Intuitionistic Fuzzy Soft Γ-Semigroup

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Abstract

Objectives: This study deals with the algebraic structure intuitionistic fuzzy soft (IFS) Γ -semi group is defined that is a natural extension of IFS semi group and fuzzy soft Γ -semi groups. The notion of IFS Γ -ideals over Γ -semi groups are introduced. Also, the basic concepts of the ideals are studied. **Methods:** Using approaches given in IFS semi groups to enlarge fuzzy soft Γ -semi groups offer the required algebraic structure. **Findings:** The ideal F Ssoft Γ -semi group is the natural consequence of viewing collectively the IFSsemigroup and fuzzy soft Γ -semi groups. **Applications:** Fuzzy soft set theory, IF set theory, etcetera are well celebrated mathematical concepts handling the uncertainties in the fields of engineering, economy, social science.

Keywords: Fuzzy Set, Intuitionistic Fuzzy Soft Set, Soft Set, Γ-Semi Group

1. Introduction

Fuzzy sets have important applications in control theory, data analysis, artificial intelligence, computational intelligence, decision theory, medicine, logic, management science, and expert systems, see¹. Fuzzy sets were even generalized and these new structures are called soft set. It was a new approach to counter the uncertainty in the modeling problems.

The notion of fuzzy subset was initiatedin² in 1965, also see^{3.4}. The concept of a Γ -semi group was presented in⁵⁻⁸ to generalize the notion of semi group. In 1986⁹ introduced the IF subset as generalization of fuzzy subsets, see also²⁻¹³. Soft set was familiarizedin¹⁴ in 1999. Fuzzy soft set and IFSset were introduced by¹² in 2001 gave IFS semi groups in¹³ 2014 defined Fuzzy soft Γ -semi groups, see¹⁵. This work is the extension of the structures. Here we revise some important algebraic structures that are necessary for our paper.

2. Preliminaries

2.1 Γ-semigroup⁶

A nonempty set G is said to be semi group if it is closed with respect to an associative binary operation. A subset $N \subseteq G$ is known as a left ideal in G when $GN \subseteq N$, provided N is non-empty. Similarly, N is said to be right ideal when $NG \subseteq N$. If a subset of G is left in addition to right ideal then it is called ideal in G. **Definition:** Suppose that G and Γ are non-empty sets. We write the image of (x, ψ, γ) under the function

$$\mathbf{G} \times \mathbf{\Gamma} \times \mathbf{G} \to \mathbf{G}$$

as $x\psi y$, then G is said to be Γ -semi group if following holds.

$$(x\psi\gamma)\rho z = p\psi(q\rho z),$$

$$\forall p,q,z \in G \text{ and } \psi, \rho \in \Gamma.$$

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Example: Suppose \mathbb{Z} is the set of integers. Denoted by $M_2(\mathbb{Z}) = G$ the set of all 2×2 matrices over \mathbb{Z} . The $M_2(\mathbb{Z})$ is a Γ -semigroup, Γ where is the following set

$$I_2 = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}.$$

The law of composition is defined as $DI_2E = DE$, for all $D, E \in M_2(\mathbb{Z})$ and $I_2 \in \Gamma$.

Definition: A subset N of a Γ -semi group G is known as a left Γ -ideal in G when $G\Gamma N \subseteq N$, provided N is non-empty. Similarly, N is said to be right Γ -ideal when $N\Gamma G \subseteq N$. If a subset of G is left as well as right Γ -ideal then it is called Γ -ideal in Γ -semi group.

Example: Consider the above example. Let

$$\tilde{L} = \left\{ \begin{bmatrix} w & 0 \\ t & 0 \end{bmatrix} : w, t \in \mathbb{Z} \right\}.$$

and

$$\tilde{R} = \left\{ \begin{bmatrix} w & t \\ 0 & 0 \end{bmatrix} : w, t \in \mathbb{Z} \right\}.$$

Here \tilde{R} is a right and \tilde{L} is a left ideal of $M_2(\mathbb{Z})$, respectively.

2.2 Soft Set^{12,14}

Definition: Suppose we have a universe given by the symbol \tilde{U} and the set of parameters E. The $P(\tilde{U})$ represents the collection of the subsets of \tilde{U} . Also suppose A is contained in E. Then we call (F, A) a soft set over \tilde{U} , here F is a function $F: A \rightarrow P(\tilde{U})$.

Definition: Suppose we have soft sets (J, C) and (K, D) over \tilde{U} .

