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The Design Control of Airplane's Movement by the Influence of Leader for Tracking a Desired Path

S. Achmadi[1], Miswanto[2*] and Marjono[3]

[1]National Institute of Technologi, Malang, Indonesia; sentot_achmadi@yahoo.co.id
[2]Depatement Matematika, Universitas Airlangga, Surabaya, Indonesia; miswanto@fst.unair.ac.id
[3]Depatement Matematika, Universitas Brawijaya, Malang, Indonesia; marjono@ub.ac.id

Abstract

Airplane should flight in compliance with the predetermined flight track to avoid the accident. This paper discusses how to design the optimal tracking control from airplanes with the influence of leader. There are four steps in designing the control of airplane's movement by leader influence. Firstly, we will modify the airplane model by adopting Fossen model[1]. Secondly, we determine the trajectory that will be tracked by the movement of airplanes. Thirdly, we design the control of airplane's movement by using tracking error dynamics methods to the leader and the agents. Finally, the numerical simulation output is shown by error tracking of airplane's movement towards desired path followed by agents and leader of the planes.

Keywords: Error Tracking, Numerical Simulation, Plane Model, Tracking,

1. Introduction

One of the fascinating phenomenon to be modeled and studied in mathematics and physics domain are the swarming movement phenomenon namely swarm. This phenomenon is the movement phenomenon that is swarming from the initial position to the end destination. This phenomenon is imitate natural behavior, for instance, the group of swans create reverse V formation while flight. In this formation, one of the swan's group member will be act as a leader, and the remains as the agents. This circumstance depicts the group of swan conducting the tracking of leader path.

Many previous studies investigate the various tracking problems with different method. For example, Hirschorn[2] discuss the tracking problem that utilize singularity method. Further, Cuevas et al.[3] probe the tracking problem by Kalman filter method. Then, Manfredi et al.[4] predict the tracking of networking in a meteorological sensing by implementing least square method and switching on Kalman filter method.

Furthermore, Defoort et al.[5] employed the study of sliding mode control algorithm to solve tracking problem and stabilize Heisenberg system expansion by augmenting integrator factor at the path input. Moreover, Yao et al.[6]; Gazi and Ordonez[7] examine tracking problem toward desired path by combining swarm model and artificial potential method, as well as sliding mode control. The artificial potential method is to design kinematic system control. Meanwhile sliding mode control is to design the dynamic system. Tang et al.[8] also discuss the optimal tracking to solve bilinear system problem in the functional of quadratic cost. They used Successive Approximation Approach (SAA) methods which transform optimation nonlinear problem into the row of non-homogeneous linear problem with two point of constraint. In 2011, Miswanto et al.[9] proposed Swarm behavior in solving tracking problem include the factors that attract all agents to follow the desired path by a specified distance. They also discuss the optimal tracking control in swarm problem by applying geometry approach at multiple dubin's car system.

**Author for correspondence*

This article is organized as follows. Section 2 the problem statement is described of the dynamic airplane system. In section 3, we design the control of the leader using tracking error dynamics methods. In section 4, we design the control of each agent follower using tracking error dynamics methods with tracing error attractor and repellent function. Finally, the numerical simulation output is shown by error tracking of airplane's movement towards desired path followed by agents and leader of the planes.

2. Materials and Methods

2.1 Dynamic Airplane System

Consider the Dynamic Airplane system1 described as:

$$\begin{aligned} \dot{x}_i &= u_i \cos\theta_i + v_i \sin\theta_i \\ \dot{y}_i &= -u_i \sin\theta_i + v_i \cos\theta_i \\ \dot{\theta}_i &= \omega_i \\ \dot{u}_i &= a_i \\ \dot{v}_i &= b_i \\ \dot{\omega}_i &= c_i \end{aligned} \tag{1}$$

where, $(\dot{x}_i, \dot{y}_i) \in R^2$ denote the position of airplane, in this case the airplane move at the constant height is assumed.a_i, b_i, and c_i are positive constans with i = 1, 2, 3 and $\theta_i \in [0,2\pi]$ denote the orientation of plane. u_i and v_i are respectively forward, side and yawing velocities. Further, this movement is tracking γ path which has been defined. Then, it is followed by the members (agents) movement. The path that followed by member is defined by the formula as follow.$y(t) = (yx(t),yy(t))$.

