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Fixed Points in A Complex Valued Metric Space

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Abstract

Objectives/Aim: To find fixed points for integral types of contraction mappings in a complex valued metric spaces. **Methods**: We used mathmatical analysis to prove the results as a generalization of weakly contraction self mappings defined in a complex metric space. **Findings**: We are finding by weakly integral types of contraction self mappings that there are common fixed points. In this paper we used commutative self mappings for our conclusions . **Application/ Improvements:** In future it can also be used for the exestense and uniquence of the differential equations solutions.

Keywords: Complex Valued Metric Space, Contraction Mapping, Fixed Point, Integral Type, Weakly Commuting

1. Introduction

A fixed point theory contains a great number of generalizations of Banach contraction principle by using different form of contraction condition in various spaces. But majority of such generalizations are obtained by improving underlying contraction conditions which also includes contraction conditions dscribed by rational expressions. The concept of complex valued metric space was introduced by[1] in 2011. The complex valued b-metric space is introducedby[2], which is more general than well-known complex valued metric space. There are many fixed point results in complex valued metric spaces[3-13] also in complex valued b-metric spaces[2,12]. In this study we present a common fixed point result for two self-mappings satisfying a contractive condition in complex valued metric spaces .This idea is intended to define rational expressions which are not meaningful in cone metric spaces and thus many such results of analysis cannot be generalized to cone metric spaces but to complex valued metric spaces. A contractive condition of integral type was introduced by [6] in 2002. After the paper of Branciari, a lot of research works have been carried out on generalizing contractiveconditions of integral type for different contractive mappings satisfying various known properties. There are various typies of weakly contractive self mappings in the last thirty years.

2. Preliminaries

Let C be the set of complex numbers, and for any two complex numbers z_1 and z_2 in C **is denoted by** $z_1, z_2 \in C$ **.If Real part and imaginary part of the complex number are denoted by Re(z) and Im(z)**respectively. Define a **partial order on** C **as follows**: z_1 z_2 (that is

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mean **subordinationif and only if** $\mathrm{Re}(z_1) \le \mathrm{Re}(z_2)$ and $\mathrm{Im}(z_1) \le \mathrm{Im}(z_2)$.

Thus $z_1 \precsim z_2$ if one of the following holds:

(1) Real parts z_1 and z_2 are equal and imaginary parts and z_1 are z_2 equal.

(2) Real part z_1 is less than Real part z_2 and imaginary parts z_1 and z_2 are equal.

(3) Real parts z_1 and z_2 are equal and imaginary part z_1 is less than imaginary part of z_2

(4) Real part z_1 is less than real part and imaginary part z_1 is less than imaginary part z_2

We will write $z_1 \lnsim z_2$ (**subordination** and isn't equal z_2) if z_1 and z_2 are n't equal and one of (2), (3) and (4) is satisfied. Also, we will write $z_1 \prec z_2$ (**z_1 is strictly subordination**if only (4) is satisfied. It follows that:

(i) $0 \prec z_1 \lnsim z_2$ **implies** $|z_1| < |z_2|$

(ii) $z_1 \prec z_2$ **and** $z_2 \prec z_3$ **imply** $z_1 \prec z_3$

(ii) $0 \precsim z_1 \precsim z_2$ **Implies** $|z_1| \le |z_2|$

(iv) For any two real numbers a and b, $0 \le a \le b$ and $z_1 \precsim z_2$, then $a.z_1 \precsim b.z_2$.

Definition 2.1[1]. Let X be a nonempty set and C be the set of complex numbers. Suppose that the mapping $d: X \times X \to C$ satisfies the following conditions:

(1) $d(x,y)$ is nonnegative for all $x,y \in X$ and $x=y$ if and only if $d(x,y) = 0$

(2) $d(x,y)$ and $d(y,x)$ are equal for all x and y in C

(3) d(x,y) is subordination of the sum of $d(x,z)$ and d(x,y) for all x, y and z in X.

Then d is called a complex valued metric on X, and $(X;d)$ is called a complex valued metric space.