1) The \land operation is (J, C) \land (K, D) = (H, C \times D) here

$$H(\alpha,\beta) = J(\alpha) \cap K(\beta) \ \forall \ (\alpha,\beta) \in C \times D$$

2) The \lor operation is (J, C) \lor (K, D) = (H, C \times D) here

$$H(\gamma, \theta) = J(\gamma) \cup K(\theta) \,\forall (\gamma, \theta) \in C \times D^{-1}$$

3) The union is $(J,C) \cup (K,D) = (\mathfrak{U},L)$,

where $L = C \cup D$ and for every $n \in L$,

$$\mathfrak{U}(i) = \begin{cases} J(i) & i \in C \setminus D \\ K(i) & i \in D \setminus C \\ J(i) \cup K(i) & i \in C \cap D. \end{cases}$$

4) The intersection is the soft set (*H*,*L*)

Where $L = C \cap D$ an or K(n) (as both are same set).

Definition: We call soft set (J,C) over a semi group G a soft ideal (respectively, left or right soft ideal) over G, if $J(\alpha)$ is an ideal (respectively, left or right ideal) of $G \forall \alpha \in C$ provided $J(\alpha) \neq \phi$.

Definition: We call a soft set (*J*, *C*) on a Γ-semi group *G* a soft Γ-ideal (respectively, left or right Γ-ideal) over *G*, if $J(\alpha)$ is an Γ-ideal (respectively, left or right Γ-ideal) of *G* for all $\alpha \in C$ with $(\alpha) \neq \phi$.

2.3 Fuzzy soft Γ-ideal over Γ-semigroup15

Definition: A fuzzy soft set (J, A) on a Γ -semi group G is fuzzy soft Γ -ideal over G if

$$J_{a}(p\zeta q) \ge \max\{J_{a}(p), J_{a}(q)\}$$
$$\forall a \in A, p, q \in G \text{ and } \zeta \in \Gamma$$

2.4 IFS semigroup13

IF set generalizes fuzzy set. In fuzzy set addition of values of inclusion and exclusion equals to one, but in intuitionistic Fuzzy Set the addition is always less than or equal to 1. It is defined as follows:

Definition: Suppose we have a non-empty set G. An IF Set in G iswritten as

$$A = \{ (p, \rho_A(p), \sigma_A(p)) \colon p \in G \}.$$

the functions $\rho_A(p), \sigma_A(p): G \to [0,1]$ are the inclusion and exclusion mappings of the element in A respectively. They satisfy $0 \ge \rho_A(p) + \sigma_A(p) \le 1$

Definition: We call (\tilde{f}, X) as IFS set on a universe \tilde{U} , here \tilde{f} is a function

$$\tilde{f}: X \to \mathfrak{IF}(\tilde{U})$$

where \tilde{U} be a universal and $\Im \mathfrak{F}(\tilde{U})$ be the collection of IFSsets of \tilde{U} .

Definitions: Suppose we are given (\tilde{f}, S) and (\tilde{g}, T) given IFS sets over a universe \tilde{U} s. t. $A \cap B \neq \phi$

(1) The $\tilde{\lor}$ operation is

$$(\tilde{f}, S) \tilde{\vee} (\tilde{g}, T) = (\tilde{H}, S \times T),$$

where

$$\tilde{H}(\gamma,\theta) = \tilde{f}(\gamma) \cup \tilde{g}(\theta) \,\,\forall \,\,(\gamma,\theta) \in \mathbf{S} \times \mathbf{T}.$$

(2) The union is

$$(\tilde{H},C) = (\tilde{f},S)\tilde{\cup}(\tilde{g},T)$$

here $C = S \cup T$ and \tilde{H} is given as

$$\tilde{H}(\gamma) = \begin{cases} \tilde{f}(\gamma) & \text{if } \gamma \in S \setminus T \\ \tilde{g}(\gamma) & \text{if } \gamma \in T \setminus S \\ \max\{\tilde{f}(\gamma), \tilde{g}(\gamma)\} & \text{if } \gamma \in S \cap T \end{cases}$$

 $\forall \gamma \in C.$

(3) Their bi-union is

$$(\tilde{H},C) = (\tilde{f},S)\tilde{\coprod}(\tilde{g},T)$$

the $C = A \cap B$ and

$$\tilde{H}(\gamma) = \tilde{f}(\gamma) \cup \tilde{g}(\gamma) \forall \ \gamma \in \mathbf{C}.$$

