Note on Contraino Semipre Continuous Functions

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Abstract
This study is devoted to introducing and also it looks into the properties of nano semipre-cont, nano semipre-open func, nano semipre-closed func, Contraino semiprecont.funcs and obtains some relationship between the existing sets.

Keywords: Almost Nβ-cont, Contraino β-cont, Nano β-cont, Nano β-closed, Nano β-open, NP-β-open, Nβ-regular

1. Introduction and Preliminaries
In¹ introduced generalized closed sets in topological spaces. The notion of Nano topology was introduced by². The basic definitions are referred from the following study²⁻³⁹. Throughout this study, func represents the function, image as ima, continuous as cont, inverses as invrs.

2. Nano Semiprecont, Nano semipreopen and Nano semipreclosed Funcs
In this section, we study some additional properties of Nβ-cont.func, Nβ-open and Nβ-closed funcs.

Definition 2.1. A func k is called Nβ-open if the ima of each nano open set A of U is Nβ-open in V.

Definition 2.2. A func is called Nβ-closed if the ima of nano closed set A of U is Nβ-closed in V.

Theorem 2.2. Let be a Nβ-contand NaNβ-open func then the invrsima of each nano open set in V is Nβ-open in U.

Theorem 2.3. Let be a Nβ-contand nano open mapping then the following statements hold.
(a) The invrsima of each NP-open set in V is Nβ-open in U
(b) The invrsima of each NS-open set in V is Nβ-open in U

Theorem 2.4. Let be bijective Nβ-contand l: V → W be bijectivenanocont.funcs then l∩k: U → W is Nβ-cont.func.

Proof: Let V be any nano open subset of Z then l⁻¹(V) be nano open in Y and as f is Nβ-contk⁻¹(l⁻¹(V)) is Nβ-open in X i.e., (l∩k)⁻¹(V) is Nβ-open in X implies l∩k is Nβ-cont.func.

Theorem 2.5. Each NS-open (NP-open) func is Nβ-open but not conversely.

Let k: U → V be NS-open (NP-open) and A be any nano open subset of U then (A) is NS-open (NP-open) in Y, as every NS-open (NP-open) set is Nβ-open, k(A) is Nβ-open in X. Hence f is Nβ-open function.

Theorem 2.6. A bijective func is Nβ-open iff it is Nβ-closed.

Let k: U → V be NS-open (NP-open) and A be any nano open subset of U then (A) is NS-open (NP-open) in Y, as every NS-open (NP-open) set is Nβ-open, k(A) is Nβ-open in X. Hence f is Nβ-open function.

Theorem 2.7. Let k: U → V be bijective Nβ-open (Nβ-closed) func. W ⊂ V and F ⊂ U is a nanoclosed(nano-open) set containing k⁻¹(W) then Nβ-closed (Nβ-open) set H of V containing W such that k⁻¹(H) ⊂ F.
3. Contranano Semipre Contfunctions

In this section, we study a new class of func s called Contranano semi pre cont. funcs and its related properties.

Definition 3.1. A func k: U → V is called Contranano semi pre (or Contranano-β) cont. func if the invrsmas of each nano open set of V is Nβ-closed set in U.

Definition 3.2. A func k: U → V is said to be nano pre-semipre (or NPβ) open if the f(B) is Nβ-open in V for each Nβ-open set B in U.

Definition 3.3. A subset A of U is said to be nano semipre regular, if it is both Nβ-open and Nβ-closed set and set of all Nβ regular sets of U is denoted by NβR(U).

Lemma 3.5. In a NTSU, Nβcl(A) ⊆ Npcl(A) \cap Nscl(A) and hence we have Nβcl(A) ⊆ Npcl(A), Nβcl(A) ⊆ Nscl(A).

Lemma 3.6. Each Nβ-open and Nβ-closed is nano-closed and Nβ-closed and Nβ-open is nano-open.

Lemma 3.7. A func k: U → V is nano open and nanocont then for any nano open subset A of V , k⁻¹(A) is Nβ-open in U and hence it is Nβ-closed as it is Nβ regular. Thus, invrsima of nano open set is Nβ-open implies f is Nβ-cont.

Conversely: Let a subset of U be Nβ regular and let k: U → V be nano-β-cont then for each nano open set A of V, k⁻¹(A) is Nβ-open in U and hence it is Nβ-closed as it is Nβ regular. Thus, invrsima of nano open set is Nβ-closed implies f is Nβ-cont.

Theorem 3.15. Each Contranano semipre (contranano pre-cont) func is Nβ-cont.

But converse of the above theorem need not be true in general.

Theorem 3.16. If the space U is nanoextremally disconnected, then each contra Nβ-cont. func is contranano pre-cont.

Lemma 3.17. Let A be a subset of a nano topological space U. Then each Nβ-open (Nβ-closed) set is nano semi-open ((nano semi-closed) if Nint (Ncl(A)) ⊆ Ncl(Nint(A)).

4. Conclusion

The properties of nano semiprecont, nano semipreopenfunc, nano semipre closed func, Contranano semiprecont.funcs are investigated.

5. References