Definition 2.2[1]**.** Let $(X;d)$ be a complex valued metric space. Then

(1) A point x in X is called an interior point of a set A in X whenever there exists positive real numberr such that the open set $B(x,r)$ in A

(2) A point x in X is called a limit point of a set A whenever there exists positive real numberr such that $B(x,r)$ and $(A\text{-}x)$ are nonempty set.

(3) A subset A in X is called closed whenever each element of A belongs to A.

(4) A subcbasis for a Hausdorff topology τ on X is a family $F=\{B(x,r): x \in X \text{ and } 0<r\}$.

Definition 2.3[1]. Let $(X;d)$ be a complex valued metric space, x_n be a sequence in X and $x \in X$. Then

(1) If for every $0 \prec c \in C$ there is $n \in N$ such that for all $n \in N, d(x_n,x) \prec c$,then x_n is said to be convergent to x and x is called a limit point of x_n. We denote this by $\lim_{n\to\infty} x_n = x$ or $\{x_n\} \to x$ as $n \to \infty$

(2) If for every $0 \prec c \in C$ there is $n \in N$ such that for all $n \in N$, $d(x_n, x_{n+m}) \prec c$ where $m \in N$, then x_n is said to be Cauchy sequence.

(3) If every Cauchy squence in X is convergent, then (X,d) is said to bea complete complex valued metric space .

Lemma 2.4 [1]**.** Let (X,d) be a complex valued metric space and let x_n be a sequence in X. Then x_n converges to x if and only if $|d(x_n, x)| \to 0$ as $n \to \infty$.

Lemma 2.5 [1]**.** Let (X,d) be a complex valued metric space and let x_n be a sequence in X. Then x_n is a Cauchy sequence if and only if $|d(x_n, x_{n+m})| \to 0$ as $n \to \infty$ where $m \in N$.

In 2002, a general contractive condition of integral type is given and analyzed by[6] in the following theorem.

Theorem 2.6 [5]**.** Let $(X;d)$ be a complete metric space, $c \in (0,1)$ and let $T: X \to X$ be a mapping such that for each $x, y \in X$,

$$\int_0^{d(Tx,Ty)} \varphi(t) \le c \int_0^{d(x,y)} \varphi(t)$$

where $\varphi:[0;+\infty) \to [0;+\infty)$ is a Lesbesgue-integrable mapping which is summable (i.e.with finite integral) on each compact subset of $[0;+\infty)$, non-negative, and such that for each $\grave{o} > 0$, $\int_0^{\grave{o}} \ddot{o}(t) > 0$, then T has a unique fixed point $a \in X$ such that for each $x \in X$, $lim_{n\to\infty} T^n x = a$.

Also, the pairwise commutating is defined.

Definition 2.7 [14]**.** The two families of self mappings $\{f_i\}_1^m$ and $\{g_j\}_1^n$ are said to be pairwise commuting if

(1) $f_i f_j = f_j f_i$, for $i,j \in \{1,2,...,m\}$

(2) $g_k g_l = g_l g_k$, for $k,l \in \{1,2,...,n\}$

(3) $f_i g_l = g_l f_i$, for $i \in \{1,2,...,m\}$ and $l \in \{1,2,...,n\}$

The concepts of weakly commuting, weak* commuting and weak** commuting are introduced by[14-17] respectively. Next, we introduce the weakly contraction self mappings in the complex valued metric space.

Definition 2.8. Let (X,d) be a complex valued metric space. Then

(1) The two self maps f and g are weakly commuting if

$$d(fgx, gfx) \precsim d(fx, gx),$$

(2) The two self maps f and g are weak* commuting if

$$d(fgx, gfx) \precsim d(f^2x, g^2x)$$

(3) The two self maps f and of a complete space (X,d) are weak** commutingif

$f(x) \subseteq g(x)$ for $x \in X$

$$d(f^2g^2x, g^2f^2x) \precsim d(f^2gx, gf^2x) \precsim d(fg^2x, g^2fx) \precsim d(fgx, gfx) \precsim d(f^2x, g^2x)$$

