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Solution of Fuzzy Multi Criteria Project Management Problem by Fuzzy Programming Technique with Possibilistic Approach

Riddhi K. Rekh[1*] and Jayesh M. Dhodiya[2]

[1]RNGPIT, Isroli, Bardoli – 394620, Gujarat, India; rkr.fetr@gmail.com
[2]AMHD, SVNIT, Surat – 395007, Gujarat, India; jdhodiya2002@yahoo.com

Abstract

Objective: This study contains a linear and exponential membership function based fuzzy programming technique with possibilistic approach and its application to find the critical path in project network. **Methods:** It contains four criteria; cost, quality, time and risk of the project activities for project management. For finding the solution of this multi-criteria project management problem in fuzzy programming technique for better decision by Decision Maker (DM) alpha level set concept is utilized. We have provided numerical illustration to demonstrate working of the proposed methodology. **Findings:** To analyse the performance of the proposed approach, we have compared it with closely related fuzzy group multi-criteria decision making related Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method for the critical path selection. **Application:** Further, a case study from manufacturing engineering industry is also presented to justify the applicability and potentials of proposed methodology in a better way. Degree of satisfaction is calculated for different values of alpha levels to validate the applicability of this new approach.

Keywords: α-level, Critical Path, Fuzzy Programming, Project Management**,** Possibilistic Approach

1. Background

Planning is essential for scheduling and controlling the various activities (or tasks) involved in the project, before commencing any project. This will help undertaking the project, identifying probable bottlenecks and if necessary, preparing an alternate work-plan for the project[1]. The techniques of operations research used for planning, scheduling and controlling large and complex projects are often referred as Network Analysis, Network Planning or Network planning and scheduling techniques. All these techniques are based on the representation of the project as a Network of activities. To help US Navy's Polaris Nuclear Submarine Missile project involving thousands of activities in the planning and scheduling and for that a research team developed PERT in 1956-58. The objective of the team was to efficiently plan and develop the Polaris missile system. This technique was useful since 1958 for all jobs or projects having an element of uncertainty in the estimation of duration, just like with new types of projects. Such approach has never been taken up before[2]. Critical Path Method (CPM) was developed independently, by E.I. Du Pont Company with Remington Rand Corporation at the same time. The aim behind its development was to provide a technique for control of the maintenance of company's chemical plants. The main objective before starting any project is to schedule all required activities in an efficient manner so as to complete it within a specified time limit and with minimised cost for completion. Kelly[3] developed and solved the time-cost trade-off problem by heuristic algorithm and mathematical modelling by assuming a linear relation between time and cost of an activity. A special parametric linear program for CPM that can be effectively solved by network flow methods was developed by the author. The model provides solutions to concerning project budget, labor requirements, procurement and plan restrictions, the results of slowdowns and conveyance problems. Several researcher

**Author for correspondence*

has developed mathematical programming model for the price and time trade-off issues few of them are as follows[4] establish the link between project's total duration and project's total cost. In project network[5] describe an exact procedure for the discrete time cost trade-off problem.

The Multi Objective Linear Programming (MOLP) technique for conflicting objectives to be view as without the necessity of creating specific rankings for the objectives.A multi objective non- linear programming approach for project planning was presented by[6]. In[7-9]during 1978-1990 have presented the project planning problem with multiple criteria which are dealing with the activity durations which is vary with the level of resources allocated to the activity. In[10]presented a LP model to obtain solution of such problemswith interactive calculative method.First time robust optimization was implemented by[11] for resources allocation time cost tradeoff problem. In[12] also formulated a fuzzy logic theory based approach to solve time-cost trade-off problem[13] presented a method to calculate intervals of possible values of the latest starting times and floats of the activities in networks with imprecise durations[14] developed an approach to solve project scheduling problem to compute earliest and latest events time by LP. Many project activities may be executed first time and therefore it's tough to get precise estimates of resource consumption for them. Moreover, at the time of designing projects, it's tough to exactly assume information concerning the duration of activities. In such things, fuzzy set theory concept is applied[15]. Fuzzy set theory concepts will handle inexact input information containing feelings and emotions mistreatment subjective judgements of the choice manufacturers while not posterior frequency distributions. In[16] have utilized activities time as a fuzzynumber in project network and developed an approach to the critical path analysis. In[17] utilized fuzzy number ranking method for find the critical path and its analysis in project network problem. In[18] develop statistical confidence interval estimates and a signed-distance ranking based a fuzzy critical method for solution of project network problem. An analytical approach for measurement of criticalcriteria in a project network with fuzzy activity times was presented by[19]. In[20] considered activity duration as a trapezoidal fuzzy numbers and proposed an approach to find critical path. For find the solution of full fuzzy critical path problem[21] developed an approach by using L-R flat fuzzy numbers to find the fuzzy optimal solution. In[22] find a solution approach for the fuzzy critical path problem using L - L fuzzy numbers. The fuzzy ranking methods to determine critical path were proposed by[23]. In[24] also developed fuzzy ranking based approach to find critical path in project network related problem. In most this discussed literature only fuzzy time criterion was considered to determine the critical path. The fact is that to find the critical path time is not only one criterion but other criteria like cost, risk and quality may be of equal or some time with greater importance. Sometime environmental conditions will reduce the project scope, increase its cost and duration and/or compromise its quality hence for very realistic critical path it is necessary to developed an approach that include the possibility of prevalence of such events should be thought-about within the project analysis for overall satisfaction.

Multi Criteria Decision Making (MCDM) are utilized by few studies of project management like contractor's prequalification[25], competition bidding[26], project selection[27–29] and contractor selection[30]. In[31] demonstrate risk, cost, time and quality criteria with fuzzy environment based project network problem and its solution by developed algorithm. Cristobal[7] applied PROMETHEE methodology for fuzzy criteria's (safety, quality, cost and time) to obtain the critical path. In[32] also proposed an approach to find critical path for project network problem with four criteria quality, risk, time and cost. Sometime vague estimates are obtained by authors when least information is available about the project. As Quality depends on expert's attitudes and beliefs, which differs frequently over the complete life-cycle of the project development, the Quality criterion was considered subjective in nature. Most of the time to represents the quality the triangular fuzzy numbers are used by authors because they offer a good compromise between accuracy and computational time[33-35]. In[32] calculated the advantage and disadvantage scores of each path relative to all the other paths on each criterion and also fuzzy strength scores by advantage scores for all paths for disadvantage scores were obtained fuzzy weakness scores. In this work for each project path by using the fuzzy strength scores and fuzzy weakness scores, they also calculated the strength index and weakness index end at last total performance score of each path is considered to obtain the critical path.

The following limitations are noticed in existing literature which motivated us to carry out this research work to find the critical path under fuzzy environment. 1. To determine the critical path most of the earlier literature mention only one criterion as a time however, quality cost and risk parameters also play a significant role for finding a critical path of the project. 2. In most of the big industries their project network is complex and very difficult to take the decision in multi criteria's problem so they want number of alternatives with respect to criteria which are not included in existing literature. 3. In most of the project analysis under fuzzy environment utilized defuzzification techniques and because of that some information is lost on the uncertainties of fuzzy numbers; so it is difficult to captured uncertainty in real sense. (4) In literature, very few integrated methods exist to analyse fuzzy environment based project network analysis with quantitative and qualitative information. In this paper we have developed possibilistic programming based approach to find the solution of fuzzy project network analysis which maintain the uncertainty of fuzzy numbers in the real sense with multiple criteria decision making without calculating weakness index, strength index and performance ranking for each path of the network for the multi-criteria critical path selection under fuzzy environment and also provide multiple alternatives by fuzzy programming techniques to take the decisions in fuzzy based environment.

2. Fuzzy Multi Objective Critical Path Problem Formulation (FMOCPP)

The main assumptions and characteristics of the FMOCPP are as follows:

- Each path of the project network will be considered.
- Dummy activity is considered with all objective values as zero.
- The decision making matrix should minimize Time, Cost, and Risk and maximize Quality.
- Triangular fuzzy numbers are considered for Linguistic variables.

3. Fuzzy Multi Objective Critical Path Problem Model

The mathematical formulation of FMOCPP is made by using the following variables, parameters and the indices[32–35] (2016).

- Indices i and j defines path joining node i and j.
- Decision variables $y_{ij} = 1, \text{if } ij \text{ is optimal path}$
  $y_{ij} = 0, \text{otherwise}$
- E = Set of arcs of the project network, $(i, j) \in E$

4. Formulation of Objective Functions

The total consumed time, total cost; total quality level and total risk are given as follows:

$$\tilde{z}_1 = \sum_{ij \in E} \tilde{t}_{ij} y_{ij},\ \tilde{z}_2 = \sum_{ij \in E} \tilde{c}_{ij} y_{ij},\ \tilde{z}_1 = \sum_{ij \in E} \tilde{q}_{ij} y_{ij},\ \tilde{z}_1 = \sum_{ij \in E} \tilde{r}_{ij} y_{ij}$$

In this problem, the quality of the linguistic variable are rated as "very low", "low", "medium low", "medium", "medium high", "high" and "very high", which are represented as (0,1,1), (0,1,3), (1,3,5), (3,5,7), (5,7,9), (7,9,10) and (9,9,10), respectively. The six levels represent the quality of project completion, where "very high" and "very low" levels denote the most efficient and least efficient, respectively, that is, a shift from "very high" to "very low" indicates that quality decreases whereas the related fuzzy values increase. Here quality objective functions are converting in minimum form to maintain uniformity of objective functions.

5. Model Constraints

The constraints of FMOCPP are formulated as follows:

$$\sum_j y_{1j} = 1 \tag{1}$$

$$\sum_j y_{ij} = \sum_k y_{kj}, i = 2, 3, \ldots, n-1. \tag{2}$$

$$\sum_k y_{kn} = 1 \tag{3}$$

$$y_{ij} \geq 0, \forall (i, j) \in A \tag{4}$$

6. Decision Problem

The FMOCPP is now formulated as follows:
(Model - 1)

$$\left(\tilde{z}_1,\tilde{z}_2,\tilde{z}_3,\tilde{z}_4\right)=\left(\sum_{ij\in E}\tilde{t}_{ij}y_{ij},\sum_{ij\in E}\tilde{c}_{ij}y_{ij},\sum_{ij\in E}\tilde{q}_{ij}y_{ij},\sum_{ij\in E}\tilde{r}_{ij}y_{ij}\right)$$

Subject to the constraints (1) - (4).

7. Some Preliminaries

To find the solution of this fuzzy project management problem some are required which are as follows:

7.1 Possibilistic Programming Approach

Most of the times when we collect real-world problems related data then generally it's include some kind of unreliability which are represented using fuzzy numbers because of their nature. Possibilistic distribution is utilized to quantify such kind of fuzzy numbers[36–40]. Many crucial applications have been used possibilistic programming approach for finding the solution of multi criteria's based fuzzy optimization model with unspecific objective function. Hence in this paper we have utilized possibilistic programming based approach to solve FMOCPP which maintain the uncertainty of the problem in real sense and convert the FMOCPP in crisp MOCPP.

7.2 Triangular Possibilistic Distribution (TPD)

Triangular probability distribution is built by most possible value (m) (possibility degree = 1), the most optimistic value(o) (possibility degree = 0) and the most pessimistic value (p) (possibility degree = 0) respectively which is generally denoted by. $\left(c_i^m\right),\left(c_i^0\right) and \left(c_i^p\right)$ Figure 1 (Triangular Fuzzy Diagram) indicate that objective function time is defined at three positions as $\left(c_i^m,1\right),\left(c_1^0=0\right) and \left(c_1^p=0\right)$ which is minimized by shifting the three positions of TPD to the left because vertical coordinates of the points are fixed by 0 or 1. Thus, only the three horizontal coordinates are considered[16].

7.3 α - Level Sets

Several researchers[18,40–44] have used this α-level set concept to find the solutions for fuzzy optimization-related problems. To set up a connection between traditional and fuzzy set theories, a α-level set is the most extremely important theory which was introduced by[42]. Largest α-value indicate the greater degree of membership in the initial fuzzy sets with upper and lower bond which is useful a smaller but more optimistic judgment. Generally, α-level indicate the DM confidence with his fuzzy judgement is also named as the confidence level. An interval judgment with a large spared, which point out a high level of pessimism and uncertainty is provided by smallest α-value. We have used this concept in the present study to determine the confidence of the DM with respect to his fuzzy judgment.

