Analysis of the Thermo-Mechanical Behavior of Composite Materials based on Plasticized Cassava Starch Reinforced with Coconut Fibers

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Abstract

Objective: In our previous work, we have developed and characterized a composite material based on cassava starch reinforced with coconut fibers. We will modelyse and simulate its thermo-mechanical behavior. **Methodology**: For this, we start from Hooke's law to establish a behavior model to evaluate the Young's modulus of each phase (matrix and fibers) as a function of temperature. This is then used in different homogenization models (Voigt and Mori-Tanaka) to approximate the thermo-elastic behavior of the composite. **Finding**: It is noted that the mechanical behavior of the material deteriorates when the temperature increases. **Applications**: The type of behavior obtained is compared with other previous general results on materials. The biodegradable composite obtained can be used in the field of plastics in the concept of sustainable development.

Keywords: Cassava Starch, Coconut Fibers, Composite Materials, Thermo-Mechanical Behavior, Young Modulus

1. Introduction

This manuscript is part of the work in which we produced our previous study¹. It was a question of elaborating and characterizing a composite material based on cassava starch reinforced by coconut fibers. Several tests were carried out. The results showed that the temperature has an effect on the structure of the composite. This affects its mechanical properties. In this study, we establish a method for predicting the elastic behavior of the material as a function of temperature. The composite being bi phasic (matrix + reinforcement), the volume fraction of each phase has an influence on the resistance to a mechanical load of the material. We will also simulate the effect of the variation of the volume contribution of each constituent on the behavior. A model of behavior conceived from the Hooke law makes it possible to determine the modulus of elasticity of each phase as a function of the temperature. The homogenization models of Mori-Tanaka² and Voigt³ will allow approximating the modulus of elasticity of the composite.

2. Law of Thermoelastic Behavior

Since the material is homogeneous isotropic¹, if it is submitted to both mechanical and thermal stress, the resulting stress is written:

$$\sigma = \sigma^{meca} + \sigma^{ther} \tag{1}$$

With σ^{meca} (GPa) the mechanical stress

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And σ^{ther} (GPa) the thermal stress.

On the other hand, Hooke's law⁴ gives the relation between the mechanical stress σ^{meca} (GPa) and the deformation ε in the domain of elastic behavior:

$$\sigma^{m \dot{\epsilon} c \alpha} = E \varepsilon \tag{2}$$

Where E (GPa) is the modulus of elasticity or Young's modulus of the material.

Taking into account (1) and (2), we have:

$$\sigma - \sigma^{ther} = E\varepsilon \tag{3}$$

From where:

$$\varepsilon = \frac{\sigma - \sigma^{ther}}{E} \tag{4}$$

By definition [4]:

$$\sigma^{ther} = E_0 \alpha_{ther} \Delta T \tag{5}$$

With
$$\Delta T = T - T_0$$

Where $\mathbf{T}_{0}(^{\circ}C)$ is the initial temperature (ordinary), T (°C) is the temperature at the current moment, \mathbf{E}_{0} (GPa) is the modulus of elasticity at temperature and \mathbf{T}_{0} (/°C) is the coefficient of thermal expansion of the material \mathbf{a}_{ther} .

In our previous publication, the tensile curve shows that the material adopts an elastic behavior as long as the stress is lower than a maximum value σ_{e} (GPa) (called resistance at the elastic limit or elastic limit *Re*) determined experimentally¹⁻⁵. The greatest stress that the material can withstand while remaining elastic is this value¹⁻⁴. At the limit, we must have:

$$\sigma = R_{pe} \tag{6}$$

With:

$$R_{pe} = \frac{R_e}{C_s}$$
(7)

Where $\mathbf{R}_{\mathbf{pe}}$ (GPa) is the practical resistance to extension and $\mathbf{C}_{\mathbf{s}}$ is the factor of security.

