Cultural Algorithm based Cooperative Spectrum Sensing Optimisation in Cognitive Radio Network

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Abstract

Objectives: Optimisation of Spectrum Sensing phenomenon by improving the probability of detection using Cultural Evolutionary Algorithm (CEA) in Cognitive Radio Network (CRN). **Methods/Statistical Analysis:** Cultural Algorithm (CA) has been used for the first time to optimize the spectrum sensing phenomenon. The acceptance function calculation and belief space adjustment have been performed for commonly used evolutionary algorithms like Genetic Algorithm (GA) and Particle Swarm Optimisation (PSO). **Findings:** Various scenarios for calculation of the probability of detection for a fixed value of probability of false alarm have been simulated in MATLAB. The results obtained have been compared with GA and PSO under identical scenarios. Improvements: Simulations reveal that CA achieves a better probability of detection as compared to GA and PSO for a given probability of false alarm. It observed that detection probability improves with an increase in participating population set of cognitive radios.

Keywords: Cognitive Radio, Cooperative Spectrum Sensing, Cultural Algorithm, Genetic Algorithm, Particle Swarm Optimisation

1. Introduction

CR Technology is the future of radio communications which is aimed at judicious spectrum utilization. The problem of spectrum scarcity is becoming increasingly pronounced with the evolution of wireless technologies like Wi-Fi, WiMAX, Long Term Evolution (LTE) etc., further, with miniaturization of portable computing devices like smart phones, laptops, palmtops etc the spectrum usage is becoming more challenging. CR has emerged as a promising technology which can addresses the issues of spectrum utilization very effectively. The licensed user or Primary User (PU) is one who owns the right to exploit the frequency spot of the frequency band allocated to him. However, it is observed that the EM spectrum is grossly underutilized. This motivates the Secondary Users (SU) who do not possesses legal ownership to any of frequency band and thus exploit the spectrum opportunistically whenever the PU is not transmitting, or can even co-exist with PU without causing any harmful interference¹⁻⁴. Conventional CRs utilize a variety of spectrum sensing techniques like Energy Detection, Matched Filter Approach, Cyclostationary Process etc. to ascertain the presence or absence of a PU. The local individual sensing results are forwarded to a central entity called Fusion Centre (FC), which finally proclaims the presence or absence of a PU and either allows or prohibits an SU from accessing the PU spectrum. In order to avoid the problems due to multipath fading, shadowing or near-far problems, cooperative sensing is performed in which a network of CRs collectively sense a PU signal and forward their individual sensing results to the FC⁵.

At FC, certain Hard Decision Fusion (HDF) techniques like logical AND, logical OR or M out of K rule may be applied to ascertain the presence or absence of PU. The FC may also apply Soft Decision Fusion (SDF) like Equal Gain Combining Scheme (EGC), Maximal Ratio Combining (MRC) or Maximal Likelihood Ratio (MLR) to reach to a final conclusion regarding the existence of PU. Though these methods are quite effective in achieving high detection probability (Pd), they do not optimize the weighting criterion very efficiently especially under low SNR conditions and limited availability of sensing intervals. Evolutionary Algorithms like Particle Swarm Optimisation (PSO) have been effectively utilized for spectrum sensing⁶. This method results in higher detection probability. In^z have used Efficient Adaptive Ant Bee Colony (EA-ABC) for spectrum sensing. The results reveal that modified strategies for ABC have been very effective in enhancing the capability to search for the global optimal solution and improved convergence speed. In this paper, we propose the use of Cultural Algorithm (CA) for maximizing the detection probability. The results have also been compared with Genetic Algorithm (GA) and PSO. The balance of the paper is structured asunder. In section 2 we discuss the system model, in section 3 we discuss CA, in brief, section 4 discusses CA in spectrum sensing role and finally, in section 5 we discuss results and simulations.

2. System Model

The system model for a network of *N* CR in a cognitive radio network is shown in Figure 1. The local individual decision of each CR is forwarded to the FC as per the given binary hypothesis. For a K^{th} instant of time hypothesis H₀ represents absence of PU and hypothesis H₁ represents presence of PU⁸.

$$x_i(k) = \begin{cases} a_i(k), & i = 1, 2, \dots N : H_0 \\ h_i s(k) + a_i(k), & i = 1, 2 \dots N : H_1 \end{cases}$$
(1)

Where $x_i(k)$ denotes the received signal of the ith CR, s(k) denotes the PU signal, h_i is the complex channel gain between the PU and the ith CR and $a_i(k)$ is the complex Additive White Gaussian Noise (AWGN) with zero mean and variance { σ_i^2 }. If the detection interval is considered over *M* samples then the sum of the received signal can be expressed as

$$U_i = \sum_{i=0}^{M-1} |x_i(k)|^2, \quad i = 1, 2, \dots, M$$
(2)