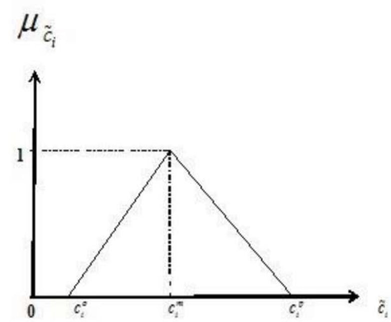

Figure 1. Triangular possibilistic distribution of. $\tilde{c}_i$

7.4 Linear Membership Function

A linear membership function can be defined as follows:

$$\mu_{z_{ij}}(x) = 1 \quad if\ z_{ij} < z_{ij}^{PIS}$$

$$\mu_{z_{ij}}(x) = 1-\frac{Z_{ij}-Z_{ij}^{PIS}}{Z_{ij}^{NIS}-Z_{ij}^{PIS}} \quad if\ Z_{ij}^{PIS}<Z_{ij}<Z_{ij}^{NIS}$$

$$\mu_{z_{ij}}(x) = 0 \quad if\ Z_{ij}>Z_{ij}^{NIS}$$

7.5 Exponential Membership Function

$$\mu_{z_{ij}}^{E}\left(x\right)=1, if z_{ij}\le z_{ij}^{PIS}$$

$$\mu_{z_{ij}}^{E}\left(x\right)=\frac{e^{-s\psi_{ij}(x)}-e^{-s}}{1-e^{-s}}, if z_{ij}^{PIS}<z_{ij}<z_{ij}^{NIS}$$

$$\mu_{z_{ij}}^{E}\left(x\right)=0, if z_{ij}\ge z_{ij}^{NIS}$$

where $\psi_{ij}=\frac{Z_{ij}-Z_{ij}^{PIS}}{Z_{ij}^{NIS}-Z_{ij}^{PIS}}$ and s is non-zero shape parameter given by DM that $0\le \mu_{z_{ij}}(x)\le 1$. For s > 0 (s < 0), the membership function is strictly concave (convex) in $\left[Z_{ij}^{PIS}, Z_{ij}^{NIS}\right]$. The value of this fuzzy membership function allows us to model the grades of precision in corresponding objective function.

8. Formulation of Multi Objective 0-1 Programming Model

To convert model 1 into auxiliary multi-objective optimization model, we used Triangular Possibilistic Distribution (TPD) strategy to treat the imprecise objectives. The cost, time, risk and quality objective functions are described as:

$$\min \widetilde{Z}_1 = \min\left(Z_1^0, Z_1^m, Z_1^p\right) = \sum_{ij\in E} \tilde{t}_{ij} y_{ij}$$
$$= \min\left(\sum_{ij\in E} t_{ij}^0 y_{ij}, \sum_{ij\in E} t_{ij}^m y_{ij}, \sum_{ij\in E} t_{ij}^p y_{ij}\right) \quad (5)$$

where $t_{ij} = \left(t_{ij}^0, t_{ij}^m, t_{ij}^p\right)$, which can be considered as follows:

$$\left(\min Z_{11}, \min Z_{12}, \min Z_{13},\right)$$
$$= \min\left(\sum_{ij\in E} t_{ij}^0 y_{ij}, \sum_{ij\in E} t_{ij}^m y_{ij}, \sum_{ij\in E} t_{ij}^p y_{ij}\right) \quad (6)$$

Similarly, objective functions for cost, risk and quality criteria are defined as follows:

$$\left(\min Z_{21}, \min Z_{22}, \min Z_{23}\right)$$
$$= \min\left(\sum_{ij\in E} c_{ij}^0 y_{ij}, \sum_{ij\in E} c_{ij}^m y_{ij}, \sum_{ij\in E} c_{ij}^p y_{ij}\right) \quad (7)$$

$$\left(\min Z_{21}, \min Z_{22}, \min Z_{23}\right)$$
$$= \min\left(\sum_{ij\in E} c_{ij}^0 y_{ij}, \sum_{ij\in E} c_{ij}^m y_{ij}, \sum_{ij\in E} c_{ij}^p y_{ij}\right) \quad (8)$$

$$\left(\min Z_{41}, \min Z_{42}, \min Z_{43}\right)$$
$$= \min\left(\sum_{ij\in E} r_{ij}^0 y_{ij}, \sum_{ij\in E} r_{ij}^m y_{ij}, \sum_{ij\in E} r_{ij}^p y_{ij}\right) \quad (9)$$

Equations (5) - (9) are associated with optimistic scenario, the most likely scenario and the pessimistic scenario respectively.

Hence the model becomes:

(Model -2)

$$(\min Z_{11}, \min Z_{12}, \min Z_{13}, \min Z_{21}, \min Z_{22}, \min Z_{23}, \min Z_{31}, \min Z_{32}, \min Z_{33}, \min Z_{41}, \min Z_{42}, \min Z_{43})$$

$$= \begin{pmatrix} \sum_{ij\in E} t_{ij}^0 y_{ij} & \sum_{ij\in E} t_{ij}^m y_{ij} & \sum_{ij\in E} t_{ij}^0 y_{ij} \\ \sum_{ij\in E} c_{ij}^0 y_{ij} & \sum_{ij\in E} c_{ij}^m y_{ij} & \sum_{ij\in E} c_{ij}^p y_{ij} \\ \sum_{ij\in E} q_{ij}^0 y_{ij} & \sum_{ij\in E} q_{ij}^m y_{ij} & \sum_{ij\in E} q_{ij}^p y_{ij} \\ \sum_{ij\in E} r_{ij}^0 y_{ij} & \sum_{ij\in E} r_{ij}^m y_{ij} & \sum_{ij\in E} r_{ij}^p y_{ij} \end{pmatrix}$$

Subject to the constraints (1)-(4).

Using the $\propto$-level set concepts ($0 \le \propto \le 1$), each t_{ij} can be stated as:

$$\left(t_{ij}\right)_{\propto}^0 = t_{ij}^0 + \propto\left(t_{ij}^m - t_{ij}^0\right), \left(t_{ij}\right)_{\propto}^m = t_{ij}^m, \left(t_{ij}\right)_{\propto}^p = t_{ij}^p - \propto\left(t_{ij}^p - t_{ij}^m\right)$$

Equation (6) can be written as:

$$\left(\min Z_{11}, \min Z_{12}, \min Z_{13}\right)$$
$$= \min\left(\sum_{ij\in E}\left(t_{ij}\right)_{\propto}^0 y_{ij}, \sum_{ij\in E}\left(t_{ij}\right)_{\propto}^m y_{ij}, \sum_{ij\in E}\left(t_{ij}\right)_{\propto}^p y_{ij}\right) \quad (10)$$

$$\left(\min Z_{21}, \min Z_{22}, \min Z_{23}\right)$$
$$= \min\left(\sum_{ij\in E}\left(c_{ij}\right)_{\propto}^0 y_{ij}, \sum_{ij\in E}\left(c_{ij}\right)_{\propto}^m y_{ij}, \sum_{ij\in E}\left(c_{ij}\right)_{\propto}^p y_{ij}\right) \quad (11)$$

Similarly, Multi Objective Optimization Problem (MOP) model of cost, risk and quality objective functions are as follows:

$$\left(\min Z_{31}, \min Z_{32}, \min Z_{33}\right)$$
$$= \min\left(\sum_{ij\in E}\left(r_{ij}\right)_{\propto}^0 y_{ij}, \sum_{ij\in E}\left(r_{ij}\right)_{\propto}^m y_{ij}, \sum_{ij\in E}\left(r_{ij}\right)_{\propto}^p y_{ij}\right) \quad (12)$$

$$\left(\min Z_{41}, \min Z_{42}, \min Z_{43}\right)$$
$$= \min\left(\sum_{ij\in E}\left(q_{ij}\right)_{\propto}^0 y_{ij}, \sum_{ij\in E}\left(q_{ij}\right)_{\propto}^m y_{ij}, \sum_{ij\in E}\left(q_{ij}\right)_{\propto}^p y_{ij}\right) \quad (13)$$

8.1 Auxiliary Multi-Objective 0-1 Programming Model

To determine the optimistic, most-likely and pessimistic scenarios by using the α-level set concept, the FMOCPP is converted into a crisp MOCPP also called as an auxiliary multi objective 0–1 programming model which is defined as follows:

(Model -3)

$$\begin{pmatrix} \min Z_{11}, \min Z_{12}, \min Z_{13}, \\ \min Z_{21}, \min Z_{22}, \min Z_{23}, \\ \min Z_{31}, \min Z_{32}, \min Z_{33}, \\ \min Z_{41}, \min Z_{42}, \min Z_{43} \end{pmatrix} = \begin{pmatrix} \sum_{ij\in E}(t_{ij})_{\propto}^{0} y_{ij}, & \sum_{ij\in E}(t_{ij})_{\propto}^{m} y_{ij}, & \sum_{ij\in E}(t_{ij})_{\propto}^{p} y_{ij}, \\ \sum_{ij\in E}(c_{ij})_{\propto}^{0} y_{ij}, & \sum_{ij\in E}(c_{ij})_{\propto}^{m} y_{ij}, & \sum_{ij\in E}(c_{ij})_{\propto}^{p} y_{ij}, \\ \sum_{ij\in E}(q_{ij})_{\propto}^{0} y_{ij}, & \sum_{ij\in E}(q_{ij})_{\propto}^{m} y_{ij}, & \sum_{ij\in E}(q_{ij})_{\propto}^{p} y_{ij}, \\ \sum_{ij\in E}(r_{ij})_{\propto}^{0} y_{ij}, & \sum_{ij\in E}(r_{ij})_{\propto}^{m} y_{ij}, & \sum_{ij\in E}(r_{ij})_{\propto}^{p} y_{ij}, \end{pmatrix}$$

Subject to the constraints (1)-(4).

9. Fuzzy Programming Technique-based Solution Approach to Solve Auxiliary Model of FMOCPP

For finding the solution of the Model 3 by fuzzy programming technique first this models are solved for single objective function and for each objective function find out the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) of the model. Now, by Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) define a membership function $\mu(Z_k)$ for the k^{th} objective function. Here, different membership functions are utilized to find an efficient solution of this multi-objective critical path problem and by using this membership functions the Model 3 is converted into the following models:

Model 3.1:

Max λ,

Subject to the constraints,

$$\lambda \le \mu_{Zij};\ 0 \le \lambda \le 1 \tag{14}$$

Subject to equation (1) to equation (4).

When we utilize fuzzy linear membership function,

$$\propto_{Z_{ij}}(y) = 1, \qquad if z_{ij} \le z_{ij}^{PIS}$$

$$\propto_{Z_{ij}}(y) = \frac{z_{ij}^{NIS} - z_{ij}}{z_{ij}^{NIS} - z_{ij}^{PIS}}, \qquad if\ z_{ij}^{PIS} < z_{ij} < z_{ij}^{NIS} \tag{15}$$

$$\propto_{Z_{ij}}(y) = 0, \qquad if z_{ij} \ge z_{ij}^{NIS}$$

then model 3.1 structure is as follows:

Model 3.2:

Max λ,

Subject to the constraints:

$$\lambda \le \frac{Z_{ij}^{NIS} - Z_{ij}}{Z_{ij}^{NIS} - Z_{ij}^{PIS}} \tag{16}$$

Equation (1) to Equation (4).

$$\propto_{z_{ij}}^{E}(y) = 1, \qquad \text{if} z_{ij} \le z_{ij}^{PIS}$$

When we utilize exponential membership function,

$$\propto_{z_{ij}}^{E}(y) = 1, \qquad \text{if} z_{ij} \le z_{ij}^{PIS}$$

$$\mu_{z_{ij}}^{E}(y) = \frac{e^{-S\psi_{ij}(y)} - e^{-s}}{1 - e^{-s}}, \qquad \text{if} z_{ij}^{PIS} < z_{ij} < z_{ij}^{NIS} \tag{17}$$

$$\mu_{z}^{E}(y) = 0, \qquad \text{if } z_{ij} \le z_{ij}^{PIS},$$

where, $\Psi_{ij}(y) \le \frac{z_{ij} - z_{ij}^{PIS}}{z_{ij}^{NIS} - z_{ij}^{PIS}}$, and S is a non-zero parameter, prescribed by the decision maker, then model 4 structure is as follows:

Model 4:

Max λ,

Subject to the constraints:

$$\left(e^{-s\psi_k(y)} - e^{-s}\right) \ge \lambda\left(1 - e^{-s}\right) \tag{18}$$

where $\psi_k(y) \le \frac{z_k(y) - z_{ij}^{PIS}}{z_{ij}^{NIS} - z_{ij}^{PIS}}$, ,$k$=1,2,….$n$.

with constraints (1) to (4).

