Fuzzy Almost Bi-ideals of Near-rings

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Abstract

Objectives: To study fuzzy almost bi-ideals in near-rings. Methods/Statistical Analysis: We extend the concept of fuzzy almost bi-ideals in semigroup to near-ring. Findings: We discuss some algebraic properties of fuzzy almost bi-ideal and minimal fuzzy almost bi-ideal. Application: This fuzzy almost bi-ideal concept can be used in semiring, ternary semiring etc.

Keywords: Almost Bi-ideals (ABI), Bi-ideals (BI), Fuzzy Almost Bi-ideals (FABI), Minimal Almost Bi-ideals (MABI), Minimal Fuzzy Almost Bi-ideals (MFABI), Near-rings (NR)

1. Introduction

Fuzzy subsets introduced by has been extended to group theory in. Different types of fuzzy ideals were characterized by. FBI generated by fuzzy subsets were studied by. ABIs of semigroup were introduced by¹. In this article, ABI concept has been extended to NR and some fascinating results are provided.

2. Preliminaries

For the definition and preliminaries of NR, ideal, bi-ideal, N-subgroup of NRs etc. in crisp case and fuzzy case see²–⁹.

3. ABIs in NRs

Definition 3.1 $A(\neq \emptyset) \subseteq B$ of $N$ is called an ABI of $N$ if $(BnB) \cap (Bn*B) \cap B \neq \emptyset \ \forall \ n \in N$.

Example 3.2 For $Z_4$ under addition, let $B = \{1, 2, 3\}$.

$(B+0+B) \cap B + 0*B \cap B = \{0, 1, 2, 3\} \cap \{1, 2, 3\} \cap \{1, 2, 3\} = \{1, 2, 3\} \neq \emptyset$

$(B+1+B) \cap B + 1*B \cap B = \{0, 1, 2, 3\} \cap \{1, 2, 3\} \cap \{1, 2, 3\} = \{1, 2, 3\} \neq \emptyset$

$(B+2+B) \cap B + 2*B \cap B = \{0, 1, 2, 3\} \cap \{1, 2, 3\} \cap \{1, 2, 3\} = \{1, 2, 3\} \neq \emptyset$

$(B+3+B) \cap B + 3*B \cap B = \{0, 1, 2, 3\} \cap \{1, 2, 3\} \cap \{1, 2, 3\} = \{1, 2, 3\} \neq \emptyset$

$B(\neq \emptyset)$ is an ABI of $N$.

Example 3.3 Consider the NR $N = \{a, \beta, \gamma, s\}$ with two binary operations "+" and "." is defined as follows:

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Then $B = \{a, \beta\}$ is an ABI of $N$.

**Remark 3.4** Every BI of $N$ is an ABI of $N$.

**Proof.** Let $B$ be a BI of $N$. Then $(BnB) \cap (Bn*B) \neq \emptyset\) & $(BnB) \cap (Bn*B) \subseteq (BNB) \cap (BN*B) \subseteq B \Rightarrow (BnB) \cap (Bn*B) \cap B \neq \emptyset, \forall n \in N.$ \[\therefore B^2 \text{ is an ABI of } N.\]

**Remark 3.5** Let $B$ be an ABI of $N$. Let $B^2 (\neq \emptyset) \subseteq N$ of $N \ni B \subseteq B' \subseteq N$. Then $B^2$ is an ABI of $N$.

**Proof.** Let $B$ be an ABI of $N \ni B \subseteq B' \subseteq N$. Then

$$\emptyset \neq (BnB) \cap (Bn*B) \subseteq (B' nB') \cap (B' n*B')$$

$$\cap B' \Rightarrow (B' nB') \cap (B' n*B') \cap B' \neq \emptyset, \forall n \in N.$$

\[\therefore B^2 \text{ is an ABI of } N.\]

**Remark 3.6** The union of two ABIs of $N$ is an ABI of $N$.

**Proof.** Use Remark 3.5.

**Example 3.7** Consider $(Z_5, +, \cdot)$ Then $B_1 = \{1, 3, 4\}$ and $B_2 = \{1, 2, 4\}$ are ABIs but $B = B_1 \cap B_2 = \{1, 4\}$ is not an ABI of $Z_5$ because $B + 0 + B = \{0, 2, 3\}$. $(B + 0 + B) \cap B = \emptyset$.

\[\therefore \text{ intersection of two ABIs need not be an ABI.}\]

### 4. FABIs

FABIs in NRs are defined and its relations with ABIs are obtained here.

**Definition 4.1** Let $f$ be a fuzzy subset of $N \ni f \neq 0$. $f$ is called a FABI of $N$ if $\forall n \in N, (fC_n f) \cap (fC_n * f) \cap f \neq 0$.

**Theorem 4.2** For $f$, a FABI of $N$ and a fuzzy subset $g$ of $N \ni f \subseteq g$, $g$ also a FABI of $N$.

**Proof.** For $f$, a FABI of $N$ and $g$ a fuzzy subset of $N \ni f \subseteq g$, $$(fC_n f) \cap (fC_n * f) \cap f \subseteq (gC_n g) \cap (gC_n g * f) \cap g$$

& $$(fC_n f \cap fC_n * f) \cap f \neq 0, \forall n \in N.$$ \[\Rightarrow (gC_n g \cap gC_n * g) \cap g \neq 0. \forall n \in N.\]

\[\therefore g \text{ is a FABI.}\]
Corollary 4.3 For FABIs \( f \) and \( g \) of \( N \), \( f \cup g \) is a FABI of \( N \).

Proof. Since \( f \subseteq f \cup g \) by Theorem (4.2), \( f \cup g \) is a FABI of \( N \).