Taking into account (4), (5) and (6), it follows that:

$$\varepsilon = \frac{1}{C_s E} \left(R_e - C_s E_0 \alpha_{ther} \Delta T \right) \tag{8}$$

Where:

$$E = \frac{1}{C_s \varepsilon} (R_s - C_s E_0 \alpha_{ther} \Delta T)$$
⁽⁹⁾

Assuming the following initial conditions:

at
$$T_0: E = E_0$$

and
$$\varepsilon^{ther} = \alpha_{ther} (T_0 - T_0) = 0$$
 (No thermal

deformation)

We obtain :

$$\varepsilon = \frac{R_{\varepsilon}}{C_{\varepsilon}E_{0}} \tag{10}$$

Finally, (9) and (10) give:

$$E = E_0 \frac{R_e - C_s E_0 \,\alpha_{ther} \Delta T}{R_e} \tag{11}$$

Equation (11) allows determining the modulus of elasticity of each phase of the composite as a function of temperature.

3. Homogenization Models

3.1 Methodology

The purpose of the homogenization process is to predict the mechanical behavior of the multiphase material by using the information related to the microstructure. The composite is composed of a matrix, denoted by the index "M" and inclusions (reinforcements) denoted by the index "r". The expression of stiffness tensor C of the material is given by⁶:

$$C = \sum_{r=0}^{n} f^{r} C^{r} A^{r}$$
⁽¹²⁾

Where f^r is the volume fraction of the phase r, C^r is the average stiffness tensor of r, n is the number of subfamilies of inclusions and A^r is called the deformation localization tensor.

The knowledge A^r of is sufficient to determine the following approximations:

3.2Mori-Tanaka Model (MT)

This approximation considers a volume fraction F^r of inclusions of the same shape and orientation in the matrix. The volume fraction F^r of inclusions can be subdivided into n sub-families f^r with different constants of elasticity C^r as:

$$\sum_{r=1}^{n} f^r = F^r \tag{13}$$

The principle of approximation of the effective modulus of elasticity of the material by this model consists in considering, for each phase, an equivalent unique inclusion, in the supposed infinite matrix phase, by supposing that this inclusion is subjected infinitely to a field of homogeneous deformation equal to the average field in the matrix.

The tensors retained in this approximation are those established by Bourgeois²:

$$C_{MT} = C^{M} \left\{ I + \left(\sum_{r=1}^{n} f^{r} Q^{r} \right) \left[I + \sum_{r=1}^{n} f^{r} (S^{Esh} - I) Q^{r} \right]^{-1} \right\}^{-1}$$
(14)

With

$$Q^{r} = [(C^{M} - C^{r})S^{Esh} - C^{M}]^{-1}(C^{r} - C^{M})$$
(15)

unit of order four.

The expressions of the different components of the Eshelby tensor are listed in several references, including the book of Mura⁸.

3.3 Voigt Model (VT)

The Voigt approximation, first of all, assumes that the deformation in each phase is identical to the applied macroscopic deformation, so homogeneous³. This is an approach that leads to a framing of the tensor of effective stress C_{ur} of the composite. We maintain the previous

"M" and "r" indexes for matrix and reinforcement.

If the composite is subjected to a deformation of components E_{ii} , we will have:

$$E_{ij} = \frac{1}{V} \int_{V}^{c} \varepsilon_{ij}(x) dv \qquad (16)$$

Where:

$$V = V^M + V^r \tag{17}$$

With V (m³) the volume of the composite, V^M the volume of the matrix, V^r the volume of the rienforcement et $e_{ii}(x)$ the microscopic deformation at the point x.

Similarly, from the application of E_{ij} results a stress $\sum_{ij} u$ which is the average volume of the local stress $\sigma_{ij}(x)$ in the material:

$$\sum_{ij} u = \frac{1}{V} \int_{V}^{r} \sigma_{ij}(x) dv$$
⁽¹⁸⁾

The generalized Hooke law gives:

$$\sum_{ij} u = C_{ijkl} \varepsilon_{kl} \tag{19}$$

With C_{ijkl} the homogenized coefficients of the modulus of elasticity which remains to be determined.

