

$\Rightarrow$ *r* m-satisfying set of multivalued positive Boolean dependency F.

From the proposition 3.1 and 3.2 we have the following necessary and sufficient conditions:

Theorem 3.2

Let R = (id; A_1,A_2,...,A_n), r(R) is a block on R, $U = \bigcup_{i=1}^{n} id^{(i)}$, $f = \bigcup_{x\in id} f_x$. Khi đó:

i) $r_x(f_x,m), \forall x \in i\,d \Leftrightarrow$ r is m-satisfying the multivalued positive Boolean dependency f: r(f,m).

ii) $r_x(F_x,m), \forall x \in i\,d \Leftrightarrow$ r m-satisfying set of multivalued positive Boolean dependency F: r(F,m).

For the set PTBDĐT F and PTBDĐT f:, $m \in [0;1]$:

- We say F m-deduced f by the block and denoted $F\,|_{-m}\ f$ if: $\forall r:\ r(F,m)\ \Rightarrow r(f,m)$.
- We say F m-deduced f by the block contains no more than 2 elements and denoted $F\,|_{-2,m}\ f$ if: $\forall r_2:\ r_2(F,m) \Rightarrow r_2(f,m)$.

We have the following equivalent theorem:

Theorem 3.3

For the set PTBDĐT F and PTBDĐT f, R = (id; A_1,A_2,...,A_n), r(R) is α block on R, m ∈ B. Then the following three propositions are equivalent:

(i) F $|=_m$ f (m-deduction by logic),

(ii) F $|\text{-}_m$ f (m-deduction by block),

(iii) F $|\text{-}_{2,m}$ f (m-deduction by block has no more than 2 elements).

Proof

(i) => (ii): Under the assumption we have $F\ |=_m\ f \Rightarrow T_{F,m}\ \subseteq\ T^{(1)}_{f,m}$. Let r be an arbitrary block and r(F,m), then by definition: $T_r \subseteq T^{(2)}_{F,m}$. From (1) and (2) we infer: $T_r \subseteq T_{f,m}$, so we have: r(f,m).

(ii) => (iii): Obviously, because inference by the block has no more than 2 elements is the special case of inference by block.

(iii) => (i): Suppose $t = \left(t^{(1)}_x, t^{(2)}_x, \ldots, t^{(n)}_x\right)\ x \in id,\ t \in T_{F,m}$ we need proof : $t \in T_{f,m}$.

Indeed, if t = e then we have $t \in T_{f,m}$ because as we know f is a positive Boolean formula. If $t\ \neq e$, we built the block r including 2 elements u and v as follows: $u = \left(u^{(1)}_x, u^{(2)}_x, \ldots, u^{(n)}_x\right)_{x\,\in\,id}, V = \left(V^{(1)}_x, V^{(2)}_x, \ldots, V^{(n)}_x\right)_{x\in id}$ satisfy $\alpha(u,v) = t\left(\text{mean}\ \alpha_i\left(u^{(i)}_x, v^{(i)}_x\right) = t^{(i)}_x, 1 \leq i \leq$
The existence of the u and v elements as above is due to the properties of the mappings α_i mentioned above. Thus r is a block with 2 elements and $T_r = \{e,t\} \subseteq T_{F,m}$, with e is a element of block whose all component values are equal to 1.

Thence inferred r (F, m). Under the assumption we have r (F, m) => r (f,m), so that $T_r \subseteq {T_{f,m}}^{(1).}$

From (1) we infer $t \in T_{f,m}$.

Consequence 3.2

For the set PTBDĐT F and PTBDĐT f, R = (id; A_1,A_2,...,A_n), r(R) is a block on R, m ∈B. Then on r_x the following three propositions are equivalent:

(i) *F_x $|=_m$ f_x (m-deduction by logic),*

(ii) *F_x $|\text{-}_m$ f_x (m-deduction by slice r_x),*

(iii) *F_x $|\text{-}_{2,m}$ f_x (m-deduction by the slice r_{2x} has no more than 2 elements).*

In the case id = {x}, then the block degenerated into a relation and the above m-equivalence theorem becomes the m-equivalent theorem in the relational data model. Specifically, we have the following consequences:

Consequence 3.3

For the set PTBDĐT F and PTBDĐT f, R = (id; A_1,A_2,...,A_n), r(R) is a block on R, m ∈ B. Then if id = {x} then block r degenerates into relation and the following three propositions are equivalent:

(i) F $|=_m$ f (m-deduction by logic),

(ii) F $|\text{-}_m$ f (m-deduction by relation),

(iii) F $|\text{-}_{2,m}$ f (m-deduction by relation has no more than 2 elements).