(4) The $\tilde{\wedge}$ operation is

$$(\tilde{f},S) \tilde{\wedge} (\tilde{g},T) = (\tilde{H},S \times T),$$

where

$$\tilde{H}(\gamma,\theta) = \tilde{f}(\gamma) \cap \tilde{g}(\theta) \,\,\forall \,\,(\gamma,\theta) \in \mathbf{S} \times \mathbf{T}.$$

(5) The intersection is

$$(\tilde{H},C) = (\tilde{f},S) \tilde{\cap} (\tilde{g},T)$$

 $C = S \cup T$ and *H* is given as below.

$$\tilde{H}(\gamma) = \begin{cases} \tilde{f}(\gamma) & \text{if } \gamma \in S \setminus T \\ \tilde{g}(\gamma) & \text{if } \gamma \in T \setminus S \\ \min\{\tilde{f}(\gamma), \tilde{g}(\gamma)\} & \text{if } \gamma \in S \cap T \end{cases}$$

 $\forall \gamma \in C.$

(6) Their bi-intersection is

$$(\tilde{H}, C) = (\tilde{f}, S)\tilde{\Pi}(\tilde{g}, T).$$

the $C=A\cap B\;$ and

$$\tilde{H}(\gamma) = \tilde{f}(\gamma) \cap \tilde{g}(\gamma) \forall \ \gamma \in \mathbf{C}.$$

If the universe \tilde{U} is semi group G then we can define the ideal as follows.

Definition: IFS set (\tilde{f}, S) on a semi group *G* is IFS ideal over *G* if the following inequalities are true:

$$\rho_{\tilde{f}(\alpha)}(st) \ge \max\{\rho_{\tilde{f}(\alpha)}(s), \rho_{\tilde{f}(\alpha)}(t)\}$$

and

$$\lambda_{\tilde{f}(\alpha)}(st) \le \min\{\lambda_{\tilde{f}(\alpha)}(s), \lambda_{\tilde{f}(\alpha)}(t)\}$$

for all s, $t \in G$ and $\alpha \in S$.

Here we give two theorems from. The proof of the first theorem is also given, because it is the motivation for the proof of the same theorems in the case of IFS Γ -semi group.

Theorem: Suppose G is semi group and (\tilde{f},S) and (\tilde{g},T) , are IFS ideals over G. Then the intersection $(\tilde{f},S) \wedge (\tilde{g},T)$ and bi-intersection $(\tilde{f},S)\Pi(\tilde{g},T)$ are also IFS ideals over G.

Proof: We can write $(\tilde{f}, S) \wedge (\tilde{g}, T) = (\tilde{I}, P)$, with $P = S \times T$ and $I(\gamma, \theta) = f(\gamma) \cap g(\theta)$ for all $(\gamma, \theta) \in P$. Since (\tilde{f}, S) and (\tilde{g}, T) are IFS ideals over G, it follows that for any $(\gamma, \theta) \in P$, we have

$$\rho_{\tilde{I}(\gamma,\theta)}(st) = \min\{\rho_{\tilde{g}(\gamma)}(st), \rho_{\tilde{g}(\theta)}(st)\}$$

$$\geq \min\{\max\{\rho_{\tilde{f}(\gamma)}(s), \rho_{\tilde{f}(\gamma)}(t)\}, \max\{\rho_{\tilde{g}(\theta)}(s), \rho_{\tilde{g}(\theta)}(t)\}\}$$

$$= \max\{\min\{\rho_{\tilde{f}(\gamma)}(s), \rho_{\tilde{g}(\theta)}(s)\}, \min\{\rho_{\tilde{f}(\gamma)}(t), \rho_{\tilde{g}(\theta)}(t)\}\}$$

$$= \max\{\rho_{\tilde{I}(\gamma,\theta)}(s), \rho_{\tilde{I}(\gamma,\theta)}(t)\}$$

and

$$\begin{split} \lambda_{\tilde{l}(\gamma,\theta)}(st) &= \max\{\lambda_{\tilde{g}(\gamma)}(st), \lambda \tilde{g}(\theta)(st)\} \\ &\leq \max\{\min\{\lambda_{\tilde{f}(\gamma)}(s), \lambda_{\tilde{f}(\gamma)}(t)\}, \qquad \min\{\lambda_{\tilde{g}(\theta)}(s), \lambda_{\tilde{g}(\theta)}(t)\}\} \\ &\leq \min\{\max\{\lambda_{\tilde{f}(\gamma)}(s), \lambda_{\tilde{g}(\theta)}(s)\}, \max\{\lambda_{\tilde{f}(\gamma)}(t), \lambda_{\tilde{g}(\theta)}(t)\}\} \\ &= \min\{\lambda_{\tilde{l}(\gamma,\theta)}(s), \lambda_{\tilde{l}(\gamma,\theta)}(t)\} \end{split}$$

for all s, $t \in P$. This proves that the intersection $(\tilde{f}, S) \wedge (\tilde{g}, T)$ is an IFS ideal over G.