2.2 The Control Design of the Leader for Tracking of Desired Path

We consider a Dynamic Airplane system, such as (1). We design the control (of the leader) of the Airplane system by tracking error dynamics for minimizing the tracking error to keep the position of the Airplane close to the desired path. We define a tracking error e(t) as the difference between the actual Airplane path and the desired path:

$$e_1(t) = \left[e_{1x}(t), e_{1y}(t)\right]^T = \left[x_1(t) - \gamma_x(t), y_1(t) - \gamma_y(t)\right]^T \tag{2}$$

Differentiating the error equation (2) with respect to time and substitute $\dot{\theta}, \dot{u}$ dan $\dot{v}$ from equation (1) yields.

$$\begin{aligned} \dot{e}_1(t) &= \left[\dot{x}_1(t) - \dot{\gamma}_x(t), \dot{y}_1(t) - \dot{\gamma}_y(t)\right]^T, \\ &= \left[u_1 \cos\theta_1 + v_1 \sin\theta_1 - \dot{\gamma}_x(t), -u_1 \sin\theta_1 + \quad \cos \right. \end{aligned} \tag{3}$$

and

$$\begin{aligned} \ddot{e}_1(t) &= \left[\ddot{x}_1(t) - \ddot{\gamma}_x(t), \ddot{y}_1(t) - \ddot{\gamma}_y(t)\right]^T, \\ &= \begin{bmatrix} \dot{u}_1 \cos\theta_1 - u_1\dot{\theta}_1 \sin\theta_1 + \dot{v}_1 \sin\theta_1 + v_1\dot{\theta}_1 \cos\theta - \ddot{\gamma}_x(t), -\dot{u}_1 \sin\theta_1 \\ -u_1\dot{\theta}_1 \cos\theta_1 + \dot{v}_1 \cos\theta_1 - v_1\dot{\theta}_1 \sin\theta_1 - \ddot{\gamma}_y(t) \end{bmatrix}^T, \\ &= \begin{bmatrix} a_1 \cos\theta_1 - u_1\omega_1 \sin\theta_1 + b_1 \sin\theta_1 + v_1\omega_1 \cos\theta - \ddot{\gamma}_x(t), \\ -a_1 \sin\theta_1 - u_1\omega_1 \cos\theta_1 + b_1 \cos\theta_1 - v_1\omega_1 \sin\theta_1 - \ddot{\gamma}_y(t) \end{bmatrix}^T. \end{aligned} \tag{4}$$

Now, we define the tracking error dynamics $F1$ where $F1=[f1x, f1y]^T$ and $f_{1i}(e_{1j}, \dot{e}_{1j}) = 0$, $j = x, y$

$$\begin{aligned} f_{1x}(t) &= \dot{e}_{1x}(t) + k_{1x} e_{1x}(t) \\ f_{1y}(t) &= \dot{e}_{1y}(t) + k_{1y} e_{1y}(t) \end{aligned} \tag{5}$$

where, k_{1x} and k_{1y} are positive constans.

Differentiating the system (5) with respect to time t, one obtains

$$\begin{aligned} \dot{f}_{1x}(t) &= \ddot{e}_{1x}(t) + k_{1x} \dot{e}_{1x}(t) \\ \dot{f}_{1y}(t) &= \ddot{e}_{1y}(t) + k_{1y} \dot{e}_{1y}(t) \end{aligned} \tag{6}$$

From equations (1), (2), (3) and (5), can be determinate the control u_i and v_1

$$\begin{aligned} u_1 &= \left(\dot{\gamma}_x(t) + k_{1x}\gamma_x(t) - k_{1x}x_1(t)\right)\cos\theta_1 - \left(\dot{\gamma}_y\right. \\ v_1 &= \left(\dot{\gamma}_x(t) + k_{1x}\gamma_x(t) - k_{1x}x_1(t)\right)\sin\theta_1 + \left(\dot{\gamma}_y\right. \end{aligned} \tag{7}$$