(4) The two self maps f and g of a complete space (X,d) are $m-$weak** commute,if

$f(x) \subseteq g(x)$ for any $x \in X$

$$d(f^m g^m x, g^m f^m x) \preccurlyeq d(f^m gx, gf^m x)$$
$$\preccurlyeq d(fg^m x, g^m fx) \preccurlyeq d(fgx, gfx) \preccurlyeq d(f^m x, g^m x).$$

In this article, we denote X to the complex metric space (X,d), and to a self mapping $f:(X,d) \to (X,d)$ by a self mapping f on X. And we define new concept of weak commuting in the complex valued metric space as follows.

Definition 2.9. For a real number $\alpha \geq \frac{1}{2}$, the two self mappings f_1 and f_2 defined in a complex valued metric space (X,d) are said to be 2α-weak commuting if $f_1(X) \subset f_2(X)$ and

$$d\left(f_1^{2\alpha} f_2^{2\alpha} z, f_2^{2\alpha} f_1^{2\alpha} z\right) \preccurlyeq d\left(f_1^{2\alpha} f_2 z, f f_1^{2\alpha}{}_2 z\right)$$
$$\preccurlyeq d\left(f_1 f_2^{2\alpha} z, f_2^{2\alpha} f_1 z\right)$$
$$\preccurlyeq d(f_1 f_2 z, f_2 f_1 z) \preccurlyeq d\left(f_1^{2\alpha} z, f_2^{2\alpha} z\right).$$

The 2α-weak commuting mapping is a generlization of various typies of weak commutating maps.

3. The Main Results

By a rational expression of contraction mapping defined in a complex valued metric space X, we prove the next resulte of a common fixed point.

Theorem 3.1 Let f_1 and f_2 be two self sequentially contінous mappings on a complete complex valued metric space X such that $f_1(X) \subset f_2(X)$ and the order pair $\{f,g\}$ is $2\alpha -$ weakly commuting, also the condition is satisfied

$$d\left(f_1^{2\alpha} f_2^{2\alpha} z, f_2^{2\alpha} f_1^{2\alpha} w\right) \preccurlyeq \lambda d\left(f_1^{2\alpha} z, f_2^{2\alpha} w\right)$$

$$+\delta\left(\frac{d(f_1^{2\alpha} w, f_2^{2\alpha} f_1^{2\alpha} w)[1+d(f_2^{2\alpha} z, f_1^{2\alpha} f_2^{2\alpha} z)]}{1+d(f_2^{2\alpha} z, f_1^{2\alpha} w)}\right)$$

$$+\gamma\left(\frac{d(f_1^{2\alpha} w, f_1^{2\alpha} f_2^{2\alpha} z)[1+d(f_2^{2\alpha} z, f_2^{2\alpha} f_1^{2\alpha} w)]}{1+d(f_2^{2\alpha} z, f_1^{2\alpha} w)}\right)$$

where, $á,ä,ã \geq 0$ and $á+ä+ã < 1$. Then f_1 and f_2 have a unique common fixed point

Proof. Let z_0 be an arbtrary point in X, and costruct a sequence $\{z_n\}$ such that

$$f_1^{2\alpha} z_{2n} = z_{2n+1} \text{ and } f_2^{2\alpha} z_{2n+1} = z_{2n+2}, \text{ for } n=0,1,2,...$$

To show that $\{z_n\}$ is a Cauchy sequence in X. Let $f_2^{2\alpha} f_1^{2\alpha} z_{2n-1} = z_{2n+1}$. Then

$$d(z_{2n+1}, z_{2n+2}) = d\left(f_1^{2\alpha} f_2^{2\alpha} z_{2n-1}, f_2^{2\alpha} f_1^{2\alpha} z_{2n}\right) \preccurlyeq \lambda d\left(f_1^{2\alpha} z_{2n-1}, f_2^{2\alpha} z_{2n}\right)$$