Similarly, the other claim for bi-intersection $(\tilde{f}, S)\Pi(\tilde{g}, T)$ can be proved

Theorem: Suppose G is semi group and (\tilde{g},T) , and (\tilde{f},S) are IFS ideals over G, then their union $(\tilde{f},S)\tilde{\vee}(\tilde{g},T)$ and bi-union $(\tilde{f},S)\tilde{\amalg}(\tilde{g},T)$ are also IFS ideals over G.

Proof: See <u>15</u>.

3. IFS Г-Semi group

This section is dealing with the IFS Γ -ideal over Γ -semi group. The concept of IFSset over semi group and fuzzy soft set over Γ -semi group is given in the section of preliminaries. The proofs of the theorem related to IFS Γ -ideals over Γ -semi group are proved in this section.

Definition: An IFS set (\tilde{f}, S) over a Γ -semi group G is called an IFS Γ -ideal over G if it satisfies:

$$\rho_{\tilde{f}(a)}(p\zeta q) \ge \max\{\rho_{\tilde{f}(a)}(p), \rho_{\tilde{f}(a)}(q)\}$$
 and

$$\lambda_{\tilde{f}(a)}(p\zeta q) \leq \min\{\lambda_{\tilde{f}(a)}(p), \lambda_{\tilde{f}(a)}(q)\},\$$

for every $p, q \in G$, $a \in A$ and $\zeta \in \Gamma$. Here note that \tilde{f} is the function given by

 $\tilde{f}: A \to \mathfrak{IF}(S),$

where $\Im \mathfrak{F}(S)$ is the set of all IF subset of G and $\rho_{\tilde{f}(a)}$ and $\lambda_{\tilde{f}(a)}$ are the inclusion and exclusion mappings, correspondingly.

Definition: Suppose we are given (\tilde{f},S) IFS set over a Γ -semigroup G, then (\tilde{f},S) is termed as an IFS Γ -subsemigroup over G if it satisfies:

$$\rho_{\tilde{f}(a)}(p\zeta q) \ge \min\{\rho_{\tilde{f}(a)}(p), \rho_{\tilde{f}(a)}(q)\}$$

and

$$\lambda_{\tilde{f}(a)}(p\zeta q) \leq \max\{\lambda_{\tilde{f}(a)}(p), \lambda_{\tilde{f}(a)}(q)\},\$$

for every $p,q \in G$, $a \in A$, and $\zeta \in \Gamma$

Definition: Suppose we are given two (\tilde{g}, Y) and (\tilde{f}, X) IFS sets over a Γ -semigroup. Their product $(\tilde{f} \Gamma \tilde{g}, Z)$, here $Z = X \cup Y$ and

$$\rho_{\tilde{f}(s)}(p) \qquad \qquad if \quad s \in X \setminus Y$$

$$\rho_{(\tilde{f}\Gamma\tilde{g})(a)}(p) = \begin{cases} \rho_{\tilde{g}(s)}(p) & \text{if } s \in Y \setminus X \\ \sup_{p=m\zeta n} \min\{\rho_{\tilde{f}(s)}(m), \rho_{\tilde{g}(s)}(n)\} & \text{if } s \in X \cap Y \end{cases}$$

and

$$\lambda_{(\tilde{f} \cap \tilde{g})(s)}(p) = \begin{cases} \lambda_{\tilde{f}(s)}(p) & \text{if } s \in X \setminus Y \\ \lambda_{\tilde{g}(s)}(p) & \text{if } s \in Y \setminus X \\ \inf_{p = m\zeta_n} \max\{\lambda_{\tilde{f}(s)}(m), \lambda_{\tilde{g}(s)}(n)\} & \text{if } s \in X \cap Y \end{cases}$$

$$\forall s \in \mathbb{Z}, p \in G$$
, and $\zeta \in \Gamma$. Note that

$$(\tilde{f}\Gamma\tilde{g},Z) = (\tilde{f},X)\Gamma(\tilde{g},Y).$$

Theorem: If (\tilde{g},T) and (\tilde{f},S) are two IFS Γ -ideals on a Γ -semi group G, then AND operation $(\tilde{f},S) \wedge (\tilde{g},T)$ as well as their bi-intersection $(\tilde{f},S) \Pi(\tilde{g},T)$ are also fuzzy soft ideals over G.