By applying the law of mixtures to (16) and (18), it follows that:

$$E_{ij} = \frac{V^r}{V} \varepsilon_{ij}^r + \frac{V^M}{V} \varepsilon_{ij}^M \tag{20}$$

$$\sum_{ij} u = \frac{V^r}{V} \sigma_{ij}^r + \frac{V^M}{V} \sigma_{ij}^M \tag{21}$$

By defining the volume fraction of the reinforcement in the composite by the ratio:

$$f^r = \frac{V^r}{V} \tag{22}$$

Equations (20) and (21) become:

$$E_{ij} = f^r \varepsilon_{ij}^r + (1 - f^r) \varepsilon_{ij}^M$$
(23)

$$\sum_{ij} y = f^r \sigma_{ij}^r + (1 - f^r) \sigma_{ij}^M$$
(24)

The tensor A^r of components A^r_{ijkl} \wedge defined in Equation (12) allows writing that:

$$\varepsilon_{ij}^r = A_{ijkl}^r E_{kl} \tag{25}$$

The law of behavior at the local level is:

$$\sigma_{ij}^{r} = C_{ijkl}^{r} \varepsilon_{kl}^{r}$$
⁽²⁶⁾

Equations (25) and (26) give:

$$\sigma_{ij}^{r} = C_{ijkl}^{r} A_{klmn}^{r} E_{mn}$$
⁽²⁷⁾

The constitutive law of the matrix is:

$$\sigma_{ij}^{M} = C_{ijkl}^{M} \varepsilon_{kl}^{M}$$
(28)

Taking into account Equations (23) and (25), we obtain:

$$\varepsilon_{ij}^{M} = \frac{1}{\left(1 - f^{r}\right)} \left(E_{ij} - f^{r} A_{ijmn}^{r} E_{mn} \right) \qquad (29)$$

From Equations (28) and (29), it follows that:

$$\sigma_{ij}^{M} = \frac{1}{(1-f^{r})} C_{ijkl}^{M} (I - f^{r} A_{ijmn}^{r}) E_{mn}$$
(30)

With I the tensor unit of order four.

We therefore note that from Equations (24), (27) and (30) and after simplification, we arrive at:

$$\sum_{ij} u = \left[C_{ijmn}^{M} + f^{r} (C_{ijkl}^{r} - C_{ijkl}^{M}) A_{klmn}^{r} \right] E_{mn}$$

$$(31)$$

By identifying Equations (19) and (23), we find the expression of C_{iikl} :

$$C_{ijkl} = C_{ijmn}^{M} + f^{r} (C_{ijkl}^{r} - C_{ijkl}^{M}) A_{klmn}^{r}$$

$$(32)$$

We obtain the model of Voigt with the hypothesis:

$$\begin{bmatrix} A_{ijkl}^r \end{bmatrix} = I \tag{33}$$

In Equation (32):

$$C_{ijkl}^{VT} = C_{ijmn}^{M} + f^r (C_{ijkl}^r - C_{ijkl}^M)$$
(34)

or

$$C_{VT} = C^M + f^r (C^r - C^M) \tag{35}$$

4. Simulations

For simulations, the elastic behaviors of the two phases

Table 1.Matrix and fibers properties

are determined from the established constitutive law (11). The Young modules obtained are then introduced into each model (14) and (15) and (35) as appropriate to approximate the effective modulus of the composite. This allows us to predict the elastic behavior of the material. This prediction will be done separately according to the temperature and the volume fraction of fibers. During the work of elaboration and characterization¹, we determined the properties of plasticized starch (without reinforcement) and fibers necessary for simulation. The following table summarizes the properties of the matrix and the fibers.

The factor of security is taken equal to: $C_s = \frac{2}{3}$ and for the rest, we suppose $f^r = f$.