Algorithm

Input: Parameters: $(Z_1, Z_2, \ldots, Z_m, n)$

Output: Solution of FMOCPP

Solve FMOCPP ($Z_k \downarrow, X \uparrow$)

begin

read: problem

while problem = FMOCPP do

for k = 1 to m do

enter matrix Z_k

end

-| Find triangular possibilities distribution for each objective function.

-| Define the crisp multi-objective critical path problem according to α- level

-| Determine the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for each objective.

for *k = 1 to m do*

$$z_{ij}^{\mathrm{PIS}} = \min\left(z_i\right)_\alpha^0, i,j = 1,2,3$$

Subject to constraints (1) to (4),

end

for *k = 1 to m do*

$$z_{ij}^{\mathrm{NIS}} = \max\left(z_i\right)_\alpha^0,\ i,j = 1,2,3$$

Subject to constraints (1) to (4),

end

-| **Define linear or exponential membership function for each objective.**

for *k = 1 to m do*

$$\mu_{Z_{ij}}\left(y\right) = 1, \qquad if\ z_{ij} \le z_{ij}^{PIS}$$

$$\mu_{Z_{ij}}\left(y\right) = \frac{z_{ij}^{NIS} - z_{ij}}{z_{ii}^{NIS} - z_{ii}^{PIS}}, \qquad if z_{ij}^{PIS} < z_{ij} < z_{ij}^{NIS}$$

$$\mu_{Z_{ij}}\left(y\right) = 0, \qquad if\ z_{ij} \ge z_{ij}^{NIS}$$

or

$$\mu_{z_{ij}}^{E}\left(y\right) = 1, \text{if } z_{ij} \le z_{ij}^{PIS}$$

$$\mu_{z_{ij}}^{E}\left(y\right) = \frac{e^{-S\psi_{ij}(y)} - e^{-s}}{1 - e^{-s}}, \qquad \text{if } z_{ij}^{PIS} < z_{ij} < z_{ij}^{NIS}$$

$$\mu_{z}^{E}\left(y\right) = 0, \qquad \text{if} z_{ij} \le z_{ij}^{PIS},$$

end

-| ***find single objective optimization model under given constraints from MOP model.***

For*k = 1 to m do*

Maximize $= \lambda$

Subject to:

Constraints (1)-(4) and model 3.2 or model 4

$\lambda \ge 0;$

end

|- *find the solution SOP usingLINGOsoftware*

9.1 Flowchart

Flowchart of the solution procedure of FMOCPP is shown in Figure 2.

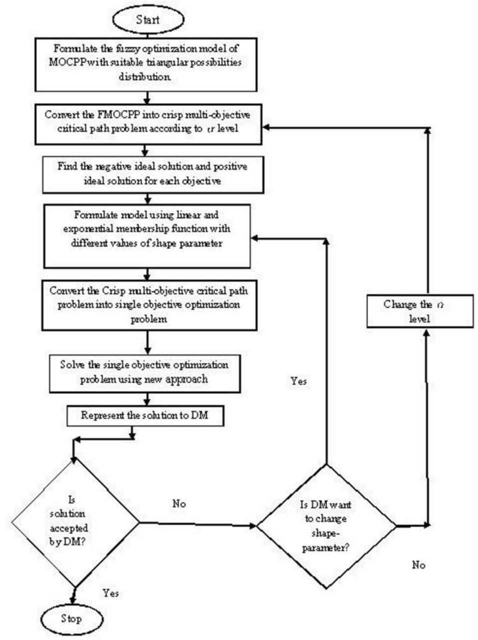

Figure 2. Flowchart of the solution procedure of FMOCPP.

10. Numerical Illustration

In this section, multi objective multi criteria critical path problem is considered[32]. A project network having13 major activities handled by a team of three experts D_1, D_2, *and* D_3 is given in Figure 3. Mapping of linguistic variable in to triangular fuzzy numbers and importance weight of criteria is described in section 4. Aggregated fuzzy values for each criteria Time, Cost, Risk and Quality with corresponding weights are given in Table 1. Aggregated fuzzy values for time criteria multiplied by corresponding weight is described in Table 2. Aggregated fuzzy values for each criteria time, cost, risk and quality for confidence level $\alpha = 0.1$, $\alpha = 0.5$ *and* $\alpha = 0.9$ multiplied by corresponding weight are described in Tables 3-5 respectively.

The mathematical formulation of FMOCPP is as follows:

Formulation of Model 2:

Minimize Z_1 (Time)=(7,39,70) × y_{01} + (21,45,70) × y_{02} + (35,81,140) × y_{03} + (21,60,100) × y_{14} + (63,114,180) × y_{24} + (42,84,120) × y_{35} + (84,144,200) × y_{78} + (70,123,170) × y_{25} + (63,126,190) × y_{46} + (35,75,110) × y_{37} + (42,93,160) × y_{59} + (91,159,210) × y_{69} + (98,153,200) × y_{89},

Minimize Z_2 (Cost) = (1200,7938.333,21600) × y_{01} + (400,3969.167,13050) × y_{02} + (250,3175.333,11250) × y_{03} + (200,3175.333,11700) × y_{14} + (1300,8515.667,24300) × y_{24}

$+ (5000,26701.667,63000) \times y_{35} + (1200,8660,25200) \times y_{78} + (1400,8948.667,24300) \times y_{25} + (700,5051.667,15750) \times y_{46} + (900,6350.667,18900) \times y_{37} + (1400,8732.167,23400) \times y_{59} + (3000,17320,45000) \times y_{69} + (2400,12990,32400) \times y_{89}$

Minimize Z_3 (Risk) = $(0,13.883,50) \times y_{01} + (5,30.543,70) \times y_{02} + (15,58.31,100) \times y_{03} + (5,36.097,70) \times y_{14} + (15,52.757,90) \times y_{24} + (5,36.097,70) \times y_{35} + (5,24.99,50) \times y_{78} + (5,36.097,70) \times y_{25} + (15,52.757,90) \times y_{46} + (15,47.203,90) \times y_{37} + (25,63.863,100) \times y_{59} + (25,69.417,100) \times y_{69} + (25,58.31,90) \times y_{89}$,

Minimize Z4 (Quality)=$(15,42.000,50) \times y_{01} + (12,28.000,30) \times y_{02} + (6,14.000,10) \times y_{03} + (12,32.666,30) \times y_{14} + (0,4.666,0) \times y_{24} + (6,18.666,10) \times y_{35} + (12,18.666,10) \times y_{78} + (12,28.000,30) \times y_{25} + (12,23.333,10) \times y_{46} + (6,14.000,10) \times y_{37} + (0,0.000,0) \times y_{59} + (0,0.000,0) \times y_{69} + (0,4.666,0) \times y_{89}$,

Subject to the constraints,

$$\begin{aligned} & y_{01} + y_{02} + y_{03} = 1 \\ & y_{01} = y_{14}, \\ & y_{02} = y_{24} + y_{25}, \\ & y_{03} = y_{35} + y_{37}, \\ & y_{14} + y_{24} = y_{46}, \\ & y_{25} + y_{35} = y_{59}, \\ & y_{46} = y_{69}, \\ & y_{37} = y_{78}, \\ & y_{78} = y_{89}, \\ & y_{59} + y_{69} + y_{89} = 1, \\ & y_{01}, y_{02}, y_{03}, y_{14}, y_{24}, y_{25}, y_{35}, y_{37}, y_{14}, y_{24}, y_{46}, y_{59}, y_{69}, \\ & y_{78}, y_{89} \geq 0. \end{aligned} \tag{14}$$

10.1 Solution

For finding the solution of this fuzzy project network analysis problem the fuzzy programming technique based developed approach utilized and for that at different level α the value of each objective PIS and NIS is as Table 6.

Substituting the values acquired in Table 6 in Model 3.2, we get:

Maximize = λ

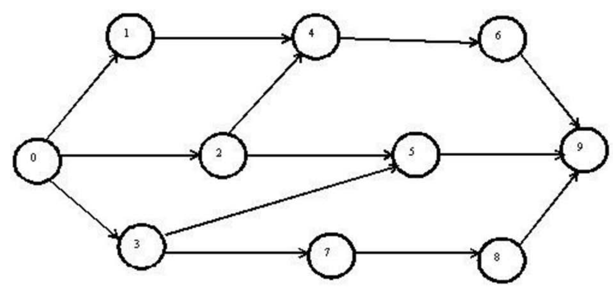

Figure 3. The project network.

Table 1. Aggregated fuzzy values of all four criteria

Activity	Criteria			
	C1 (Time)	C2 (Cost)	C3 (Risk)	C4 (Quality)
0-1	(1,4.333,7)	(1200,1833.33,2400)	(0,1.67,5)	(0,2.33,5)
0-2	(3,5,7)	(400,916.67,1450)	(1,3.67,7)	(1,4.33,7)
0-3	(5,9,14)	(250,733.33,1300)	(3,7,10)	(3,6.33,9)
1-4	(3,6.667,10)	(200,733.33,1300)	(1,4.33,7)	(1,3.67,7)
2-4	(9,12.667,18)	(1300,1966.67,2700)	(3,6.33,9)	(5,7.67,10)
3-5	(6,9.333,12)	(5000,6166.67,7000)	(1,4.33,7)	(3,5.67,9)
7-8	(12,16,20)	(1200,2000,2800)	(1,3,5)	(1,5.67,9)
2-5	(10,13.667,17)	(1400,2066.67,2700)	(1,4.33,7)	(1,4.33,7)
4-6	(9,14,19)	(700,1166.67,1750)	(3,6.33,9)	(1,5,9)
3-7	(5,8.33,11)	(900,1466.67,2100)	(3,5.67,9)	(3,6.33,9)
5-9	(6,10.333,16)	(1400,2016.16,2600)	(5,7.67,10)	(5,8.33,10)
6-9	(13,17.667,21)	(3000,4000,5000)	(5,8.33,10)	(5,8.33,10)
8-9	(14,17,20)	(2400,3000,3600)	(5,7,9)	(5,7.67,10)
Weight	**(7,9,10)**	**(1,4.33,9)**	**(5,8.33,10)**	**(3,7,10)**

Table 2. Aggregated fuzzy values of all criteria's multiplied by corresponding weights at α = 0

Activity	Criteria			
	C1 (Time)	C2 (Cost)	C3 (Risk)	C4 (Quality)
0-1	(7,39,70)	(1200,7938.33,21600)	(0,13.88,50)	(15,42,50)
0-2	(21,45,70)	(400,3969.17,13050)	(5,30.54,70)	(12,28,30)
0-3	(35,81,140)	(250,3175.33,11250)	(15,58.31,100)	(6,14,10)
1-4	(21,60,100)	(200,3175.33,11700)	(5,36.09,70)	(12,32.67,30)
2-4	(63,114,180)	(1300,8515.67,24300)	(15,52.75,90)	(0,4.67,0)
3-5	(42,84,120)	(5000,26701.67,63000)	(5,36.09,70)	(6,18.67,10)
7-8	(84,144,200)	(1200,8660,25200)	(5,24.99,50)	(12,18.67,10)
2-5	(70,123,170)	(1400,8948.67,24300)	(5,36.09,70)	(12,28,30)
4-6	(63,126,190)	(700,5051.67,15750)	(15,52.76,90)	(12,23.33,10)
3-7	(35,75,110)	(900,6350.67,18900)	(15,47.20,90)	(6,14,10)
5-9	(42,93,160)	(1400,8732.17,23400)	(25,63.86,100)	(0,0,0)
6-9	(91,159,210)	(3000,17320,45000)	(25,69.42,100)	(0,0,0)
8-9	(98,153,200)	(2400,12990,32400)	(25,58.31,90)	(0,4.67,0)