Definition 3.7

Let R = (id; A_1,A_2,...,A_n), r(R) is a block on R, $U = \bigcup_{i=1}^{n} i\,d^{(i)}$, m ∈B. ∑ is a subset of PTBDĐT on U, we denoted (∑ ,m)⁺ is the set of all PTBDĐT are m-deduction from ∑ , in other words:

$$(\Sigma, m)^+ = \{g \in MVP(U)|\Sigma| =_m g\} = \left\{g \in MVP(U)\mid T_{\Sigma,m} \subseteq T_{g,m}\right\}$$

Definition 3.8

Let R = (id; A_1,A_2,...,A_n), r(R) is a block on R, $U = \bigcup_{i=1}^{n} i\,d^{(i)}$, m ∈ B, we denoted MBDĐT(r,m) is the set of all PTBDĐT m-right in r, In other words:

$$\text{MBDĐT}(r,m) = \{g \in \text{MVP}(U) \mid r(g,m)\}$$

So, we have:

$$g \in \text{MB}D\text{Đ}T(r,m) \Leftrightarrow g \in \text{MVP}(U) \wedge T_r \subseteq T_{g,m}$$

Theorem 3.4

Let R = (id; A_1,A_2,...,A_n), r(R) is a block on R, $U = \bigcup_{i=1}^{n} id^{(i)}$, $m \in$ B. Then we have:

$$(MBDĐT(r,m),m)^{+} = MBDĐT(r,m)$$

Proof

By definition, we have:

$$(MBDĐT(r,m),m)^{+} = \{g \in MVP(U) | MBDĐT(r,m)| = mg\} \tag{1}$$

Apply the result of the theorem of three equivalent propositions for PTBDĐT, we have:

$$\{\mathrm{g} \in \mathrm{MVP(U)}|\,\mathrm{MBD}ĐT(r,m)| =_m g\} = \{\mathrm{g} \in \mathrm{MVP(U)}|\,\mathrm{MBD}ĐT(r,m)|_{-m}\, g\} \tag{2}$$

From (1) and (2) we infer: $(\text{MBDĐT(r,m),m})^{+}$ = MBDĐT(r,m).

So two sets $(\text{MBDĐT(r,m),m})^{+}$ and MBDĐT(r,m) are two sets PTBDĐT m-equivalents on blocks.

Consequence 3.4

Let R = (id; A_1,A_2,...,A_n), r(R) is a block on R, $U = \bigcup_{i=1}^{n} id^{(i)}$, $m \in$ B . Then on r_x we have:

$$(MBDĐT\,(r_x,m)\,,m)^{+} = MBDĐT\,(r_x,m)\,, x \in id$$

Consequence 3.5

Let R = (id; A_1,A_2,...,A_n), r(R) is a block on R, m ∈ B. Then we have, if id = {x} then block r degenerates into relation and we have in the relational data model:

$$(MBDĐT(r,m),m)^{+} = MBDĐT(r,m)$$

Definition 3.9

Let R = (id; A_1,A_2,...,A_n), r(R) is a block on R, $U = \bigcup_{i=1}^{n} id^{(i)}$, $m \in$ B, Σ is the subset of PTBDĐT on U. We say block r is m-representation set Σ if MBDĐT $(r,m) \supseteq (\Sigma,m)^{+}$ *and we say block r is m-tight representation set Σ if* MBDĐT$(r,m) = (\Sigma,m)^{+}$.

If r is m-tight representation set PTBDĐT Σ then we say r is the block m-Armstrong of set PTBDĐT Σ .