$$+\delta\left(\frac{d(f_1^{2\alpha} z_{2n}, f_2^{2\alpha} f_1^{2\alpha} z_{2n})[1+d(f_2^{2\alpha} z_{2n-1}, f_1^{2\alpha} f_2^{2\alpha} z_{2n-1})]}{1+d(f_2^{2\alpha} z_{2n-1}, f_1^{2\alpha} z_{2n})}\right)$$

$$)+\gamma\left(\frac{d(f_1^{2\alpha} z_{2n}, f_1^{2\alpha} f_2^{2\alpha} z_{2n-1})[1+d(f_2^{2\alpha} z_{2n-1}, f_2^{2\alpha} f_1^{2\alpha} z_{2n})]}{1+d(f_2^{2\alpha} z_{2n-1}, f_1^{2\alpha} z_{2n})}\right)$$

But $d(f_1^{2\alpha} z_{2n}, f_1^{2\alpha} f_2^{2\alpha} z_{2n-1}) = 0$ since $f_2^{2\alpha} z_{2n-1} = z_{2n}$. So that

$$d(z_{2n+1}, z_{2n+2}) \preccurlyeq \lambda d\left(f_1^{2\alpha} z_{2n-1}, f_2^{2\alpha} z_{2n}\right)$$

$$+\delta\left(\frac{d(f_1^{2\alpha} z_{2n}, f_2^{2\alpha} f_1^{2\alpha} z_{2n})[1+d(f_2^{2\alpha} z_{2n-1}, f_1^{2\alpha} f_2^{2\alpha} z_{2n-1})]}{1+d(f_2^{2\alpha} z_{2n-1}, f_1^{2\alpha} z_{2n})}\right).$$

Therefore,

$$|d(z_{2n+1},z_{2n+2})| \le \lambda |d(f_1^{2\alpha} z_{2n-1}, f_2^{2\alpha} z_{2n})|$$

$$+\delta \frac{|d(f_1^{2\alpha} z_{2n}, f_2^{2\alpha} f_1^{2\alpha} z_{2n})||1+d(f_2^{2\alpha} z_{2n-1}, f_1^{2\alpha} f_2^{2\alpha} z_{2n-1})|}{|1+d(f_2^{2\alpha} z_{2n-1}, f_1^{2\alpha} z_{2n})|}.$$

Since

$$|1+d(f_2^{2\alpha} z_{2n-1}, f_1^{2\alpha} f_2^{2\alpha} z_{2n-1})|$$

$$=|1+d(f_2^{2\alpha} z_{2n-1}, f_1^{2\alpha} z_{2n})| > |d(f_2^{2\alpha} z_{2n-1}, f_1^{2\alpha} z_{2n})|$$

which implies that

$$|d(z_{2n+1},z_{2n+2})| \le \lambda |d(f_1^{2\alpha} z_{2n-1}, f_2^{2\alpha} z_{2n})|$$

$$+2\delta |d(f_1^{2\alpha} z_{2n}, f_2^{2\alpha} f_1^{2\alpha} z_{2n})|$$

and

$$|d(z_{2n+1},z_{2n+2})| \le \frac{\lambda}{1-2\delta} |d(f_1^{2\alpha} z_{2n-1}, f_2^{2\alpha} z_{2n})|.$$

Moreover,

$$+\delta\left(\frac{d(f_1^{2\alpha} z_{2n+1}, f_2^{2\alpha} f_1^{2\alpha} z_{2n+1})[1+d(f_2^{2\alpha} z_{2n}, f_1^{2\alpha} f_2^{2\alpha} z_{2n})]}{1+d(f_2^{2\alpha} z_{2n}, f_1^{2\alpha} z_{2n+1})}\right)$$

$$+\delta\left(\frac{d(f_1^{2\alpha} z_{2n+1}, f_2^{2\alpha} f_1^{2\alpha} z_{2n+1})[1+d(f_2^{2\alpha} z_{2n}, f_1^{2\alpha} f_2^{2\alpha} z_{2n})]}{1+d(f_2^{2\alpha} z_{2n}, f_1^{2\alpha} z_{2n+1})}\right)$$

$$+\gamma\left(\frac{d(f_1^{2\alpha} z_{2n+1}, f_1^{2\alpha} f_2^{2\alpha} z_{2n})[1+d(f_2^{2\alpha} z_{2n}, f_2^{2\alpha} f_1^{2\alpha} z_{2n+1})]}{1+d(f_2^{2\alpha} z_{2n}, f_1^{2\alpha} z_{2n+1})}\right).$$