Table 3. Aggregated fuzzy values at0.1 multiplied by corresponding weights

Activity	Criteria			
	C1 (Time)	C2 (Cost)	C3 (Risk)	C4 (Quality)
0-1	(9.6,39,66.66)	(1684.02,7938.33,19995.66)	(0.89,13.88,45.88)	(17.34,42,49.47)
0-2	(23.04,45,67.32)	(602.07,3969.17,11917.76)	(6.76,30.54,65.55)	(13.6,28,30.07)
0-3	(38.88,81,133.65)	(379.68,3175.33,10225.38)	(18.13,58.31,95.38)	(6.8,14,10.67)
1-4	(24.24,60,95.7)	(337.69,3175.33,10609.36)	(7.11,36.09,66.21)	(13.83,32.67,30.72)
2-4	(67.44,114,172.92)	(1821.77,8515.67,22413.35)	(17.78,52.76,85.86)	(0.23,4.67,0.65)
3-5	(45.60,84,116.16)	(6820.52 ,26701.67,59019.92)	(7.11,36.09,66.21)	(7.03,18.67,11.32)
7-8	(89.28,144,194.04)	(1706.24,8660,23209.76)	(6.4,24.99,47.20)	(13.15,18.67,11.32)
2-5	(74.64,123,165)	(1955.07,8948.67,22498.68)	(7.11,36.09,66.21)	(13.6,28,30.07)
4-6	(68.4,126,183.15)	(995.31,5051.67,14434.99)	(17.78,52.76,85.88)	(13.37,23.33,11.96)
3-7	(38.4,75,106.26)	(1275.24,6350.67,17378.88)	(17.42,47.20,85.22)	(6.8,14,10.67)
5-9	(46.32,93,152.79)	(1948.4,8732.17,21688.04)	(28.09,63.86,96.04)	(0.00,0.00,0.00)
6-9	(96.96,159,204.6)	(4132.3,17320,41811.7)	(28.44,69.42,96.69)	(0.00,0.00,0.00)
8-9	(102.96,153,195.03)	(3279.18,12990,30206.82)	(27.73,58.31,86.53)	(0.23,4.67,0.65)

Table 4. Aggregated fuzzy values at 0.5 multiplied by corresponding weights

Activity	Criteria			
	C1 (Time)	C2 (Cost)	C3 (Risk)	C4 (Quality)
0-1	(21.33,39,53.83)	(4041.92,7938.33,14107.58)	(5.55,13.88,30.55)	(27.50,42,46.75)
0-2	(32,45,57)	(1754.46,3969.17,7886.92)	(15.55,30.54,48.88)	(20,28,29.75)
0-3	(56,81,109.25)	(1310.29,3175.33,6609.46)	(33.32,58.31,77.90)	(10,14,12.75)
1-4	(38.67,60,79.17)	(1243.67,3175.33,6776.08)	(17.77,36.09,51.94)	(21.6,32.67,32.58)

2-4	(86.67,114,145.67)	(4352.83,8515.67,15551.67)	(31.10,52.76,70.26)	(1.67,4.70,2.83)
3-5	(61.33,84,101.33)	(14879.58,26701.67,43877.92)	(17.77,36.09,51.94)	(11.67,18.67,15.5)
7-8	(112,144,171)	(4264,8660,15996)	(13.33,24.99,36.66)	(16.67,18.67,15.58)
2-5	(94.67,123,145.67)	(4619.33,8948.67,15884.92)	(17.77,36.09,51.94)	(20.00,28.00,29.75)
4-6	(92,126,156.75)	(2487.33,5051.67,9719.79)	(31.10,52.76,70.26)	(18.34,23.33,18.42)
3-7	(53.33,75,91.83)	(3153.58,6350.67,11885.92)	(28.88,47.20,67.21)	(10.00,14.00,12.75)
5-9	(65.33,93,125.08)	(4552.71,8732.17,15385.04)	(42.21,63.86,80.96)	(0.00,1.00,10.00)
6-9	(122.67,159,183.67)	(9327.5,17320,29992.5)	(44.43,69.42,84.01)	(0.00,1.00,10.00)
8-9	(124,153,175.75)	(7195.5,12990,21994.5)	(39.99,58.31,73.32)	(1.67,4.70,2.83)

Table 5. Aggregated fuzzy values at0.9 multiplied by corresponding weights

Activity	Criteria			
	C1 (Time)	**C2 (Cost)**	**C3 (Risk)**	**C4 (Quality)**
0-1	(35.20,39.00,41.86)	(7074.69,7938.33,9066.33)	(12.00,13.88,16.99)	(38.94,42.00,43.07)
0-2	(42.24,45.00,47.32)	(3457.41,3969.17,4653.09)	(27.19,30.54,33.99)	(26.40,28.00,28.47)
0-3	(75.68,81.00,86.45)	(2737.95,3175.33,3765.65)	(52.79,58.31,62.03)	(13.20,14.00,13.87)
1-4	(55.44,60.00,63.70)	(2717.96,3175.33,3789.63)	(31.99,36.10,39.09)	(30.36,32.67,32.85)
2-4	(108.24,114.00,120.12)	(7594.30,8515.67,9785.88)	(47.99,52.76,56.08)	(3.96,4.67,4.38)
3-5	(79.20,84.00,87.36)	(24181.85,26701.67,29981.25)	(31.99,36.10,39.09)	(17.16,18.67,18.25)
7-8	(137.28,144.00,149.24)	(7674.24,8660.00,9977.76)	(22.40,24.99,27.19)	(18.48,18.67,18.25)
2-5	(117.04,123.00,127.40)	(7994.00,8948.67,10217.61)	(31.99,36.10,39.09)	(26.40,28.00,28.47)
4-6	(118.80,126.00,131.95)	(4476.64,5051.67,5876.33)	(47.99,52.76,56.08)	(22.44,23.33,22.63)
3-7	(70.40,75.00,78.26)	(5635.77,6350.67,7339.41)	(43.19,47.20,50.98)	(13.20,14.00,13.87)
5-9	(87.12,93.00,99.19)	(7814.14,8732.17,9953.78)	(59.18,63.86,67.13)	(0.00,0.00,0.00)
6-9	(151.36,159.00,163.80)	(15588.30,17320.00,19667.70)	(63.98,69.42,72.22)	(0.00,0.00,0.00)
8-9	(146.96,153.00,157.43)	(11751.18,12990.00,14678.82)	(54.38,58.31,61.18)	(3.96,4.67,4.38)

Subject to the constraints:

$133 \times \lambda + 7 \times y_{01} + 21 \times y_{02} + 35 \times y_{03} + 21 \times y_{14} + 63 \times y_{24} + 42 \times y_{35} + 84 \times y_{78} + 70 \times y_{25} + 63 \times y_{46} + 35 \times y_{37} + 42 \times y_{59} + 91 \times y_{69} + 98 \times y_{89} \leq 252,$

$3450 \times \lambda + 1200 \times y_{01} + 400 \times y_{02} + 250 \times y_{03} + 200 \times y_{14} + 1300 \times y_{24} + 5000 \times y_{35} + 1200 \times y_{\backslash 78} + 1400 \times y_{25} + 700 \times y_{46} + 900 \times y_{37} + 1400 \times y_{59} + 3000 \times y_{69} + 2400 \times y_{89} \leq 6650,$

$25 \times \lambda + 0 \times y_{01} + 5 \times y_{02} + 15 \times y_{03} + 5 \times y_{14} + 15 \times y_{24} + 5 \times y_{35} + 5 \times y_{78} + 5 \times y_{25} + 15 \times y_{46} + 15 \times y_{37} + 25 \times y_{59} + 25 \times y_{69} + 25 \times y_{89} \leq 60,$

$27 \times \lambda + 15 \times y_{01} + 12 \times y_{02} + 6 \times y_{03} + 12 \times y_{14} + 0 \times y_{24} + 6 \times y_{35} + 12 \times y_{78} + 12 \times y_{25} + 12 \times y_{46} + 6 \times y_{37} + 0 \times y_{59} + 0 \times y_{69} + 0 \times y_{89} \leq 39,$

$195 \times \lambda + 39 \times y_{01} + 45 \times y_{02} + 81 \times y_{03} + 60 \times y_{14} + 114 \times y_{24} + 84 \times y_{35} + 114 \times y_{78} + 123 \times y_{25} + 126 \times y_{46} + 75 \times y_{37} + 93 \times y_{59} + 159 \times y_{69} + 153 \times y_{89} \leq 453,$

$16959.17 \times \lambda + 7938.34 \times y_{01} + 3969.17 \times y_{02} + 3175.34 \times y_{03} + 3175.34 \times y_{14} + 8515.67 \times y_{24} + 26701.67 \times y_{35} + 8660 \times y_{78} + 8948.67 \times y_{25} + 5051.67 \times y_{46} + 6350.67 \times y_{37} + 8732.17 \times y_{59} + 17320 \times y_{69} + 12990 \times y_{89} \leq 38609.18,$

$74.97 \times \lambda + 13.89 \times y_{01} + 30.55 \times y_{02} + 58.31 \times y_{03} + 36.1 \times y_{14} + 52.76 \times y_{24} + 36.1 \times y_{35} + 24.99 \times y_{78} + 36.1 \times y_{25} + 52.76 \times y_{46} + 47.21 \times y_{37} + 63.87 \times y_{59} + 69.42 \times y_{69} + 58.31 \times y_{89} \leq 205.49,$

$65.333 \times \lambda + 41.9997 \times y_{01} + 27.9997 \times y_{02} + 13.9997 \times y_{03} + 32.6664 \times y_{14} + 4.6664 \times y_{24} + 18.6664 \times y_{35}$

$+ 18.6664 \times y_{78} + 27.9997 \times y_{25} + 23.333 \times y_{46} + 13.9997 \times y_{37} + 0.000 \times y_{59} + 0.000 \times y_{69} + 4.6664 \times y_{89} \leq 97.9991,$

$250 \times \lambda + 70 \times y_{01} + 70 \times y_{02} + 140 \times y_{03} + 100 \times y_{14} + 180 \times y_{24} + 120 \times y_{35} + 200 \times y_{78} + 170 \times y_{25} + 190 \times y_{46} + 110 \times y_{37} + 160 \times y_{59} + 210 \times y_{69} + 200 \times y_{89} \leq 650,$

$37350 \times \lambda + 21600 \times y_{01} + 13050 \times y_{02} + 11250 \times y_{03} + 11700 \times y_{14} + 24300 \times y_{24} + 63000 \times y_{35} + 25200 \times y_{78} + 24300 \times y_{25} + 15750 \times y_{46} + 18900 \times y_{37} + 23400 \times y_{59} + 45000 \times y_{69} + 32400 \times y_{89} \leq 98100,$

$110 \times \lambda + 50 \times y_{01} + 70 \times y_{02} + 100 \times y_{03} + 70 \times y_{14} + 90 \times y_{24} + 70 \times y_{35} + 50 \times y_{78} + 70 \times y_{25} + 90 \times y_{46} + 90 \times y_{37} + 100 \times y_{59} + 100 \times y_{69} + 90 \times y_{89} \leq 350,$

$70 \times \lambda + 50 \times y_{01} + 30 \times y_{02} + 10 \times y_{03} + 30 \times y_{14} + 0 \times y_{24} + 10 \times y_{35} + 10 \times y_{78} + 30 \times y_{25} + 10 \times y_{46} + 10 \times y_{37} + 0 \times y_{59} + 0 \times y_{69} + 0 \times y_{89} \leq 90$

with additional constraints (14).

The solution of this mode by developed approach is given in Table 7.

Table 7 indicate the solution of illustrated FMOCPP, which shows that at α level 0, 0.1, 0.5 and 0.9 the optimal degree of satisfaction are 0.4286, 0.4563, 0.5509 and 0.6264 respectively. Table 7 also indicate that critical path remain same at each α level. Comparing the developed solution approach with other existing solution approach shows that the developed solution approach provides additional optimal degree of satisfaction to take the decision to decision makers. The Figure 4 indicates shows the distribution of objective values with respect to liner membership function at different *α* level.