Proof: Write $(\tilde{f},S) \wedge (\tilde{g},T) = (\tilde{H},C)$, here $C = S \times T$ and $\tilde{H}(\gamma,\theta) = \tilde{f}(\gamma) \cap \tilde{g}(\theta) \quad \forall (\gamma,\theta) \in C$. Since (\tilde{f},S) and (\tilde{g},T) are IFS Γ -ideals over a Γ -semi group G, it follows for any $(\gamma,\theta) \in C$, we have

$$\rho_{\tilde{H}(\gamma,\theta)}(p\zeta q) = \min\{\rho_{\tilde{f}(\gamma)}(p\zeta q), \rho_{\tilde{g}(\theta)}(p\zeta q)\}$$

$$\geq \min\{\max\{\rho_{\tilde{f}(\gamma)}(p), \rho_{\tilde{f}(\gamma)}(q)\}, \max\{\rho_{\tilde{g}(\theta)}(p), \rho_{\tilde{g}(\theta)}(q)\}\}$$

$$= \max\{\min\{\rho_{\tilde{f}(\gamma)}(p), \rho_{\tilde{g}(\theta)}(p)\}, \min\{\rho_{\tilde{f}(\gamma)}(q), \rho_{\tilde{g}(\theta)}(q)\}\}$$

$$= \max\{\rho_{\tilde{H}(\gamma,\theta)}(p), \rho_{\tilde{H}(\gamma,\theta)}(q)\}$$

and

$$\begin{split} \lambda_{\tilde{H}(\gamma,\theta)}(p\zeta q) &= max\{\lambda_{\tilde{f}(\gamma)}(p\zeta q),\lambda\tilde{g}(\theta)(pq)\}\\ &\leq \max\{\min\{\lambda_{\tilde{f}(\gamma)}(p),\lambda_{\tilde{f}(\gamma)}(q)\},\min\{\lambda_{\tilde{g}(\theta)}(p),\lambda_{\tilde{g}(\theta)}(q)\}\}\\ &\leq \min\{\max\{\lambda_{\tilde{f}(\gamma)}(p),\lambda_{\tilde{g}(\theta)}(p)\},max\{\lambda_{\tilde{f}(\gamma)}(q),\lambda_{\tilde{g}(\theta)}(q)\}\}\\ &= \min\{\lambda_{\tilde{H}(\gamma,\theta)}(p),\lambda_{\tilde{H}(\gamma,\theta)}(q)\} \end{split}$$

 $\forall p,q \in C$. This proves that AND operation $(\tilde{f},S) \wedge (\tilde{g},T)$ is an IFS Γ -ideal on a Γ -semigroup G.

The other claim about bi-intersection $(\tilde{f},S) \wedge (\tilde{g},T)$ can also be proved in similar fashion.

Theorem: If (\tilde{f}, S) and (\tilde{g}, T) are IFS-ideals on a Γ -semi group G, then their OR operation $(\tilde{f}, S)\tilde{\vee}(\tilde{g}, T)$ and bi-union $(\tilde{f}, S)\tilde{\amalg}(\tilde{g}, T)$ are also fuzzy soft Γ -ideals over a Γ -semigroup G.

Proof: Write $(\tilde{f}, S) \lor (\tilde{g}, T) = (\tilde{O}, C)$, the $C = S \times T$ and $\tilde{O}(\gamma, \theta) = \tilde{f}(\gamma) \cup \tilde{g}(\theta) \forall (\gamma, \theta) \in C$. Since (\tilde{f}, S) and (\tilde{g}, T) have some properties given above, it implies that $\forall p, q \in G$ and $(\gamma, \theta) \in C$ we can write