But $d(f_1^{2\alpha} z_{2n+1}, f_1^{2\alpha} f_2^{2\alpha} z_{2n}) = 0$ since $f_2^{2\alpha} z_{2n} = z_{2n+1}$. So that

$$d(z_{2n+2}, z_{2n+3}) \leqslant \lambda d(f_1^{2\alpha} z_{2n}, f_2^{2\alpha} z_{2n+1})$$

$$+\delta\left(\frac{d(f_1^{2\alpha} z_{2n+1}, f_2^{2\alpha} f_1^{2\alpha} z_{2n+1})[1+d(f_2^{2\alpha} z_{2n}, f_1^{2\alpha} f_2^{2\alpha} z_{2n})]}{1+d(f_2^{2\alpha} z_{2n}, f_1^{2\alpha} z_{2n+1})}\right).$$

Therefore,

$$|d(z_{2n+2},z_{2n+3})| \le \lambda |d(f_1^{2\alpha} z_{2n}, f_2^{2\alpha} z_{2n+1})|$$

$$+\delta \frac{|d(f_1^{2\alpha} z_{2n+1}, f_2^{2\alpha} f_1^{2\alpha} z_{2n+1})||1+d(f_2^{2\alpha} z_{2n}, f_1^{2\alpha} f_2^{2\alpha} z_{2n})|}{|1+d(f_2^{2\alpha} z_{2n}, f_1^{2\alpha} z_{2n+1})|}.$$

Since

$$|1+d(f_2^{2\alpha} z_{2n}, f_1^{2\alpha} f_2^{2\alpha} z_{2n})| = |1+d(f_2^{2\alpha} z_{2n}, f_1^{2\alpha} z_{2n+1})|$$

$$> |d(f_2^{2\alpha} z_{2n}, f_1^{2\alpha} z_{2n+1})|$$

which implies that

$$|d(z_{2n+2},z_{2n+3})| \le \lambda |d(f_1^{2\alpha} z_{2n}, f_2^{2\alpha} z_{2n+1})|$$

$$+2\delta |d(f_1^{2\alpha} z_{2n+1}, f_2^{2\alpha} f_1^{2\alpha} z_{2n+1})|$$

and

$$|d(z_{2n+2},z_{2n+3})| \le \frac{\lambda}{1-2\delta} |d(f_1^{2\alpha} z_{2n}, f_2^{2\alpha} z_{2n+1})|.$$

Let $\dot{o} = \frac{\lambda}{1-2\delta}$. Then

$$|d(z_n,z_{n+1})| \le \dot{o} |d(z_{n-1},z_n)| \le \dot{o}^2 |d(z_{n-2},z_{n-1})| \le \dot{o}^3 |d(z_{n-3},z_{n-2})| \le \ldots \le \dot{o}^n |d(z_0,z_1)|.$$

So that for $m > n$,

$$|d(z_n,z_m)| \le |d(z_n,z_{n+1})| + |d(z_{n+1},z_{n+2})|$$

$$+|d(z_{n+2},z_{n+3})| + \ldots + |d(z_{m-1},z_m)|$$

$$\le (\grave{o}^{n} + \grave{o}^{n+1} + \grave{o}^{n+2} + \ldots + \grave{o}^{m-1}) \,|\, d(z_0, z_1)|$$

$$\le (\frac{\grave{o}^{n}}{1-\grave{o}}) \,|\, d(z_0, z_1) \,| \to 0 \text{ as } n \to \infty.$$