10.2 Solution Method using Model 4 (Exponential Membership Function)

Model 4 can be formulated with PIS and NIS obtained in Table 6 as follows:

Maximize $= \lambda$

Subject to the constraints:

$\exp(-s((7 \times y_{01} + 21 \times y_{02} + 35 \times y_{03} + 21 \times y_{14} + 63 \times y_{24} + 42 \times y_{35} + 84 \times y_{78} + 70 \times y_{25} + 63 \times y_{46} + 35 \times y_{37} + 42 \times y_{59} + 91 \times y_{69} + 98 \times y_{89}) - 119)/133) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$

$\exp(-s((1200 \times y_{01} + 400 \times y_{02} + 250 \times y_{03} + 200 \times y_{14} + 1300 \times y_{24} + 5000 \times y_{35} + 1200 \times y_{78} + 1400 \times y_{25} + 700 \times y_{46} + 900 \times y_{37} + 1400 \times y_{59} + 3000 \times y_{69} + 2400 \times y_{89}) - 3200)/3450)((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$

$\exp(-s((0 \times y_{01} + 5 \times y_{02} + 15 \times y_{03} + 5 \times y_{14} + 15 \times y_{24} + 5 \times y_{35} + 5 \times y_{78} + 5 \times y_{25} + 15 \times y_{46} + 15 \times y_{37} + 25 \times y_{59} + 25 \times y_{69} + 25 \times y_{89}) - 35)/25) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$

$\exp(-s((15 \times y_{01} + 12 \times y_{02} + 6 \times y_{03} + 12 \times y_{14} + 0 \times y_{24} + 6 \times y_{35} + 12 \times y_{78} + 12 \times y_{25} + 12 \times y_{46} + 6 \times y_{37} + 0 \times y_{59} + 0 \times y_{69} + 0 \times y_{89}) - 12)/27) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$

$\exp(-s((39 \times y_{01} + 45 \times y_{02} + 81 \times y_{03} + 60 \times y_{14} + 114 \times y_{24} + 84 \times y_{35} + 114 \times y_{78} + 123 \times y_{25} + 126 \times y_{46} + 75 \times y_{37} + 93 \times y_{59} + 159 \times y_{69} + 153 \times y_{89}) - 258)/195) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$

$\exp(-s((7938.34 \times y_{01} + 3969.17 \times y_{02} + 3175.34 \times y_{03} + 3175.34 \times y_{14} + 8515.67 \times y_{24} + 26701.67 \times y_{35} + 8660 \times y_{78} + 8948.67 \times y_{25} + 5051.67 \times y_{46} + 6350.67 \times y_{37} + 8732.17 \times y_{59} + 17320 \times y_{69} + 12990 \times y_{89}) - 21650.01)/16959.17) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$

$\exp(-s((13.89 \times y_{01} + 30.55 \times y_{02} + 58.31 \times y_{03} + 36.1$

Table 6. Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for each objective

α level	Solutions	Objective											
		Z_{11}	Z_{12}	Z_{12}	Z_{21}	Z_{22}	Z_{23}	Z_{31}	Z_{32}	Z_{33}	Z_{41}	Z_{42}	Z_{43}
0	PIS	119	258	400	3200	21650.01	60750	35	130.52	240	12	32.67	20
	NIS	252	453	650	6650	38609.18	98100	60	205.49	350	39	97.99	90
0.1	PIS	130.80	258	385.11	4505.56	21650.01	56104.49	41.97	130.52	227.81	13.83	32.67	21.99
	NIS	269.52	453	628.98	9166.61	38609.18	90933.35	70.77	205.49	334.02	44.55	97.99	92.16
0.5	PIS	182.680	258	327.76	10926.51	21650.01	39156.89	75.56	130.52	181.78	21.67	32.67	28.35
	NIS	345.340	453	547.84	20742.60	38609.18	65872.43	122.22	205.49	273.44	67.51	98.00	97.77
0.9	PIS	242.00	258	273.00	19265.55	21650.01	24824.48	118.36	130.52	140.21	30.36	32.67	32.12
	NIS	430.32	453	471.38	34733.94	38609.18	43700.68	187.15	205.49	218.40	91.74	97.99	98.55

Table 7. Results for α = 0,0.1,0.5 and 0.9 using LMF

α level	λ	Optimal path	(Z_1,Z_2,Z_3,Z_4)	Optimal path in[32]	(Z_1,Z_2,Z_3,Z_4)
$\alpha = 0$	0.4286	0-2-5-9	((133,261,400), (3200,21650,60750), (35,130.5,240),(21,119,240))	0-2-5-9	((133,261,400), (3200,21650,60750), (35,130.5,240), (21,119,240))
$\alpha = 0.1$	0.4563	0-2-5-9	((144,261,385.11), (4505.54,21650,56104.48), (41.95,130.5,227.8), (27.2,119 ,226.01))		
$\alpha = 0.5$	0.5509	0-2-5-9	((192,261,327.75), (10926.5,21650,39156.88), (75.54,130.50,181.77), (60,119,174.25))		
$\alpha = 0.9$	0.6264	0-2-5-9	((246.4,261,273.91), (19265.54,21650,24824.48), (118.36,130.5,140.2), (105.6,119,129.21))		

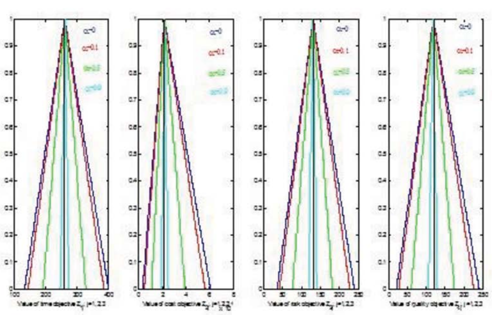

Figure 4. Time, cost, risk and quality objective at α levels 0, 0.1, 0.5 and 0.9 with linear membership function.

$\times\, y_{14} + 52.76 \times y_{24} + 36.1 \times y_{35} + 24.99 \times y_{78} + 36.1 \times y_{25} + 52.76 \times y_{46} + 47.21 \times y_{37} + 63.87 \times y_{59} + 69.42 \times y_{69} + 58.31 \times y_{89}) - 130.52)/74.97) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$

$\exp(-s((41.9997 \times y_{01} + 27.9997 \times y_{02} + 13.9997 \times y_{03} + 32.6664 \times y_{14} + 4.6664 \times y_{24} + 18.6664 \times y_{35} + 18.6664 \times y_{78} + 27.9997 \times y_{25} + 23.333 \times y_{46} + 13.9997 \times y_{37} + 0.000 \times y_{59} + 0.000 \times y_{69} + 4.6664 \times y_{89}) - 32.6661)/65.333) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$

$\exp(-s((70 \times y_{01} + 70 \times y_{02} + 140 \times y_{03} + 100 \times y_{14} + 180 \times y_{24} + 120 \times y_{35} + 200 \times y_{78} + 170 \times y_{25} + 190 \times y_{46} + 110 \times y_{37} + 160 \times y_{59} + 210 \times y_{69} + 200 \times y_{89}) - 400)/250) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$

$\exp(-s((21600 \times y_{01} + 13050 \times y_{02} + 11250 \times y_{03} + 11700 \times y_{14} + 24300 \times y_{24} + 63000 \times y_{35} + 25200 \times y_{78} + 24300 \times y_{25} + 15750 \times y_{46} + 18900 \times y_{37} + 23400 \times y_{59} + 45000 \times y_{69} + 32400 \times y_{89}) - 60750)/37350) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$

$\exp(-s((50 \times y_{01} + 70 \times y_{02} + 100 \times y_{03} + 70 \times y_{14} + 90 \times y_{24} + 70 \times y_{35} + 50 \times y_{78} + 70 \times y_{25} + 90 \times y_{46} + 90 \times y_{37} + 100 \times y_{59} + 100 \times y_{69} + 90 \times y_{89}) - 240)/110) - ((1-\exp(-s)) \times \lambda) \geq \exp(-s)$

$\exp(-s((50 \times y_{01} + 30 \times y_{02} + 10 \times y_{03} + 30 \times y_{14} + 0 \times y_{24} + 10 \times y_{35} + 10 \times y_{78} + 30 \times y_{25} + 10 \times y_{46} + 10 \times y_{37} + 0 \times y_{59} + 0 \times y_{69} + 0 \times y_{89}) - 20)/70) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$

Subject to the constraints (14).

The solution of these exponential models with different values of shape parameters by using LINGO software is given in Table 8.

Table 9 indicates the solution of FMOCPP with exponential membership function by fuzzy programming technique. It shows that at α level 0 the optimal degree of satisfaction are 0.5514, 0.5271, 0.5392, 0.5392 and 0.5514 with shape parameter (-1,-1,-1,-1), (-0.1,-0.3,-0.6,-0.8), (-0.1,-0.4,-0.8,-0.9), (-0.2,-0.4,-0.7,-0.9), (-0.1,-0.3,-0.6,-1) respectively. Similarly at α level 0.1 the optimal degree of satisfaction are 0.5796, 0.5554, 0.5676, 0.5676 and 0.5796 with shape parameter (-1,-1,-1,-1), (-0.1,-0.3,-0.6,-0.8), (-0.1,-0.4,-0.8,-0.9), (-0.2,-0.4,-0.7,-0.9), (-0.1,-0.3,-0.6,-1) respectively. For α level 0.5 the optimal degree of satisfaction are 0.6700, 0.6472, 0.6569, 0.6586 and 0.6700 with shape parameter (-1,-1,-1,-1), (-0.1,-0.3,-0.6,-0.8), (-0.1,-0.4,-0.8,-0.9), (-0.2,-0.4,-0.7,-0.9), (-0.1,-0.3,-0.6,-1) respectively. Similarly For α level 0.9 the optimal degree of satisfaction are 0.7353, 0.7147, 0.7251, 0.7251 and 0.7353 with shape parameter (-1,-1,-1,-1), (-0.1,-0.3,-0.6,-0.8), (-0.1,-0.4,-0.8,-0.9), (-0.2,-0.4,-0.7,-0.9), (-0.1,-0.3,-0.6,-1) respectively. With five cases of shape parameter we will get optimal path with different degree of satisfaction which provides opportunity to DM to take the decisions. If decision makers are not satisfied with obtain critical path they may change the different value of shape parameters to obtain desired level of satisfaction and this is one of the best advantage of this developed approach. Table 9 also compares the obtained output with existing solution approach which shows that the developed solution approach provides additional optimal degree of satisfaction to take the decision to decision makers. The Figure 5 indicates shows the distribution of objective values with respect to liner membership function at different α level.

11. Manufacturing Engineering Industry Project Management Problem

We have applied new approach to solve real-world case of an aircraft component development undertaken by a company that designs and manufactures small electronic components, particularly for the aviation, defence and space industries[32]. The details of the activities along with dependencies are provided in Table 10. The project network is shown in Figure 6.

11.1 Formulation of Model 2

$$Minimize Z_1\ (Time) = (2,21.1078,60) \times y_{12} + (1,12.66,30) \times y_{28} + (2,29.5561,70) \times y_{23} + (1,12.666,30) \times y_{35} + (1,12.66,40) \times y_{34} + (2,21.1078,60) \times y_{56} + (8,69.663,140) \times y_{57} + (3,37.998,90) \times y_{47} + (1,14.7748,40) \times y_{69} + (3,31.665,70) \times y_{610} + (1,14.7748,40) \times y_{710} + (2,18.999,40) \times y_{318} + (1,12.666,30) \times y_{812} + (5,35.887,80) \times y_{911} + (3,31.665,60) \times y_{1011} + (4,35.8891,70) \times y_{1113} + (2,21.1078,50) \times y_{1215} + (1,12.666.30) \times y_{1314} + (1,12.666,30) \times y_{1517} + (1,14.7748,40) \times y_{1417} + (10,82.329,160) \times y_{1416} + (1,12.666,30) \times y_{1718} + (1,14.7748,40) \times y_{1819} + (0,0,0) \times y_{68} + (0,0,0) \times y_{1415} + (0,0,0) \times y_{1617},$$

$$Minimize Z_2\ (Cost) = (6,60.669,130) \times y_{12} + (6,63,130) \times y_{28} + (85,723.331,1300) \times y_{23} + (70,595,1100) \times y_{35} + (25,256.669,550) \times y_{34} + (130,1061.669,1800) \times y_{56} + (350,2660,4200) \times y_{57} + (30,315,650) \times y_{47} + (75,606.669,1000) \times y_{69} + (240,1960,3200) \times y_{610} + (60,560,1050) \times y_{710} + (120,980,1700) \times y_{318} + (140,1085,1800) \times y_{812} + (280,2146.669,3500) \times y_{911} + (190,1540,2600) \times y_{1011} + (270,2018.331,3050) \times y_{1113} + (140,1085,1700) \times y_{1215} + (30,315,650) \times y_{1314} + (30,315,700) \times y_{1517} + (60,560,1050) \times y_{1417} + (330,2450,3700) \times y_{1416} + (7,67.669,120) \times y_{1718} + (310,2345,3600) \times y_{1819} + (0,0,0) \times y_{68} + (0,0,0) \times y_{1415} + (0,0,0) \times y_{1617},$$

$$Minimize Z_3\ (Risk) = (0,13.891,50) \times y_{12} + (5,30.557,70) \times y_{28} + (5,30.557,70) \times y_{23} + (5,47.223,100) \times y_{35} + (15,52.773,90) \times y_{34} + (5,36.107,70) \times y_{56} + (5,24.999,50) \times y_{57} + (15,52.773,90) \times y_{47} + (15,47.223,90) \times y_{69} + (25,63.889,100) \times y_{610} + (25,69.439,100) \times y_{710} + (25,58.331,90) \times y_{318} + (0,19.441,50) \times y_{812} + (15,58.331,100) \times y_{911} + (5,36.107,70) \times y_{1011} + (5,30.557,70) \times y_{1113} + (5,36.107,70) \times y_{1215} + (5,30.557,70) \times y_{1314} + (25,69.438,100) \times y_{1517} + (0,24.999,70) \times y_{1417} + (25,63.889,100) \times y_{1416} + (0,19.441,50) \times y_{1718} + (0,19.441,50) \times y_{1819} + (0,0,0) \times y_{68} + (0,0,0) \times y_{1415} + (0,0,0) \times y_{1617},$$

$$Minimize Z_4\ (Quality) = (15,37.998,45) \times y_{12} + (12,25.332,27) \times y_{28} + (6,12.666,9) \times y_{23} + (12,29.550,27) \times y_{35} + (0,0.000,0) \times y_{34} + (6,16.884,9) \times y_{56} + (12,16.884,9) \times$$