From equations (3), (4) and (6), can be determinate the control ω_1

$$\omega_1 = \frac{\left(k_{1x}\dot{\gamma}_x(t)+\ddot{\gamma}_x(t)\right)\cos\theta_1}{v_1} - \frac{\left(k_{1y}\dot{\gamma}_y(t)+\ddot{\gamma}_y(t)\right)\sin\theta_1}{v_1} - \frac{u_1\left(k_{1x}\cos^2\theta_1+k_{1y}\sin^2\theta_1\right)}{v_1} - \frac{v_1\sin\theta_1\cos\theta_1\left(k_{1x}-k_{1y}\right)+a_1}{v_1} \quad (8)$$

Substitute equation (7) to (8) yields

2.3 The Control Design of the Following Agents

We define a tracking error as the agent to leader:

$e_i(t)=[e_{ix}(t), e_{iy}(t)]^T$, i = 2, 3 the number of agent. (10)

with

$$e_{ix}(t) = -(x_1-x_i)\left(a-\frac{r}{b+c\,\|\boldsymbol{x}_1-\boldsymbol{x}_i\|^2}\right)$$
$$e_i y(t) = -(y_1-y_i)\left(a-\frac{r}{b+c\,\|x_1-x_i\|^2}\right) \quad (11)$$

and

a, b, and c are positive constans

$\|(x_1,y_1)-(x_i,y_i)\| = \sqrt{(x_1-x_i)^2+(y_1-y_i)^2}$

Differentiating the error equation (11) with respect to time and substitute θ, u, and v from equation (1) yields.

$$\dot{e}_{ix}(t) = -(\dot{x}_1-\dot{x}_i)\left(a-\frac{r}{b+c\,\|\boldsymbol{x}_1-\boldsymbol{x}_i\|^2}\right)$$
$$\dot{e}_{iy}(t) = -(\dot{y}_1-\dot{y}_i)\left(a-\frac{r}{b+c\,\|\boldsymbol{x}_1-\boldsymbol{x}_i\|^2}\right) \quad (12)$$

and

$$\dot{e}_{ix}(t) = -(\dot{x}_1-\dot{x}_i)\left(a-\frac{r}{b+c\,\|\boldsymbol{x}_1-\boldsymbol{x}_i\|^2}\right)$$
$$\dot{e}_{iy}(t) = -(\dot{y}_1-\dot{y}_i)\left(a-\frac{r}{b+c\,\|\boldsymbol{x}_1-\boldsymbol{x}_i\|^2}\right) \quad (13)$$

f_{ij} (e_ij, $e\,\dot{e}_{ij}$) = 0; $j = x,y$

$f_{ix}(t) = e\,\dot{e}_{ix}(t) + k_{ix}\, e_{ix}(t)$

$f_{iy}(t) = e\,\dot{e}_{iy}(t) + k_{iy}\, e_{iy}(t)$ (14)

where, k_{ix} and k_{iy} are positive constans.

Differentiating the system (14) with respect to time t, one obtains

$\dot{f}_{ix}(t) = \ddot{e}_{ix}(t) + \mathrm{k}_{ix}\,\dot{e}_{ix}(t)$

$\dot{f}_{iy}(t) = \ddot{e}_{iy}(t) + \mathrm{k}_{iy}\,\dot{e}_{iy}(t)$ (15)

From equations (1), (12), and (14), can be determinate the control u_i and v_i:

$$\begin{aligned} u_i &= \left(\dot{\gamma}_x(t)+k_{1x}\gamma_x(t)+(k_{ix}-k_{1x})x_1(t)-k_{ix}x_i(t)\right)\cos\theta_i \\ &\quad -\left(\dot{\gamma}_y(t)+k_{1y}\gamma_y(t)+(k_{iy}-k_{1y})y_1(t)-k_{iy}y_i(t)\right)\sin\theta_i \\ v_i &= \left(\dot{\gamma}_x(t)+k_{1x}\gamma_x(t)+(k_{ix}-k_{1x})x_1(t)-k_{ix}x_i(t)\right)\sin\theta_i \\ &\quad +\left(\dot{\gamma}_y(t)+k_{1y}\gamma_y(t)+(k_{iy}-k_{1y})y_1(t)-k_{iy}y_i(t)\right)\cos\theta_i \end{aligned} \quad (16)$$