Hence, the sequence $\{z_n\}$ is Cauchy. By the completness of the complex valued metric space X, there exists a number $t \in X$ such that $x_n \to 0$ as $n \to \infty$. Assume by the contradiction that $t \ne f_1^{\hat{a}}\, t$ and $0 \precsim z = d\left(t, f_1^{\,2\alpha} t\right)$ so that

$$z = d\left(t, f_1^{\,2\alpha} t\right) \precsim d\left(t, f_1^{\,2\alpha} z_{2n+1}\right) + d\left(f_2^{\,2\alpha} z_{2n+1}, f_1^{\,2\alpha} t\right)$$

$$l(t, z_{2n+2}) + \lambda d(t, z_{2n+1}) + \delta(\frac{d(t, f_1^{2\alpha} t)[1 + d(z_{2n+1}, f_2^{2\alpha} z_{2n+1})]}{1 + d(t, f_1^{2\alpha} z_{2n})})$$

$$+\gamma(\frac{d(z_{2n+1}, f_1^{2\alpha} t)[1 + d(t, f_2^{2\alpha} z_{2n+1})]}{1 + d(t, f_1^{2\alpha} z_{2n})})$$

Hence,

$$|d(t, f_1^{2\alpha} t)| \le \;|\, d(t, z_{2n+2})| + \lambda |d(t, z_{2n+1})|$$

$$+ \;\delta(\frac{|z|[1 + |d(z_{2n+1}, z_{2n+2})|]}{|1 + d(t, z_{2n+1})|})$$

$$+\gamma(\frac{|d(z_{2n+1}, f_1^{2\alpha} t)|[1 + |d(t, z_{2n+2})|]}{|1 + d(t, z_{2n+1})|})$$

Therefore, $|\, d(t, f_1^{2\alpha} t)\,| = 0$ As $n \to \infty$ which is a contradiction. Hence $t = f_1^{\hat{a}}\, t$. In similar arguemnts we find that $t = f_2^{2\alpha} t$. Also $t = f_1^{2\alpha} t = f_2^{2\alpha} t$. Now since $f_1^{2\alpha}$ is sequentially continuous, then

$$f_1^{2\alpha}\left(\lim_{n\to\infty} (f_2^{2\alpha} f_1 z_n)\right) = \lim_{n\to\infty}\left(f_1^{2\alpha}(f_2^{2\alpha} f_1 z_n)\right).$$

So that $f_1^{2\alpha} f_2^{2\alpha}\, t = f_1^{2\alpha}\, t$.

Since $f_1^{2\alpha} f_2^{2\alpha} f_1(t) = f_1^{2\alpha} f_1(t)$, and $f_2^{2\alpha} f_1^{2\alpha} f_1(t) = f_2^{2\alpha} f_1(t)$ then

$$d(t, f_1 t) = d(f_1^{\,2\alpha} f_2^{\,2\alpha} t, f_2^{\,2\alpha} f_1^{\,2\alpha}(f_1 t)) \precsim \lambda d\left(f_1^{\,2\alpha} t, f_2^{\,2\alpha}(f_1 t)\right)$$

$$+\delta(\frac{d(f_2^{2\alpha}(f_1 t), f_2^{2\alpha} f_1^{2\alpha}(f_1 t))[1 + d(f_1^{2\alpha} t, f_1^{2\alpha} f_2^{2\alpha} t)]}{1 + d(f_1^{2\alpha} t, f_2^{2\alpha}(f_1 t))})$$

$$+\gamma(\frac{d(f_2^{2\alpha}(ft), f_2^{2\alpha} f_1^{2\alpha}(f_1 t))[1 + d(f_1^{2\alpha} t, f_1^{2\alpha} f_2^{2\alpha} t)]}{1 + d(f_1^{2\alpha} t, f_2^{2\alpha}(f_1 t))}).$$