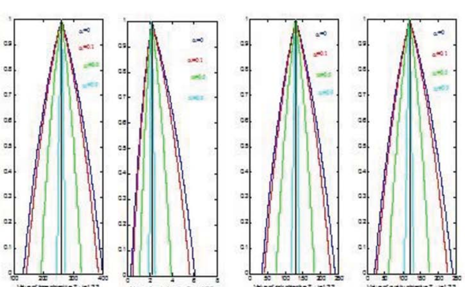

Figure 5. Time, cost, risk and quality objective α at level 0, 0.1, 0.5 and 0.9 for shape parameter (-1,-1,-1,-1).

Table 8. Shape parameters

Case	Shape parameter (s_1,s_2,s_3, s_4)
Case -1	(-1,-1,-1,-1)
Case -2	(-0.1,-0.3,-0.6,-0.8)
Case -3	(-0.1,-0.4,-0.8,-0.9)
Case -4	(-0.2,-0.4,-0.7,-0.9)
Case -5	(-0.1,-0.3,-0.6,-1)

Table 9. Results for α = 0,0.1,0.5 and 0.9 using exponential membership model

α level	Case	λ	Optimal path	(Z_1,Z_2,Z_3,Z_4)	Optimal path in[32]	(Z_1,Z_2,Z_3,Z_4)
α = 0	Case -1	0.5514	0-2-5-9	((133,261,400), (3200,21650,60750), (35,130.5,240), (21,119,240))	0-2-5-9	((133,261,400), (3200,21650,60750), (35,130.5,240), (21,119,240))
	Case -2	0.5271	0-2-5-9			
	Case -3	0.5392	0-2-5-9			
	Case -4	0.5392	0-2-5-9			
	Case -5	0.5514	0-2-5-9			
α = 0.1	Case -1	0.5796	0-2-5-9	((144,261,385.11), (4505.54,21650,56104.48), (41.95,130.50,227.80), (27.2,119 ,226.01))		
	Case -2	0.5554	0-2-5-9			
	Case -3	0.5676	0-2-5-9			
	Case -4	0.5676	0-2-5-9			
	Case -5	0.5796	0-2-5-9			
α = 0.5	Case -1	0.6700	0-2-5-9	((192,261,327.75), (10926.5,21650,39156.86), (75.54,130.50,181.77), (60,119,174.25))		
	Case -2	0.6472	0-2-5-9			
	Case -3	0.6569	0-2-5-9			
	Case -4	0.6586	0-2-5-9			
	Case -5	0.6700	0-2-5-9			
α = 0.9	Case -1	0.7353	0-2-5-9	((246.4,261,273.91), (19265.54,21650,24824.48), (118.36,130.50,140.20), (105.6,119,129.21))		
	Case -2	0.7147	0-2-5-9			
	Case -3	0.7251	0-2-5-9			
	Case -4	0.7251	0-2-5-9			
	Case -5	0.7353	0-2-5-9			

Table 10. Aggregated fuzzy values

Activities	Criteria			
	C1(Time)	**C2(Cost)**	**C3(Risk)**	**Q(Quality)**
GCS	(2,3.333,6)	(6,8.67,13)	(0,1.667,5)	(0,2.333,5)
DEP	(1,2,3)	(6,9,13)	(1,3.667,7)	(1,4.333,7)
DBTC	(2,4.667,7)	(85,103.333,130)	(1,3.667,7)	(3,6.333,9)
DPBL	(1,2,3)	(70,85,110)	(1,5.667,10)	(1,3.667,7)
SLEC	(1,2,4)	(25,36.667,55)	(3,6.333,9)	(5,8.333,10)
DC	(2,3.333,5)	(130,151.667,180)	(1,4.333,7)	(3,5.667,9)
BCB	(8,11,14)	(350,380,420)	(1,3,5)	(1,5.667,9)
BCP	(3,6,9)	(30,45,65)	(3,6.333,9)	(1,4.333,7)
DEM	(1,2.333,4)	(75,86.667,100)	(3,5.667,9)	(3,6.333,9)
MPC	(3,5,7)	(240,280,320)	(5,7.667,10)	(3,6.333,9)
APCB	(1,2.333,4)	(60,80,105)	(5,8.333,10)	(3,5.667,9)
WPTP	(2,3,4)	(120,140,170)	(5,7,9)	(5,7.667,10)
DVTC	(1,2,3)	(140,155,180)	(0,2.333,5)	(3,7,10)
MM	(5,6.667,8)	(280,306.667,350)	(3,7,10)	(0,2.333,5)
ATPU	(3,5,6)	(190,220,260)	(1,4.333,7)	(1,4.333,7)
CU	(4,5.667,7)	(270,288.333,305)	(1,3.667,7)	(1,4.333,7)
MVTC	(2,3.333,5)	(140,155,170)	(1,4.333,7)	(1,3.667,7)
RUAC	(1,2,3)	(30,45,65)	(1,3.667,7)	(3,5.667,9)
VSTU	(1,2,3)	(30,45,70)	(5,8.333,10)	(1,5.667,9)
CTU	(1,2.333,4)	(60,80,105)	(0,3,7)	(1,4.333,7)

ALTU	(10,13,16)	(330,350,370)	(5,7.667,10)	(3,7,10)
ATR	(1,2,3)	(7,9.667,1)	(0,2.333,5)	(1,4.333,7)
FDP	(1,2.333,4)	(310,335,360)	(0,2.333,5)	(1,5.667,9)
Weights	**(1,6.333,10)**	**(1,7,10)**	**(5,8.333,10)**	**(3,6.333,9)**

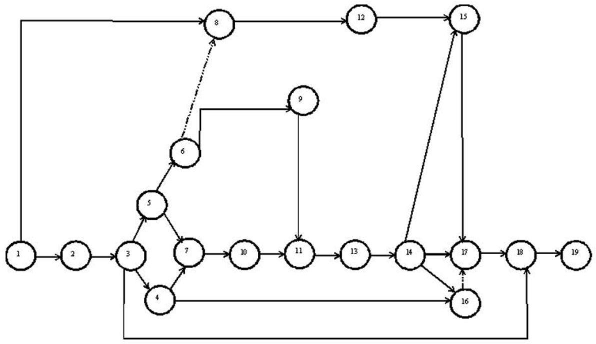

Figure 6. The project network.

$y_{57} + (12,25.332,27) \times y_{47} + (6,12.666.9) \times y_{69} + (6,12.666.9) \times y_{61}0 + (6,16.884,9) \times y_{710} + (0,4.218,0) \times y_{318} + (6,8.442,0) \times y_{812} + (15,37.998,45) \times y_{911} + (12,25.332,27) \times y_{1011} + (12,25.332,27) \times y_{1113} + (12,29.550,27) \times y_{1215} + (6,16.884,9) \times y_{1314} + (12,16.884,9) \times y_{1517} + (12,25.332,27) \times y_{1417} + (6,8.442,0) \times y_{1416} + (12,25.332,27) \times y_{1718} + (12,16.884,9) \times y_{1819} + (15,52.773,90) \times y_{68} + (15,52.773,90) \times y_{1415} + (15,52.773,90) \times y_{1617}$

Subject to the constraints

$y_{12} = 1$

$y_{12} = y_{23} + y_{28}$

$y_{23} = y_{34} + y_{35} + y_{318}$

$y_{34} = y_{47}$

$y_{35} = y_{56} + y_{57}$

$y_{56} = y_{68} + y_{69} + y_{610}$

$y_{57} + y_{47} = y_{710}$

$y_{28} + y_{68} = y_{812}$

$y_{69} = y_{911}$

$y_{610} + y_{710} = y_{1011}$

$y_{911} + y_{1011} = y_{1113}$

$y_{812} = y_{1215}$

$y_{1113} = y_{1314}$

$y_{1314} = y_{1415} + y_{1416} + y_{1417}$

$y_{1215} + y_{1415} = y_{1517}$

$y_{1416} = y_{1617}$

$y_{1417} + y_{1517} + y_{1617} = y_{1718}$

$y_{318} + y_{1718} = y_{1819}$

$y_{12} \ge 0, y_{23} \ge 0, y_{28} \ge 0, y_{34} \ge 0, y_{35} \ge 0, y_{318} \ge 0, y_{47} \ge 0, y_{56} \ge 0, y_{57} \ge 0, y_{68} \ge 0, y_{69} \ge 0, y_{610} \ge 0, y_{710} \ge 0, y_{812} \ge 0, y_{911} \ge 0,$

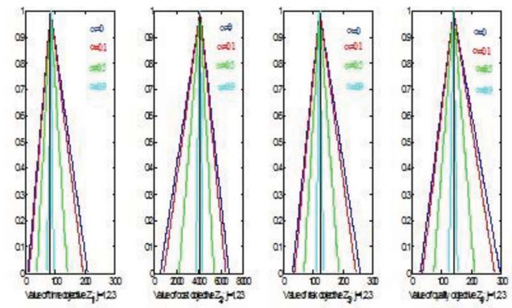

Figure 7. Time, cost, risk and quality objective at α levels 0, 0.1, 0.5 and 0.9 with linear membership function.

$y_{\_1011} \ge 0, y_{\_1113} \ge 0, y_{1215} \ge 0, y_{1314} \ge 0, y_{1415} \ge 0, y_{14}16 \ge 0, y_{1417} \ge 0, y_{1517} \ge 0, y_{1617} \ge 0, y_{1718} \ge 0, y_{1819} \ge 0.$

11.2 Solution

For finding the solution of this problem the fuzzy programming technique based developed approach is utilized and for that at different α level the value of each objective PIS and NIS is as Table 11.