$$\begin{aligned} \rho_{\tilde{\rho}(\gamma,\theta)}(p\zeta q) &= (\rho_{\tilde{f}(\gamma)} \cup \rho_{\tilde{g}(\theta)})(p\zeta q) \\ &= max\{\rho_{\tilde{f}(\gamma)}(p\zeta q), \rho_{\tilde{g}(\theta)}(p\zeta q)\} \\ &\geq \max\{max\{\rho_{\tilde{f}(\gamma)}(p), \rho_{\tilde{f}(\gamma)}(q)\}, max\{\rho_{\tilde{g}(\theta)}(p), \rho_{\tilde{g}(\theta)}(q)\}\} \\ &\geq \max\{max\{\rho_{\tilde{f}(\gamma)}(p), \rho_{\tilde{f}(\gamma)}(q)\}, max\{\rho_{\tilde{g}(\theta)}(p), \rho_{\tilde{g}(\theta)}(q)\}\} \\ &= \max\{(\rho_{\tilde{o}(\gamma,\theta)}(p), \rho_{\tilde{o}(\gamma,\theta)}(q))\} \end{aligned}$$

and

$$\begin{aligned} \lambda_{\tilde{o}(\gamma,\theta)}(p\zeta q) &= (\lambda_{\tilde{f}(\gamma)} \cap \lambda_{\tilde{g}(\theta)})(p\zeta q) \\ &= \min\{\lambda_{\tilde{f}(\gamma)}(p\zeta q), \lambda_{\tilde{g}(\theta)}(p\zeta q)\} \\ &\leq \min\{\min\{\lambda_{\tilde{f}(\gamma)}(p), \lambda_{\tilde{f}(\gamma)}(q)\}, \min\{\lambda_{\tilde{g}(\theta)}(p), \lambda_{\tilde{g}(\theta)}(q)\}\} \end{aligned}$$

$$= \min\{(\lambda_{\tilde{f}(\gamma)} \cap \lambda_{\tilde{g}(\theta)})(p), (\lambda_{\tilde{f}(\gamma)} \cap \lambda_{\tilde{g}(\theta)})(q))\}$$
$$= \min\{(\lambda_{\tilde{O}(\gamma,\theta)}(p), \lambda_{\tilde{O}(\gamma,\theta)}(q))\}$$

It gives required result that the OR operation $(\tilde{f}, S) \tilde{\vee} (\tilde{g}, T)$ is an IFS Γ -ideal over a Γ -semigroupG. The claim for bi-union $(\tilde{f}, S) \tilde{\amalg} (\tilde{g}, T)$ can also be proved in the similar manner.

Theorem: If (\tilde{g},T) and (\tilde{f},S) are IFS Γ-ideals on a Γ-semigroup G, the intersection $(\tilde{f},S) \cap (\tilde{g},T)$ is also an IFS Γ-ideal on Γ-semigroup G.

Proof: By definition intersection, $(\tilde{f}, S) \cap (\tilde{g}, T) = (\tilde{H}, C)$, here $C = A \cup B$ and

$$\tilde{H}(\gamma) = \begin{cases} \tilde{f}(\gamma) & \text{if} \quad \gamma \in A \setminus B \\ \tilde{g}(\gamma) & \text{if} \quad \gamma \in B \setminus A \\ \tilde{f}(\gamma) \cap \tilde{g}(\gamma) & \text{if} \quad \gamma \in A \cap B. \end{cases}$$

 $\forall \ \gamma \in C$. Now $\forall \ \gamma \in C$. and $p,q \in G$, the following situations are considered.

Case 01: If $\gamma \in A \setminus B$. Then

$$\rho_{\tilde{H}(\gamma)}(p\zeta q) = \rho_{\tilde{f}(\gamma)}(pq)$$

$$\geq max\{\rho_{\tilde{f}(\gamma)}(p), \rho_{\tilde{f}(\gamma)}(q)\}$$

$$= max\{\rho_{\tilde{H}(\gamma)}(p), \rho_{\tilde{H}(\gamma)}(q)\}$$

and

$$\begin{split} \lambda_{\tilde{H}(\gamma)}(p\zeta q) &= \lambda_{\tilde{f}(\gamma)}(p\zeta q) \\ &\leq \min\{\lambda_{\tilde{f}(\gamma)}(p),\lambda_{\tilde{f}(\gamma)}(q)\} \\ &= \min\{\lambda_{\tilde{H}(\gamma)}(p),\lambda_{\tilde{H}(\gamma)}(q)\}. \end{split}$$