Theorem 3.5

Let R = (id; A_1,A_2,...,A_n), m ∈ B. Then, with every block r(R) different from the empty set on R we have:

$$\mathrm{T_r} = \mathrm{T}_{\mathrm{MBDĐ}T(\mathrm{r,m}),\mathrm{m}}$$

Proof

Suppose g ∈ MBDĐT(r, m) ⇒ r is m-satisfying $g \Leftrightarrow T_r \subseteq T_{g,m}$. From T_r and value m, According to theorem 3.1 we find a multivalued boolean formula f satisfying conditions: f(e) = 1 và $T_{f,m} = T_r$. So: e ∈ $\mathrm{T_r} = \mathrm{T_{f,m}}$ infer f is one CTBDĐT and and more due $\mathrm{T_r} = \mathrm{T_{f,m}}$ ⇒ r is m-satisfying f, mean: f ∈ MBDĐT (r, m).

We denoted: F = MBDĐT(r,m), from the above proof we have:

$$\forall \mathrm{g} \in \mathrm{MBD}ĐT(\mathrm{r,m}) \Rightarrow \mathrm{T_r} \subseteq \mathrm{T_{g,m}} \Rightarrow \mathrm{T_r} \subseteq \bigcap_{g \in F} T_{g,m} \tag{3}$$

$$\exists \mathrm{f} \in \mathrm{MBDĐ}T(\mathrm{r,m}) : \mathrm{T_r} = \mathrm{T_{f,m}} \Rightarrow \mathrm{T_r} \supseteq \bigcap_{g \in F} T_{g,m} \tag{4}$$

From (3) and (4) we infer:

$$\mathrm{T_f} = \bigcap_{g \in F} T_{g,\bar{m}} = \mathrm{T_{F,m}}$$

Thus: $\mathrm{T_t} = \mathrm{T}_{\mathrm{MBDĐ}T(\mathrm{r,m}),\mathrm{m}}$.

Consequence 3.6

Let R = (id; A_1,A_2,...,A_n), r(R) is a block on R, m ∈ B . Then, on the slice r_x we have:

$$\mathrm{T_{Tx}} = \mathrm{T}_{\mathrm{MBD\text{Đ}}T\,(\mathrm{r,m}),\,\mathrm{m}}\ ,\ x \in id$$

Consequence 3.7

Let R = (id; A_1,A_2,...,A_n), r(R) is a block on R, m ∈ *B . Then we have, if id = {x} then block r degenerates into relation and we have in the relational data model: $T_r = T_{MBD\text{Đ}T(r,m),m.}$*

Theorem 3.6

Let R = (id; A_1,A_2,...,A_n), $U = \bigcup_{i=1}^{n} id^{(i)}$, $m \in \mathrm{B}$, Σ is the subset of PTBDĐT on U. Then, with every block r(R) is otherwise empty on R we have: r is m-tight representation set PTBDĐT Σ if and only if $\mathrm{T_r} = \mathrm{T}_{\Sigma,\mathrm{m}}$.

Proof

By definition, we have: r is m-tight representation set $\Sigma \Leftrightarrow \mathrm{MBD\text{Đ}}T(\mathrm{r,m}) = (\Sigma,\mathrm{m})^+ \Leftrightarrow \quad \mathrm{MBD\text{Đ}}T(\mathrm{r,m}) \equiv_{\mathrm{m}} \sum$.

Other way:

$$\mathrm{MBD\text{Đ}}T(\mathrm{r,m}) \equiv_{\mathrm{m}} \Sigma \Leftrightarrow \mathrm{T}_{\mathrm{MBD\text{Đ}}T(\mathrm{r,m}),\mathrm{m}} = \mathrm{T}_{\Sigma,\mathrm{m}} \tag{5}$$

Apply the results of theorem 3.4 we obtain:

$$\mathrm{T_r} = \mathrm{T}_{\mathrm{MBD\text{Đ}}T(\mathrm{r,m}),\mathrm{m}}$$

So from (5) and (6) we infer: $\mathrm{T}_r = \mathrm{T}_{S,m}$.

Therefore: r is m-tight representation set $\Sigma \Leftrightarrow \mathrm{T}_x = \mathrm{T}_{\Sigma,\mathrm{m}}$.

Consequence 3.8

Let R = (id; A_1,A_2,...,A_n), $U = \bigcup_{i=1}^{n} id^{(i)}$, $m \in \mathrm{B}$, *Σis the subset of PTBDĐT on U. Then we have, if id = {x} then block r degenerated into relation and we have in the relational data model: every relation r is different from the empty set on R is m-tight representation set PTBDĐT Σif and only if $T_r = T_{\sum,m}$.*

This consequence is exactly what we already know in the relational data model.