Therefore,

$$|d(t, f_1 t)| \le \lambda |d(f_1^{2\alpha} t, f_2^{2\alpha}(f_1 t))|$$

$$+\delta \frac{|d(f_2^{2\alpha}(f_1 t), f_2^{2\alpha} f_1^{2\alpha}(f_1 t))||1 + d(f_1^{2\alpha} t, f_1^{2\alpha} f_2^{2\alpha} t)|}{|1 + d(f_1^{2\alpha} t, f_2^{2\alpha}(f_1 t))|}$$

$$\le [\delta + \lambda] |d(t, f_1 t)|.$$

So that $|\, d(t, f_1 t)\,| = 0$ and $f_1 t = t$ since $\ddot{a} + \ddot{e} < 1$. Similarly we get $f_2 t = t$. Thus t is a common fixed point of f and g. Let $s \in X$ be another common fixed point of f_1 and f_2 (i.e. $s = f_1 s = f_2 s$). Then

$$d(s, t) = d(f_1^{\,2\alpha} f_2^{\,2\alpha} s, f_2^{\,2\alpha} f_1^{\,2\alpha} t) \precsim \lambda d(f_1^{\,2\alpha} s, f_2^{\,2\alpha} t)$$

$$+\delta(\frac{d(f_2^{2\alpha}t,f_2^{2\alpha}f_1^{2\alpha}(f_1t))[1+d(f_1^{2\alpha}s,f_1^{2\alpha}f_2^{2\alpha},(f_1s))]}{1+d(f_1^{2\alpha}s,f_2^{2\alpha}t)})$$

$$+\gamma(\frac{d(f_2^{2\alpha}s,f_2^{2\alpha}f_1^{2\alpha}(f_1t))[1+d(f_1^{2\alpha}t,f_1^{2\alpha}f_2^{2\alpha},(f_1s)]}{1+d(f_1^{2\alpha}s,f_2^{2\alpha}t)})$$

Hence,

$$|d(s,t)|\le\lambda|d(f_1^{2\alpha}s,f_2^{2\alpha}t)|+\delta(\frac{|d(f_2^{2\alpha}t,f_2^{2\alpha}f_1^{2\alpha}(f_1t))||1+d(f_1^{2\alpha}s,f_1^{2\alpha}f_2^{2\alpha},(f_1s))|}{|1+d(f_1^{2\alpha}s,f_2^{2\alpha}t)|}$$

$$\le[\delta+\lambda]|d(s,t)|$$

which is a contradicion and $s=t$ since ä+ë < 1. This complete the proof of the theorem. W

Let $\{f_i\}_1^m$ and $\{g_j\}_1^n$ be two families of pairwise commuting self mappings on a complete complex valued metric space X. Define F and G to be a finite products of self mappings on X. such that $F=f_1f_2\cdots f_n$ and $G=g_1g_2\cdots g_n$. Then we have the next result.

Corollary 3.2. Let $\{f_i\}_1^m$ and $\{g_j\}_1^n$ be two families of pairwise sequentially continous commuting self mappings on the complex valued matric space X. Define F and G to be a finite products of self mappings on X such that $F=f_1f_2\cdots f_n$ and $G=g_1g_2\cdots g_n$. If $F(X)\subset G(X)$, the order pair $\{F,G\}$ is $2\alpha-$ weakly commuting and the condition is satisfied

$$d(F^{2\alpha}G^{2\alpha}z,G^{2\alpha}F^{2\alpha}w)\preccurlyeq\lambda d(F^{2\alpha}z,G^{2\alpha}w)$$

$$+\delta(\frac{d(F^{2\alpha}w,G^{2\alpha}F^{2\alpha}w)[1+d(G^{2\alpha}z,F^{2\alpha}G^{2\alpha}z)]}{1+d(G^{2\alpha}z,F^{2\alpha}w)})$$

$$+\gamma(\frac{d(F^{2\alpha}w,F^{2\alpha}G^{2\alpha}z)[1+d(G^{2\alpha}z,G^{2\alpha}F^{2\alpha}w)]}{1+d(G^{2\alpha}z,F^{2\alpha}w)})$$

where, á,ä,ã ≥ 0 and á+2ä+ã <1. Then F and G have a unique common fixed point.