Case 02: If $\gamma \in B - A$, then in the similar manner as in the proof of case 01, we find

$$\rho_{\tilde{H}(\gamma)}(p\zeta q) \ge \max\{\rho_{\tilde{H}(\gamma)}(p), \rho_{\tilde{H}(\gamma)}(q)\}$$

and

$$\lambda_{\tilde{H}(\gamma)}(pq) \leq \min\{\lambda_{\tilde{H}(\gamma)}(p), \lambda_{\tilde{H}(\gamma)}(q)\}$$

Case 03: If $\gamma \in A \cap B$, then suppose $\tilde{H}(\gamma) = \tilde{f}(\gamma) \cap \tilde{g}(\gamma)$. Therefore, we can write

$$\rho_{\tilde{H}(\gamma)}(p\zeta q) \ge \max\{\rho_{\tilde{H}(\gamma)}(p), \rho_{\tilde{H}(\gamma)}(q)\}$$

and

$$\lambda_{\tilde{H}(\gamma)}(pq) \leq \min\{\lambda_{\tilde{H}(\gamma)}(p), \lambda_{\tilde{H}(\gamma)}(q)\}$$

Therefore, in any case we have

$$\rho_{\tilde{H}(\gamma)}(pq) \geq \max\{\rho_{\tilde{H}(\gamma)}(p), \rho_{\tilde{H}(\gamma)}(q)\};$$

and

$$\lambda_{\tilde{H}(\gamma)}(pq) \le \min\{\lambda_{\tilde{H}(\gamma)}(p), \lambda_{\tilde{H}(\gamma)}(q)\}$$

therefore, the intersection $(f,S) \cap (\tilde{g},T)$ is also IFS Γ -ideal over G.

Theorem: If (\tilde{g},T) and (\tilde{f},S) two IFS Γ-ideals over a Γ- semi group *G*, then the union $(\tilde{f},S) \tilde{\cup} (\tilde{g},T)$ is an IFS Γ-ideal over *G*.

Proof: This can be proved in the similar manner as in above theorem.

Theorem: If (\tilde{g},T) and (\tilde{f},S) are two IFS Γ -ideals on a Γ -semi group G, then the product $(\tilde{f},S)\Gamma(\tilde{g},T)$ is an IFS Γ -ideal over G.

Proof: If (\tilde{f},S) and (\tilde{g},T) are IFS Γ -ideals over a semi group *G*. Then $\forall \gamma \in A \cup B$ and $p,q \in G, \zeta \in \Gamma$ following cases are considered.

Case 01: If $\gamma \in A - B$,

$$\begin{split} \rho_{(\tilde{f} \Gamma \tilde{g})(\gamma)}(p \zeta q) &= \rho_{(\tilde{f})(\gamma)}(p \zeta q) \\ &\geq max\{\rho_{(\tilde{f})(\gamma)}(p), \rho_{(\tilde{g})(\gamma)}(q)\} \\ &= max\{\rho_{(\tilde{f} \Gamma \tilde{g})(\gamma)}(p), \rho_{(\tilde{f} \Gamma \tilde{g})(\gamma)}(q)\}. \end{split}$$

Case 02: If $\gamma \in B - A$, similar to case 01,

$$\rho_{(\tilde{f}\Gamma\tilde{g})(\gamma)}(p\zeta q) \ge \max\{\rho_{(\tilde{f}\Gamma\tilde{g})(\gamma)}(p), \rho_{(\tilde{f}\Gamma\tilde{g})(\gamma)}(q)\}.$$

Case 03: If
$$\in A \cap B$$
, then

$$\begin{split} \rho_{(\tilde{f} \Gamma \tilde{g})(\gamma)}(q) &= \sup_{y=a\zeta b} \min\{\rho_{(\tilde{f})(\gamma)}(a), \rho_{(\tilde{g})(\gamma)}(b)\} \\ &\leq \sup_{p\zeta q=p\zeta ab} \min\{\rho_{(\tilde{f})(\gamma)}(pa), \rho_{(\tilde{f} \Gamma \tilde{g})(\gamma)}(b)\}. \\ &\leq \sup_{p\zeta q=c\zeta d} \min\{\rho_{(\tilde{f})(\gamma)}(c), \rho_{(\tilde{g})(\gamma)}(d)\} \\ &= \rho_{(\tilde{f} \Gamma \tilde{g})(\gamma)}(p\zeta q). \end{split}$$