We denoted $\Sigma_{\mathrm{x}} = \Sigma \cap \bigcup_{i=1}^{n} x^{(i)}$

Theorem 3.7

Let R = (id; A,A,...,A), $U = \bigcup_{i=1}^{n} i\,d^{(i)}, m \in \mathrm{B}$, *Σis the subset of PTBDĐT on if U,* $\Sigma = \bigcup_{x \in id} \Sigma_x, \quad \Sigma_x \neq \varnothing$ *Then, with every block r(R) is otherwise empty on R we have: r is m-tight representation set PTBDĐT Σif and only if r_x is m-tight representation set $\Sigma_x, \forall x \in i\,d$.*

Proof

⇒) Suppose r is m-tight representation set PTBDĐT Σ we need proof r_x is m-tight representation set $\Sigma_{\mathrm{x}}, \forall \mathrm{x} \in \mathrm{id}$.

Indeed, under the assumption we have: r is m-tight representation set PTBDĐT Σ, using the results of theorem 3.6 we have: $\mathrm{T_r} = \mathrm{T}_{\Sigma,\mathrm{m}}$.

Thence inferred: $(\mathrm{T_r})_{\mathrm{x}} = (\mathrm{T}_{\Sigma,\mathrm{m}})_{\mathrm{x}}, \forall \mathrm{x} \in \mathrm{id}$.

Which we have: $\mathrm{T_{rx}} = (\mathrm{T_r})_{\mathrm{x}} = (\mathrm{T}_{\Sigma,\mathrm{m}})_{\mathrm{x}} = \mathrm{T}_{\Sigma\mathrm{x,m}}, \forall \mathrm{x} \in \mathrm{id} \Rightarrow \mathrm{T_{rx}} = \mathrm{T}_{\Sigma\mathrm{x,m}} \Rightarrow \mathrm{r_x}(\Sigma_{\mathrm{x}},\mathrm{m}), \forall \mathrm{x} \in \mathrm{id}$.

So r_x is m-tight representation set$\Sigma_{\mathrm{x}}, \forall \mathrm{x} \in \mathrm{id}$.

⇐) Suppose r_x is m-tight representation set $\Sigma_{\mathrm{x}}, \forall \mathrm{x} \in \mathrm{id}$ we need proof r is m-tight representation set Σ.

Indeed, under the assumption r_x is m-tight representation set $\Sigma_{\mathrm{x}}, \forall \mathrm{x} \in \mathrm{id} \Rightarrow \mathrm{T_{rx}} = \mathrm{T}_{\Sigma x,\mathrm{m}}, \forall \mathrm{x} \in \mathrm{id}$.

Inferred: $(\mathrm{T_r})_{\mathrm{x}} = \mathrm{T_{rx}} = \mathrm{T}_{\Sigma\mathrm{x,m}} = (\mathrm{T}_{\Sigma,\mathrm{m}})_{\mathrm{x}}, \quad \forall \mathrm{x} \in \mathrm{id}$

Which we have: $\mathrm{T_r} = \bigcup_{x \in \mathrm{id}} T_{rx}, \mathrm{T}_{\Sigma,\mathrm{m}} = \bigcup_{\mathrm{x} \in \mathrm{id}} T_{\Sigma x,\mathrm{m}} \Rightarrow \mathrm{T_r} = \mathrm{T}_{\Sigma,\mathrm{m}}$.

So r is m-tight representation set PTBDĐT Σ.

4 Conclusions

From the proposed new concept: multivalued positive Boolean dependence on block and slice, the article defined the truth block of the data block, prove the completeness of the set of functions $\{I, \wedge, \vee\}$. In addition, the article also proves the equivalent theorem for multivalued positive Boolean dependencies on block and slice. The necessary and sufficient condition for a block is m-tight representation ∑ … If id = {x} then the block degenerated into a relation and the results found on the block are still true on the relation.

We can further study the relationship between other types of logical dependencies on block and slice, extend the set of function dependencies on the block,... contribute to further complete design theory of the database model of block form.

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