5. Results and Discussion

5.1 Results

We chose to present the results in the form of graphs to show the evolution of mechanical behavior as a function of temperature and volume fraction. The values of the properties of the different phases summarized in the Table 1 are used in the different models to plot the graphs. Before evaluating the mechanical characteristics of the composite, we will give a representation of each phase using the established mechanical behavior model (11). The application of the model gave us the Figure 1 for the

Properties	Plasticized starch (without reinforcement)	Coconut fibers
Modulus of elasticity (25°C), E (GPa)	0.037	4,500
Resistance to the limit of elasticity, (GPa)	0.0021	0.175
Coefficient of thermal expansion, (°)	137x10 ⁻⁶	4.7x10 ⁻⁶
Poisson coefficient	0.51	



Figure 1. Variation of modulus of elasticity of plasticized starch as a function of temperature.



Figure 2. Variation of modulus of elasticity of coconut fibers as a function of temperature.

plasticized starch without reinforcement (matrix) and the Figure 2 (for the coconut fibers (reinforcements)).

Then, we use the Mori-Tanaka and Voigt models to evaluate the Young's modulus of the composite (matrix



Figure 3. Variation of Mori-Tanaka modulus of elasticity of the composite as a function of temperature.



Figure 4. Variation of Voigt modulus of elasticity of the composite as a function of temperature.

+ reinforcements) as a function of temperature at fixed volume fractions. This is illustrated by the Figure 3.

This makes it possible to predict the elastic behavior as a function of the temperature Figure 4. For both mod-



Figure 5. Variation of Mori-Tanaka modulus of elasticity of the composite as a function of volume fraction.



Figure 6. Variation of Voigt modulus of elasticity of the composite as a function of volume fraction.

els, we also determined the modulus of elasticity of the composite as a function of the volume fraction for given

temperature values. The Figure 5 and the Figure 6 show, in this case, the evolution of the mechanical behav-

ior of the composite. This makes it possible to evaluate the mechanical behavior as a function of the volume fraction.

5.2 Discussion

All figures show that the modulus of elasticity decreases with increasing temperature. This is consistent with findings found in the work of other authors^{4–10}. They specify that the Young's modulus of a material decreases with temperature. Indeed, any input of energy of thermal origin causes the agitation of the atoms causing their distance from each other: The interstitial forces diminish. This means that a rise in temperature leads to a decrease in the mechanical rigidity of the material. The consequence is the decline of Young's modulus^{1–11}. In addition, it can be seen that this variation as a function of temperature is linear in the zone of elastic behavior. This is consistent with the results and Al¹². This showed that the variation of the modulus of elasticity is generally linear in the elastic zone of the material.

The graphs also show that the elastic behavior changes with the volume fraction. Our previous study¹ in which, we have shown that the fibers supply improves the mechanical behavior of the composite confirms this. However, while the Young's modulus of Voigt varies linearly, that of Mori-Tanaka evolves in a parabolic way. This is because the Mori-Tanaka model is polynomial of degree 2 in f (volume fraction). So we can say that as the amount of fiber increases, the mechanical behavior of the composite improves. For example, for the temperature of 25°C, the modulus of elasticity in GPA is approximated to 0.42, 0.85 and 4.35 respectively for the volume fractions of 0.1, 0.5 and 0.9.

6 Conclusion

In this study, from a model, we determined the elastic behavior of a material as a function of temperature by evaluating the Young's modulus of reinforcements, matrix and composite. For all materials (fibers, plasticized starch without reinforcements, the composite) it was found that the modulus of elasticity decreases with temperature. This allows us to say that any rise in temperature leads to degradation of the mechanical properties of the material. The modulus of elasticity also increases with the volume fraction. It can be seen that its value tends towards a limit equal to that of the fibers. We can remember that the contribution of fibers improves the behavior of the material with the resistance to the mechanical load, whereas any elevation of the temperature degrades the properties.

7. References

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