Similarly, we have

$$\rho_{(\tilde{f}\Gamma\tilde{g})(\gamma)}(p) \leq \rho_{(\tilde{f}\Gamma\tilde{g})(\gamma)}(p\zeta q)$$

and so

$$\rho_{(\tilde{f}\Gamma\tilde{g})(\gamma)}(p\zeta q) \geq \max\{\rho_{(\tilde{f}\Gamma\tilde{g})(\gamma)}(p), \rho_{(\tilde{f}\Gamma\tilde{g})(\gamma)}(q)\}.$$

Thus in any case, we have

$$\rho_{(\tilde{f}\Gamma\tilde{g})(\gamma)}(p\zeta q) \ge \max\{\rho_{(\tilde{f}\Gamma\tilde{g})(\gamma)}(p), \rho_{(\tilde{f}\Gamma\tilde{g})(\gamma)}(q)\}.$$

In the parallel manner, we may obtain that

$$\lambda_{(\tilde{f} \Gamma \tilde{g})(\gamma)}(p \zeta q) \leq \min\{\lambda_{(\tilde{f} \Gamma \tilde{g})(\gamma)}(p), \lambda_{(\tilde{f} \Gamma \tilde{g})(\gamma)}(q)\}.$$

Therefore, $(\tilde{f},S)\Gamma(\tilde{g},T)$ is an IFS Γ -ideal over

4. Conclusion

The notions of IFS Γ -ideals over Γ -semi groups are defined. The theorems about the AND, OR operations of two such ideals are proved. Explicitly, AND OR operations of IFS Γ -ideals over Γ -semi groups are also IFS Γ -ideals over Γ -semi groups. Also, it is proved that the intersection, bi-intersection, union and bi-union of IFS Γ -ideals over Γ -semi groups are IFS Γ -ideals over Γ -semi groups. Moreover, the product of two IFS Γ -ideals over Γ -semi groups is also IFS Γ -ideals over Γ -semi groups. These results are true for soft Γ -semi groups and intuitionist fuzzy soft semi group. As a natural extension, these results also true for IFS Γ -ideals over Γ -semi group.

5. References

- Yang HL. Notes on generalized fuzzy soft sets. Journal of Mathematical Research and Exposition. 2011; 31(3):567-70.
- 2. ZadehLA.Fuzzyset.InformationandControl.1965;8:338-53. https://doi.org/10.1016/S0019-9958(65)90241-X
- Yang CF. Fuzzy soft semi groups and fuzzy soft ideals. Computers and Mathematics with Applications. 2011; 61(2):255-61. https://doi.org/10.1016/j.camwa.2010.10.047
- Maji P, Biswas R, Roy AR. Fuzzy soft sets. Journal of Fuzzy Mathematics. 2001; 9(3):589-602.
- 5. Sen MK. On semigroups. International Symposium on Algebra and its Applications; 1984. p. 301-8.
- Sen MK, Saha NK. On -semigroups-I. Bulletin Calcutta. Mathematics Society. 1986; 78(3):180-6.
- Sen MK, Saha NK. Orthodox -semi groups. International Journal of Mathematics and Mathematical Sciences. 1990; 13(3):527-34. https://doi.org/10.1155/S016117129000076X
- 8. Sen MK, SethA. Radical of -semigroups-III. Bulletin Calcutta of Mathematics Society. 1988; 80(3):189-96.
- Atanassov KT. Intuitionistic fuzzy sets. Fuzzy Sets and Systems. 1986; 20(1):87-96.https://doi.org/10.1016/S0165-0114(86)80034-3
- 10. Krassimir AT. Ituitionistic fuzzy sets theory. Studies in Fuzziness and Soft Computing. 2012; 1-12. https://doi.org/10.1007/978-3-642-35221-8_1
- Krassimir AT. On the intuitionistic fuzzy implications and negations. Fuzziness Soft Computing. 2010; 261:19-38. https://doi.org/10.1007/978-3-642-16629-7_2
- Maji P, Biswas R, Roy AR. Intuitionistic fuzzy soft sets. Journal of Fuzzy Mathematics. 2001; 9(3):677-92.
- Jiehua Z, Li Y, Yin Y. Intuitionistic fuzzy soft semi group. Mathematica Aeterna. 2011; 1(3):173-83.
- Molodtsov D. Soft set theory- first results. Computers and Mathematics with Applications. 1999; 37(4-5):19-31. https://doi.org/10.1016/S0898-1221(99)00056-5
- 15. AkramM, JacobKK, KhamisA. Fuzzysoft \Gamma semigroups. Applied Mathematics and Information Sciences. 2014;8(2):929-34. https://doi.org/10.12785/amis/080256