Cosider $\varphi:[0;+\infty)\to[0;+\infty)$ is a Lesbesgue-integrable mapping which is summable (i.e.with finite integral) on each compact subset of $[0;+\infty)$, non-negative, and such that for each ò >0, $\int_0^{ò}\varphi(t)>0$. Then by using the general contractive condition of integral type we have the following Corollaries.

Corollary 3.3. Let f_1 and f_2 be two self sequentially continous mappings on a complete complex valued metric space X such that $f_1(X)\subset f_2(X)$ and the order pair $\{f,g\}$ is $2\alpha-$ weakly commuting, also the condition is satisfied

$$\int_0^{d(f_1^{2\alpha}f_2^{2\alpha}z,f_2^{2\alpha}f_1^{2\alpha}w)}\varphi(t)\preccurlyeq\lambda\int_0^{d(f_1^{2\alpha}z,f_2^{2\alpha}w)}\varphi(t)$$

$$+\delta\int_0^{(\frac{d(f_1^{2\alpha}w,f_2^{2\alpha}f_1^{2\alpha}w)[1+d(f_2^{2\alpha}z,f_1^{2\alpha}f_2^{2\alpha}z)]}{1+d(f_2^{2\alpha}z,f_1^{2\alpha}w)})}\varphi(t)$$

$$+\gamma\int_0^{(\frac{d(f_1^{2\alpha}w,f_1^{2\alpha}f_2^{2\alpha}z)[1+d(f_2^{2\alpha}z,f_2^{2\alpha}f_1^{2\alpha}w)]}{1+d(f_2^{2\alpha}z,f_1^{2\alpha}w)})}\varphi(t)$$

where, á,ä,ã ≥ 0, á+ä+ã <1 and

$\ddot{o}:[0;+\infty)\to[0;+\infty)$ is a Lesbesgue-integrable mapping which is summable (i.e.with finite integral) on each compact subset of $[0;+\infty)$, non-negative, and such that for each $\dot{o}>0$, $\int_0^{\dot{o}}\ddot{o}(t)>0$. Then f_1 and f_2 have a unique common fixed point .

Corollary 3.4. Let $\{f_i\}_1^m$ and $\{g_j\}_1^n$ be two families of pairwise sequentially continous commuting self mappings on the complex valued matric space X. Define F and G to be a finite products of self mappings on X such that $F=f_1f_2\cdots f_n$ and $G=g_1g_2\cdots g_n$. If $F(X)\subset G(X)$, the order pair $\{F,G\}$ is $2\alpha-$weakly commuting and the condition is satisfied

$$\int_0^{d(f_1^{2\alpha}f_2^{2\alpha}z,f_2^{2\alpha}f_1^{2\alpha}w)}\varphi(t)\precsim\lambda\int_0^{d(f_1^{2\alpha}z,f_2^{2\alpha}w)}\varphi(t)$$

$$+\delta\int_0^{(\frac{d(F^{2\alpha}w,G^{2\alpha}F^{2\alpha}w)[1+d(G^{2\alpha}z,F^{2\alpha}G^{2\alpha}z)]}{1+d(G^{2\alpha}z,F^{2\alpha}w)})}\varphi(t)$$

$$+\gamma\int_0^{(\frac{d(F^{2\alpha}w,F^{2\alpha}G^{2\alpha}z)[1+d(G^{2\alpha}z,G^{2\alpha}F^{2\alpha}w)]}{1+d(G^{2\alpha}z,F^{2\alpha}w)})}\varphi(t)$$

where, $\acute{a},\ddot{a},\tilde{a}\geq 0$, $\acute{a}+2\ddot{a}+\tilde{a}<1$ and

$\ddot{o}:[0;+\infty)\to[0;+\infty)$ is a Lesbesgue-integrable mapping which is summable (i.e.with finite integral) on each compact subset of $[0;+\infty)$, non-negative, and such that for each $\dot{o}>0$, $\int_0^{\dot{o}}\ddot{o}(t)>0$. Then F and G have a unique common fixed point.

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