Substituting the values acquired in Table 11 in Model 3.2, we get:

For α=0,

Maximize λ ,

subject to the constraints:

$27 \times \lambda + 2 \times y_{12} + 1 \times y_{28} + 2 \times y_{23} + 1 \times y_{35} + 1 \times y_{34} + 2 \times y_{56} + 8 \times y_{57} + 3 \times y_{47} + 1 \times y_{69} + 3 \times y_{610} + 1 \times y_{710} + 2 \times y_{318} + 1 \times y_{812} + 5 \times y_{911} + 3 \times y_{1011} + 4 \times y_{1113} + 2 \times y_{1215} + 1 \times y_{1314} + 1 \times y_{1517} + 1 \times y_{1417} + 10 \times y_{1416} + 1 \times y_{1718} + 1 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617} \le 34,$

$1187 \times \lambda + 6 \times y_{12} + 6 \times y_{28} + 8 \times y_{23} + 70 \times y_{35} + 25 \times y_{34} + 130 \times y_{56} + 350 \times y_{57} + 30 \times y_{47} + 75 \times y_{69} + 240 \times y_{610} + 60 \times y_{710} + 120 \times y_{318} + 140 \times y_{812} + 280 \times y_{911} + 190 \times y_{1011} + 270 \times y_{1113} + 140 \times y_{1215} + 30 \times y_{1314} + 30 \times y_{1517} + 60 \times y_{1417} + 330 \times y_{1416} + 7 \times y_{1718} + 310 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617} \le 1708 ,$

$70 \times \lambda + 0 \times y_{12} + 5 \times y_{28} + 5 \times y_{23} + 5 \times y_{35} + 15 \times y_{34} + 5 \times y_{56} + 5 \times y_{57} + 15 \times y_{47} + 15 \times y_{69} + 25 \times y_{610} + 25 \times y_{710} + 25 \times y_{318} + 0 \times y_{812} + 15 \times y_{911} + 5 \times y_{1011} + 5 \times y_{1113} + 5 \times y_{1215} +$

Table 11. Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for each objective

α level	Solutions	Objective											
		Z_{11}	Z_{12}	Z_{13}	Z_{21}	Z_{22}	Z_{23}	Z_{31}	Z_{32}	Z_{33}	Z_{41}	Z_{42}	Z_{43}
0	PIS	7	84.439	210	521	4109	6730	30	122.22	260	33	71.77	63
	NIS	34	337.76	730	1708	13335	21500	100	424.98	820	132	301.86	315
0.1	PIS	55.1	84.439	194.91	844.16	4109	6444.68	36.62	122.22	244.52	36.78	71.77	64.92
	NIS	11.7	337.76	684.27	2764.32	13335	20617.35	123.20	424.98	773.21	147.44	301.86	316.72
0.5	PIS	37.28	84.439	140.20	2216	4109	5355	68.90	122.22	186.40	52.12	71.77	70.28
	NIS	160.13	337.76	515.87	7226	13335	17233.75	236.68	424.98	605.02	211.57	301.86	316.90
0.9	PIS	73.66	84.439	94.47	3714.56	4109	4347.88	110.40	122.22	134.30	67.79	71.77	71.93
	NIS	298.12	337.76	370.51	12065.92	13335	14085.35	383.20	424.98	458.10	283.1830	301.86	306.22

Table 12. Results for α = 0,0.1,0.5 and 0.9

α level	λ	Optimal path	(Z_1,Z_2,Z_3,Z_4)	Optimal path in[32]	(Z_1,Z_2,Z_3,Z_4)
α = 0	1	1-2-3-18-19	((7,84.44,210), (521,4109,6730), (30,122.22,260), (27,139.33,297))	1-2-3-18-19	((7,84.44,210), (521,4109,6730), (30,122.22,260), (27,139.33,297))
α = 0.1	1	1-2-3-18-19	((11.70,84.44,194.91), (844.16,4109,6444.68), (36.62,122.22,244.51), (34.33,139.33,278.58))		
α = 0.5	1	1-2-3-18-19	((37.28,84.44,140.20), (2216,4109,5355), (68.89,122.22,186.40), (72.34,139.33,210.84))		
α = 0.9	1	1-2-3-18-19	((73.66,84.44,94.47), (3714.56,4109,4347.88), (110.40,122.22,134.30), (124.2,139.33,152.46))		

$5 \times y_{1314} + 25 \times y_{1517} + 0 \times y_{1417} + 25 \times y_{1416} + 0 \times y_{1718} + 0 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617} \le 100$,

$99 \times \lambda + 15 \times y_{12} + 12 \times y_{28} + 6 \times y_{23} + 12 \times y_{35} + 0 \times y_{34} + 6 \times y_{56} + 12 \times y_{57} + 12 \times y_{47} + 6 \times y_{69} + 6 \times y_{610} + 6 \times y_{710} + 0 \times y_{318} + 6 \times y_{812} + 15 \times y_{911} + 12 \times y_{1011} + 12 \times y_{1113} + 12 \times y_{1215} + 6 \times y_{1314} + 12 \times y_{1517} + 12 \times y_{1417} + 6 \times y_{1416} + 12 \times y_{1718} + 12 \times y_{1819} + 15 \times y_{68} + 15 \times y_{1415} + 15 \times y_{1617} \le 132$,

$253.32 \times \lambda + 21.108 \times y_{12} + 12.666 \times y_{28} + 29.556 \times y_{23} + 12.666 \times y_{35} + 12.666 \times y_{34} + 21.108 \times y_{56} + 69.663 \times y_{57} + 37.998 \times y_{47} + 14.775 \times y_{69} + 31.665 \times y_{610} + 14.775 \times y_{710} + 18.999 \times y_{318} + 12.666 \times y_{812} + 35.887 \times y_{911} + 31.665 \times y_{1011} + 35.889 \times y_{1113} + 21.108 \times y_{1215} + 12.666 \times y_{1314} + 12.666 \times y_{1517} + 14.775 \times y_{1417} + 82.329 \times y_{1416} + 12.666 \times y_{1718} + 14.775 \times y_{1819} + 0.000 \times y_{68} + 0.000 \times y_{1415} + 0.000 \times y_{1617} \le 337.76$,

$9226 \times \lambda + 60.669 \times y_{12} + 63.000 \times y_{28} + 723.331 \times y_{23} + 595 \times y_{35} + 256.669 \times y_{34} + 1061.669 \times y_{56} + 2660 \times y_{57} + 315 \times y_{47} + 606.669 \times y_{69} + 1960 \times y_{610} + 560 \times y_{710} + 980.000 \times y_{318} + 1085 \times y_{812} + 2146.669 \times y_{911} + 1540.000 \times y_{1011} + 2018.331 \times y_{1113} + 1085 \times y_{1215} + 315 \times y_{1314} + 315 \times y_{1517} + 560 \times y_{1417} + 2450 \times y_{1416} + 67.669 \times y_{1718} + 2345 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617} \le 13335$,

$302.76 \times \lambda + 13.892 \times y_{12} + 30.558 \times y_{28} + 30.558 \times y_{23} + 47.224 \times y_{35} + 52.773 \times y_{34} + 36.107 \times y_{56} + 24.999 \times y_{57} + 52.773 \times y_{47} + 47.224 \times y_{69} + 63.890 \times y_{610} + 69.439 \times y_{710} + 58.331 \times y_{318} + 19.441 \times y_{812} + 58.331 \times y_{911} + 36.107 \times y_{1011} + 30.558 \times y_{1113} + 36.107 \times y_{1215} + 30.558 \times y_{1314} + 69.439$

$\times y_{1517} + 24.999 \times y_{1417} + 63.890 \times y_{1416} + 19.441 \times y_{1718} + 19.441 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617} \leq 424.98$,

$230.089 \times \lambda + 37.999 \times y_{12} + 25.333 \times y_{28} + 12.667 \times y_{23} + 29.550 \times y_{35} + 0 \times y_{34} + 16.884 \times y_{56} + 16.884 \times y_{57} + 25.333 \times y_{47} + 12.667 \times y_{69} + 12.667 \times y_{610} + 16.884 \times y_{710} + 4.218 \times y_{318} + 8.442 \times y_{812} + 37.999 \times y_{911} + 25.333 \times y_{1011} + 25.333 \times y_{1113} + 29.550 \times y_{1215} + 16.884 \times y_{1314} + 16.884 \times y_{1517} + 25.333 \times y_{1417} + 8.442 \times y_{1416} + 25.333 \times y_{1718} + 16.884 \times y_{1819} + 52.773 \times y_{68} + 52.773 \times y_{1415} + 52.773 \times y_{1617} \leq 301.857$,

$520 \times \lambda + 60 \times y_{12} + 30 \times y_{28} + 70 \times y_{23} + 30 \times y_{35} + 40 \times y_{34} + 50 \times y_{56} + 140 \times y_{57} + 90 \times y_{47} + 40 \times y_{69} + 70 \times y_{610} + 40 \times y_{710} + 40 \times y_{318} + 30 \times y_{812} + 80 \times y_{911} + 60 \times y_{1011} + 70 \times y_{1113} + 50 \times y_{1215} + 30 \times y_{1314} + 30 \times y_{1517} + 40 \times y_{1417} + 160 \times y_{1416} + 30 \times y_{1718} + 40 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617} \leq 730$

$14770 \times \lambda + 130 \times y_{12} + 130 \times y_{28} + 1300 \times y_{23} + 1100 \times y_{35} + 550 \times y_{34} + 1800 \times y_{56} + 4200 \times y_{57} + 650 \times y_{47} + 1000 \times y_{69} + 3200 \times y_{610} + 1050 \times y_{710} + 1700 \times y_{318} + 1800 \times y_{812} + 3500 \times y_{911} + 2600 \times y_{1011} + 3050 \times y_{1113} + 1700 \times y_{1215} + 650 \times y_{1314} + 700 \times y_{1517} + 1050 \times y_{1417} + 3700 \times y_{1416} + 120 \times y_{1718} + 3600 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617} \leq 21500$,

$560 \times \lambda + 50 \times y_{12} + 70 \times y_{28} + 70 \times y_{23} + 100 \times y_{35} + 90 \times y_{34} + 70 \times y_{56} + 50 \times y_{57} + 90 \times y_{47} + 90 \times y_{69} + 100 \times y_{610} + 100 \times y_{710} + 90 \times y_{318} + 50 \times y_{812} + 100 \times y_{911} + 70 \times y_{1011} + 70 \times y_{1113} + 70 \times y_{1215} + 70 \times y_{1314} + 100 \times y_{1517} + 70 \times y_{1417} + 100 \times y_{1416} + 50 \times y_{1718} + 50 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617} \leq 820$,

$252 \times \lambda + {}_{45} \times y_{12} + 27 \times y_{28} + 9 \times y_{23} + 27 \times y_{35} + 0 \times y_{34} + 9 \times y_{56} + 9 \times y_{57} + 27 \times y_{47} + 9 \times y_{69} + 9 \times y_{610} + 9 \times y_{710} + 0 \times y_{318} + 0 \times y_{812} + 45 \times y_{911} + 27 \times y_{1011} + 27 \times y_{1113} + 27 \times y_{1215} + 9 \times y_{1314} + 9 \times y_{1517} + 27 \times y_{1417} + 0 \times y_{1416} + 27 \times y_{1718} + 9 \times y_{1819} + 90 \times y_{68} + 90 \times y_{1415} + 90 \times y_{1617} \leq 315$

With additional constraints of case study.

The results of model 3.2 obtained for $\alpha = 0$, $\alpha = 0.1$, $\alpha = 0.5$, *and* $\alpha = 0.9$ by developed approach are given in Table 12.

Table 12 indicates the solution of illustrated FMOCPP, which shows that at α level 0, 0.1, 0.5 and 0.9 the optimal degree of satisfaction is 1. Table 12 also indicates that critical path remain same at each α level. Table 12 also compares the developed solution approach with other existing solution approach which shows that the developed solution approach provides additional optimal degree of satisfaction to take the decision to decision makers. The Figure 7 shows the distribution of objective values with respect to liner membership function at different α levels.