Example 4.4 Consider \( Z_5 \) under the usual addition. Let \( f : Z_5 \rightarrow [0,1] \) be defined by \( f(0) = 0, f(1) = 0.5, f(2) = 0, f(3) = 0.1, f(4) = 0.1 \) and \( g \rightarrow [0,1] \) be defined by \( g(0) = 0, g(1) = 0.2, g(3) = 0 \) and \( g(4) = 0.2 \). We have \( f \) and \( g \) are FABIs of \( Z_5 \) but \( f \cap g \) is not a FABI of \( Z_5 \).

Theorem 4.5 \((B \neq \emptyset) \subseteq N\) is an ABI of \( N \) iff \( C_B \) is a FABI of \( N \).

Proof. \((\Rightarrow) BnB \cap Bn* B \cap B \neq 0 \ \forall n \in N \). Thus \( \exists x \in BnB, x \in Bn* B \& x \in B \).

\[ (C_B n C_B)(x) = 1, (C_B n C_B)(x) = 1 \land C_B(x) = 1. \]

Hence \( (C_B n C_B) \cap (C_B n C_B) \cap C_B \neq 0 \) for all \( n \in N \).

\[ \therefore C_B \text{ is a FABI of } N. \]

\((\Leftarrow), C_B \text{ be a FABI of } N \text{ and } n \in N \). Then \( ((C_B n C_B) \cap (C_B n C_B)) \cap C_B \neq 0 \land \exists x \in N \exists x \in N \exists \]

\[ \exists x \in BnB \cap (Bn* B) \cap B. \]

\[ BnB \cap (Bn* B) \cap B \neq 0 \ \forall n \in N. \]

Consequently, \( B \) is an ABI of \( N \).

Theorem 4.6 Let \( f \) be a fuzzy subset of \( N \). Then \( f \) is a FABI of \( N \) iff \( \text{supp } f \) is an ABI of \( N \).

Proof. Let \( f \) be a FABI of \( N \) and \( n \in N \). Then \( (fC_n f) \cap (fC_n f) \cap f \neq 0. \) Hence \( \forall n \in N, \exists x \in N \exists (fC_n f) \cap (fC_n f) \cap f \neq 0. \) So \( \exists \ y_1, y_2, y_3 \in N \exists x = y y_1 y_2 y_2 \) and \( x = y y_1 y_2 + y_3 - (y y_1 y_2), f(x) \neq 0, f(y_1) \neq 0, f(y_2) \neq 0 \) \& \( f(y_3) \neq 0 \). That is \( x, y_1, y_2, y_3 \in \text{suppf} \).

\[ ((C \text{suppf } C_n C \text{suppf}) \cap (C \text{suppf } C_n C \text{suppf}) \cap C \text{suppf} \neq 0. \]

Hence \( C \text{suppf} \) be a FABI of \( N \). By Theorem (4.5), \( \text{suppf} \) be an ABI of \( N \).

Conversely, let \( \text{suppf} \) be an ABI of \( N \). By Theorem (4.5), \( C \text{suppf} \) be a FABI of \( N \). Then

\[ (C \text{suppf } C_n C \text{suppf}) \cap (C \text{suppf } C_n C \text{suppf}) \cap C \text{suppf} \neq 0 \]

for all \( n \in N \). Then \( \exists x \in N \exists (C \text{suppf } C_n C \text{suppf}) \cap (C \text{suppf } C_n C \text{suppf}) \cap C \text{suppf} \neq 0. \) Hence

\[ ((C \text{suppf } C_n C \text{suppf}) \cap (C \text{suppf } C_n C \text{suppf}) \cap C \text{suppf} \neq 0 \]

\& \( C \text{suppf } (x) \neq 0. \) Then \( \exists y_1, y_2 \in N \exists x = y y_1 y_2, f(x) \neq 0, f(y_1) \neq 0 \) \& \( f(y_2) \neq 0. \) This means \( (fC_n f) \cap (fC_n f) \cap f \neq 0. \)

\[ \therefore f \text{ is a FABI of } N. \]

5. MAFBIs

In this section, we define MAFBIs in NR and study relationship between support and MAFBIs of NRs.
Definition 5.1 A FABI $f$ is called minimal if for each FABI $g$ of $N \ni g \subseteq f$, we have $\text{supp} g = \text{supp} f$.

Theorem 5.2 $B(\neq \emptyset) \subseteq N$ is a MABI iff $C_B$ is a MFABI of $N$.

Proof. Let $B$ be a MABI of $N$. By Theorem (4.5), $C_B$ is a FABI of $N$. For FABI $g$ of $N \ni g \subseteq C_B$,

$\text{supp} g \subseteq \text{supp} C_B = B$. As $g \subseteq C_{\text{supp} g}$,

$$(gC_B g) \cap (gC_B ^* g) \cap g \subseteq C_{\text{supp} g} C_B C_{\text{supp} g}$$

$\cap C_{\text{supp} g}.$

Thus $C_{\text{supp} g}$ is a FABI of $N$. By Theorem (4.5), $\text{supp} g$ is an ABI of $N$. Since $B$ is minimal,

$\text{supp} g = B = \text{supp} C_B.$

$\therefore C_B$ is minimal.

Now, $C_B$ be a MFABI of $N$. Let $B'$ be an ABI of $N \ni B' \subseteq B$. Then $C_{B'}$ is a FABI of $N \ni C_{B'} \subseteq C_B$.

Hence $B' = \text{supp} C_{B'} = \text{supp} C_B = B$.

$\therefore B$ is minimal.

Corollary 5.3 $N$ has no proper ABI of $N$ iff $\forall$ FABI $f$ of $N$, $\text{supp} f = N$.

6. Conclusion

In this study the notion of FABI and MABI of NRs have been presented and some properties of these ideals are derived.

7. References

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