3. Results and Discussion

In this section, we present the result of numerical simulation from model (1) in the two-dimensional space, first of all, the formulation of the path that will be tracked is:

$$\dot{\gamma}_x(t) = \frac{3}{20}t^2 - \frac{7}{10}t + 1$$
$$\dot{\gamma}_y(t) = \frac{3}{50}t^2 - \frac{1}{2}t + 15 \quad (17)$$

This tracking is conducted from t = 0 to t = 20 with the initial path $\gamma(0)$ = (30, 5). Simulation is undertaken with three agents (one leader and two agents). The initial points of each agent are $x_1(0)$ = (50,5) as the leader who is tracking the desired path. Then, x_2 (0) = (125,2) and x_3 (0) = (10,2) are the members (agents) who follow the track of the leader path.

The simulation result are k_{1x}=k_{1y}=1.00 ; k_{2x}=k_{2y}=0.10 ; and k_{3x}=k_{3y}=0.05 illustrated by Figure 1 and Figure 2.

The graph in Figure 1 illustrates the implementation tracking dynamic error method. The graph shows the actual path of plane's movement is approaching the desired path. While, in Figure 2, the graph illustrates the movement of two agents is tracking the path that have been tracked by the leader.

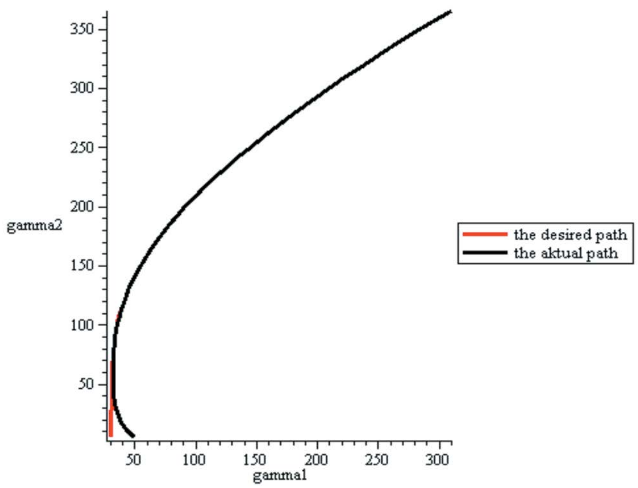

Figure 1. Graph of the desired and the actual path.

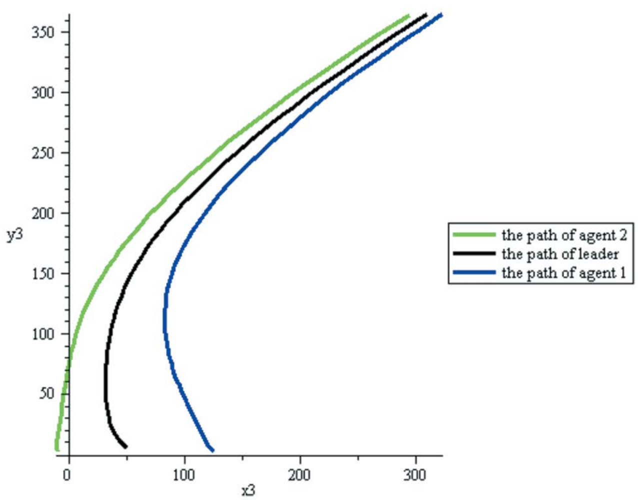

Figure 2. Graph of the leader path and the agent path.

4. Conclusion

This paper discuss the movement of airplanes that are tracking the desired path. From the numerical simulation results aforementioned, it can be seen that the tracking error of the path of the leader tracing a desired path is sufficiently small. Further, the distance between the leader path and the desired path is preserved. On the other hand, every agent move to track the leader movement by preserving the desired path to avoid the crash. This model is a simple model which include the norm as interrelatedness factor.

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