11.2 Solution Method using Model 4 (Exponential Membership Function)

Model 4 can be formulated with PIS and NIS obtained in Table 11 as follows:

Maximize $= \lambda$

Subject to the constraints:

$\exp(-s((2 \times y_{12} + 1 \times y_{28} + 2 \times y_{23} + 1 \times y_{35} + 1 \times y_{34} + 2 \times y_{56} + 8 \times y_{57} + 3 \times y_{47} + 1 \times y_{69} + 3 \times y_{610} + 1 \times y_{710} + 2 \times y_{318} + 1 \times y_{812} + 5 \times y_{911} + 3 \times y_{1011} + 4 \times y_{1113} + 2 \times y_{1215} + 1 \times y_{1314} + 1 \times y_{1517} + 1 \times y_{1417} + 10 \times y_{1416} + 1 \times y_{1718} + 1 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617}) - 7)/27) - ((1-\exp(-s)) \times \lambda) \geq \exp(-s)$,

$\exp(-s((6 \times y_{12} + 6 \times y_{28} + 85 \times y_{23} + 70 \times y_{35} + 25 \times y_{34} + 130 \times y_{56} + 350 \times y_{57} + 30 \times y_{47} + 75 \times y_{69} + 240 \times y_{610} + 60 \times y_{710} + 120 \times y_{318} + 140 \times y_{812} + 280 \times y_{911} + 190 \times y_{1011} + 270 \times y_{1113} + 140 \times y_{1215} + 30 \times y_{1314} + 30 \times y_{1517} + 60 \times y_{1417} + 330 \times y_{1416} + 7 \times y_{1718} + 310 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617}) - 521)/1187)-((1-\exp(-s)) \times \lambda) \geq \exp(-s)$,

$\exp(-s((0 \times y_{12} + 5 \times y_{28} + 5 \times y_{23} + 5 \times y_{35} + 15 \times y_{34} + 5 \times y_{56} + 5 \times y_{57} + 15 \times y_{47} + 15 \times y_{69} + 25 \times y_{610} + 25 \times y_{710} + 25 \times y_{318} + 0 \times y_{812} + 15 \times y_{911} + 5 \times y_{1011} + 5 \times y_{1113} + 5 \times y_{1215} + 5 \times y_{1314} + 25 \times y_{1517} + 0 \times y_{1417} + 25 \times y_{1416} + 0 \times y_{1718} + 0 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617}) - 30)/70) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$,

$\exp(-s((15 \times y_{12} + 12 \times y_{28} + 6 \times y_{23} + 12 \times y_{35} + 0 \times y_{34} + 6 \times y_{56} + 12 \times y_{57} + 12 \times y_{47} + 6 \times y_{69} + 6 \times y_{610} + 6 \times y_{710} + 0 \times y_{318} + 6 \times y_{812} + 15 \times y_{911} + 12 \times y_{1011} + 12 \times y_{1113} + 12 \times y_{1215} + 6 \times y_{1314} + 12 \times y_{1517} + 12 \times y_{1417} + 6 \times y_{1416} + 12 \times y_{1718} + 12 \times y_{1819} + 15 \times y_{68} + 15 \times y_{1415} + 15 \times y_{1617}) - 33)/99) - ((1-\exp(-s)) \times \lambda) \geq \exp(-s)$,

$\exp(-s((21.108 \times y_{12} + 12.666 \times y_{28} + 29.556 \times y_{23} + 12.666 \times y_{35} + 12.666 \times y_{34} + 21.108 \times y_{56} + 69.663 \times y_{57} + 37.998 \times y_{47} + 14.775 \times y_{69} + 31.665 \times y_{610} + 14.775 \times y_{710} + 18.999 \times y_{318} + 12.666 \times y_{812} + 35.887 \times y_{911} + 31.665 \times y_{1011} + 35.889 \times y_{1113} + 21.108 \times y_{1215} + 12.666 \times y_{1314} + 12.666 \times y_{1517} + 14.775 \times y_{1417} + 82.329 \times y_{1416} + 12.666 \times y_{1718} + 14.775 \times y_{1819} + 0.000 \times y_{68} + 0.000 \times y_{1415} + 0.000 \times y_{1617}) - 84.439)/253.321-((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$,

$\exp(-s((60.669 \times y_{12} + 63.000 \times y_{28} + 723.331 \times y_{23} + 595 \times y_{35} + 256.669 \times y_{34} + 1061.669 \times y_{56} + 2660 \times y_{57} + 315 \times y_{47} + 606.669 \times y_{69} + 1960 \times y_{610} + 560 \times y_{710} + 980.000 \times y_{318} + 1085 \times y_{812} + 2146.669 \times y_{911} + 1540.000 \times y_{1011} +$

$2018.331 \times y_{1113} + 1085 \times y_{1215} + 315 \times y_{1314} + 315 \times y_{1517} + 560 \times y_{1417} + 2450 \times y_{1416} + 67.669 \times y_{1718} + 2345 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617}) - 4109)/9226) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s),$

$\exp(-s((13.892 \times y_{12} + 30.558 \times y_{28} + 30.558 \times y_{23} + 47.224 \times y_{35} + 52.773 \times y_{34} + 36.107 \times y_{56} + 24.999 \times y_{57} + 52.773 \times y_{47} + 47.224 \times y_{69} + 63.890 \times y_{610} + 69.439 \times y_{710} + 58.331 \times y_{318} + 19.441 \times y_{812} + 58.331 \times y_{911} + 36.107 \times y_{1011} + 30.558 \times y_{1113} + 36.107 \times y_{1215} + 30.558 \times y_{1314} + 69.439 \times y_{1517} + 24.999 \times y_{1417} + 63.890 \times y_{1416} + 19.441 \times y_{1718} + 19.441 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617}) - 122.222)/302.757) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s),$

$\exp(-s((37.999 \times y_{12} + 25.333 \times y_{28} + 12._{667} \times y_{23} + 29.550 \times y_{35} + 0 \times y_{34} + 16.884 \times y_{56} + 16.884 \times y_{57} + 25.333 \times y_{47} + 12.667 \times y_{69} + 12.667 \times y_{610} + 16.884 \times y_{710} + 4.218 \times y_{318} + 8.442 \times y_{812} + 37.999 \times y_{911} + 25.333 \times y_{1011} + 25.333 \times y_{1113} + 29.550 \times y_{1215} + 16.884 \times y_{1314} + 16.884 \times y_{1517} + 25.333 \times y_{1417} + 8.442 \times y_{1416} + 25.333 \times y_{1718} + 16.884 \times y_{1819} + 52.773 \times y_{68} + 52.773 \times y_{1415} + 52.773 \times y_{1617}) - 71.768)/230.089) - ((1 - \exp - s)) \times \lambda) \geq \exp(-s),$

$\exp(-s((60 \times y_{12} + 30 \times y_{28} + 70 \times y_{23} + 30 \times y_{35} + 40 \times y_{34} + 50 \times y_{56} + 140 \times y_{57} + 90 \times y_{47} + 40 \times y_{69} + 70 \times y_{610} + 40 \times y_{71}0 + 40 \times y_{318} + 30 \times y_{812} + 80 \times y_{911} + 60 \times y_{1011} + 70 \times y_{1113} + 50 \times y_{1215} + 30 \times y_{1314} + 30 \times y_{1517} + 40 \times y_{1417} + 160 \times y_{1416} + 30 \times y_{1718} + 40 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617}) - 210)/520) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s),$

$\exp(-s((130 \times y_{12} + 130 \times y_{28} + 1300 \times y_{23} + 1100 \times y_{35} + 550 \times y_{34} + 1800 \times y_{56} + 4200 \times y_{57} + 650 \times y_{47} + 1000 \times y_{69} + 3200 \times y_{610} + 1050 \times y_{710} + 1700 \times y_{318} + 1800 \times y_{812} + 3500 \times y_{911} + 2600 \times y_{1011} + 3050 \times y_{1113} + 1700 \times y_{1215} + 650 \times y_{1314} + 700 \times y_{1517} + 1050 \times y_{1417} + 3700 \times y_{1416} + 120 \times y_{1718} + 3600 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617}) - 6730)/14770) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s),$

$\exp(-s((50 \times y_{12} + 70 \times y_{28} + 70 \times y_{23} + 100 \times y_{35} + 90 \times y_{34} + 70 \times y_{56} + 50 \times y_{57} + 90 \times y_{47} + 90 \times y_{69} + 100 \times y_{610} + 100 \times y_{710} + 90 \times y_{318} + 50 \times y_{812} + 100 \times y_{911} + 70 \times y_{1011} + 70 \times y_{1113} + 70 \times y_{1215} + 70 \times y_{1314} + 100 \times y_{1517} + 70 \times y_{1417} +$

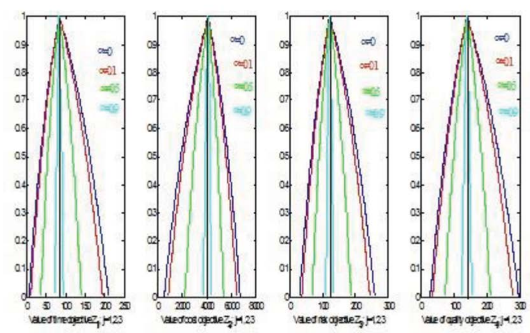

Figure 8. Time, cost, risk and quality objective functions at α levels 0, 0.1, 0.5 and 0.9 with exponential membership function for shape parameter (-1,-1,-1,-1).

Table 13. Results for α = 0,0.1,0.5 and 0.9

α level	Case	λ	Optimal path	(Z_1,Z_2,Z_3,Z_4)	Optimal path in[32]	(Z_1,Z_2,Z_3,Z_4)
α = 0	Case -1	1	1-2-3-18-19	((7,84.44,210), (521,4109,6730), (30,122.22,260), (27,139.33,297))	1-2-3-18-19	((7,84.44,210), (521,4109,6730), (30,122.22,260), (33,71.77,63))
	Case -2	1				
	Case -3	1				
	Case -4	1				
	Case -5	1				
α = 0.1	Case -1	1	1-2-3-18-19	((11.70,84.44,194.91), (844.16,4109,6444.68), (36.62,122.22,244.51), (34.33,139.33,278.58))		
	Case -2	1				
	Case -3	1				
	Case -4	1				
	Case -5	1				
α = 0.5	Case -1	1	1-2-3-18-19	((37.28,84.44,140.20), (2216,4109,5355), (68.89,122.22,186.40), (72.34,139.33,210.84))		
	Case -2	1				
	Case -3	1				
	Case -4	1				
	Case -5	1				
α = 0.9	Case -1	1	1-2-3-18-19	((73.66,84.44,94.47), (3714.56,4109,4347.88), (110.40,122.22,134.30), (124.2,139.33,152.46))		
	Case -2	1				
	Case -3	1				
	Case -4	1				
	Case -5	1				

$100 \times y_{1416} + 50 \times y_{1718} + 50 \times y_{1819} + 0 \times y_{68} + 0 \times y_{1415} + 0 \times y_{1617}) - 260)/560) - ((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$,

$\exp(-s((45 \times y_{12} + 27 \times y_{28} + 9 \times y_{23} + 27 \times y_{35} + 0 \times y_{34} + 9 \times y_{56} + 9 \times y_{57} + 27 \times y_{47} + 9 \times y_{69} + 9 \times y_{610} + 9 \times y_{710} + 0 \times y_{318} + 0 \times y_{812} + 45 \times y_{911} + 27 \times y_{1011} + 27 \times y_{1113} + 27 \times y_{1215} + 9 \times y_{1314} + 9 \times y_{1517} + 27 \times y_{1417} + 0 \times y_{1416} + 27 \times y_{1718} + 9 \times y_{1819} + 90 \times y_{68} + 90 \times y_{1415} + 90 \times y_{1617}) - 63)/252)-((1 - \exp(-s)) \times \lambda) \geq \exp(-s)$

With additional constraints of case study.

We have formulated model 4 for $\alpha = 0.1$, $\alpha = 0.5$, $\alpha = 0.9$ with different values of shape parameters and solution obtained are given in Table 13.

Table 13 indicates the solution of FMOCPP with exponential membership function by fuzzy programming technique. It shows that at α level 0 the optimal degree of satisfaction is 1 for each shape parameters (-1,-1,-1,-1), (-0.1,-0.3,-0.6,-0.8), (-0.1,-0.4,-0.8,-0.9), (-0.2,-0.4,-0.7,-0.9), (-0.1,-0.3,-0.6,-1). Similarly at α level 0.1 the optimal degree of satisfaction is 1 for each shape parameter (-1,-1,-1,-1), (-0.1,-0.3,-0.6,-0.8), (-0.1,-0.4,-0.8,-0.9), (-0.2,-0.4,-0.7,-0.9), (-0.1,-0.3,-0.6,-1). For α level 0.5 the optimal degree of satisfaction is 1 for each shape parameter (-1,-1,-1,-1), (-0.1,-0.3,-0.6,-0.8), (-0.1,-0.4,-0.8,-0.9), (-0.2,-0.4,-0.7,-0.9), (-0.1,-0.3,-0.6,-1). Similarly For α level 0.9 the optimal degree of satisfaction is 1 for each shape parameter (-1,-1,-1,-1), (-0.1,-0.3,-0.6,-0.8), (-0.1,-0.4,-0.8,-0.9), (-0.2,-0.4,-0.7,-0.9), (-0.1,-0.3,-0.6,-1). With five cases of shape parameter we get maximum degree of satisfaction that helps DM to take the decisions. Table 13 also compares the obtained output with existing solution approach which shows that the developed solution approach provides additional optimal degree of satisfaction to take the decision to decision makers. The Figure 8 shows the distribution of objective values with respect to liner membership function at different α level.

12. Conclusion

This approach provides the solution of FMOCPP using fuzzy linear membership function and exponential membership function subject to some realistic constraints to optimize the optimistic, the most likely and the pessimistic scenario of fuzzy objective functions with the triangular possibilistic distribution. The main benefit of our new approach is it provides optimum critical path according to all criteria's without calculation of all performance ranking of each path also sensitivity analysis is not necessary to perform in this approach.

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