Where the value *M* may be evaluated from the timeband width product the statistics are transmitted to the FC through a control channel in an orthogonal manner can be denoted as $\{y_i\}_{i=0}^{M-1}$ where

$$y_i = U_i + n_i \tag{3}$$

here n_i denotes the noise induced in the channel characterised by zero mean and spatially uncorrelated, independent and identically distributed Gaussian random variable with variance $\{\delta_i^2\}$; these variances are collected into vector form $\boldsymbol{\delta} = [\delta_1^2, \delta_1^2, \dots, \delta_N^2]^{T_{\hat{n}}}$. The global statistics have been calculated as under:

$$y_{fc} = \sum_{i=0}^{N} w_i y_i = w^T y \tag{4}$$



Figure 1. Framework for cooperative spectrum sensing in cognitive radio network.

Where $w = [w_1, w_2, \dots, w_N]^T (w_i \ge 0)$ signifies a weight vector assigned at the FC and $y = [y_1, y_2, \dots, y_N]^T$. The weight is allocated to each SU depending on the contribution each SU makes towards the global decision. The probability of False Alarm (P_{fa}) can thus be calculated as under:

$$P_{fa} = Q\left(\frac{\gamma_{fc} - M\sigma^T w}{\sqrt{w^T A w}}\right) \tag{5}$$

Where $Q(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} exp^{\frac{-t^2}{2}} dt$, $A = 2M diag^2(\sigma) + diag(\delta)$, and diag (•) is a diagonal matrix, $\boldsymbol{\delta} = [\delta_1^2, \delta_2^2, \dots, \delta_N^2]^T$, $\boldsymbol{\sigma} = [\sigma_1^2, \sigma_2^2, \dots, \sigma_N^2]^T$. The Probability of Detection (P_d) can be calculated as:

$$P_d = Q\left(\frac{\gamma_{fc} - (M\sigma + E_sh)^T w}{\sqrt{w^T B w}}\right) \tag{6}$$

Where $\mathbf{B} = 2M \operatorname{diag}(\sigma) + \operatorname{diag}(\delta) + 4 \mathbb{E}_s \operatorname{diag}(h) \operatorname{diag}(\sigma)$, $\mathbf{h} = [|h_1|^2, |h_2|^2, \dots, |h_N|^2]^T$ and $\mathbb{E}_s = \sum_{k=0}^{M-1} |s(k)|^2$ the test threshold can be expressed as:

$$\gamma_{fc} = M\sigma^T w + Q^{-1} (P_{fa}) \sqrt{w^T A w}$$
(7)

3. Cultural Algorithm

The conventional EAs have little or no domain knowledge of the search objective and hence the search process employed by such algorithms is totally unbiased. Very often because of limited domain knowledge the search space of such algorithms becomes quite large. However, if some domain knowledge is infused into the search process, this drastically cuts down the search space. In other words, domain knowledge serves as a mechanism to reduce the search space by pruning undesirable parts of the solution space, and by promoting desirable parts^{8.9}. Reynolds¹⁰ in 1994 proposed a concept of Cultural Algorithm in which the search process is biased with domain knowledge as well as knowledge acquired due to evolution to yield a better result. CA unlike GA enables societies to adapt to their changing environments at rates that exceed that of biological evolution. Culture has been defined as "Cumulative deposit of knowledge, experience, beliefs, values, attitudes, meanings, hierarchies, religion, notions of time, roles, spatial relations, concepts of the universe, and material objects and possessions acquired by a group of people in the course of generations through individual and group striving"².

The dual inheritance of CA maintains two search spaces; the population representing the genetic component and belief space representing the cultural component. Both these search spaces evolve in parallel and have significant influence over one another. Individual experiences of various users amongst the population space, identified through an acceptance function, are utilized for generation of a problem- solving knowledge residing within the belief space. An acceptance function determines which individual in the current population are able to impact or to be voted to contribute to the current beliefs. This knowledge is stored and manipulated in the belief space; this is called adjusting the belief space. The adjusted beliefs in-turn influences the evolution of a population. The two components, population space, and belief space; interact through a communication protocol which determines the set of acceptable individuals that are able to update that belief space as shown in Figure 2.



Figure 2. Population and Belief Space in CA.

3.1 Belief Space

The belief space is a central natural repository of information or knowledge where the collective behaviour or beliefs of individuals in population space is stored. It is sometimes also referred to as meme pool, where the meme is a generalized experience of individuals within the population space which acts as a unit of information transmitted through behavioural means 10. The information or knowledge is accumulated over multiple generations in the belief space. As the search is biased through the domain knowledge or the knowledge inherited from the previous generations, it results in the significant pruning of the population space. The knowledge residing within the belief space filters the optimal solutions, resulting in better solutions with each generation¹¹⁻¹². Updating of belief space is scheduled to occur after each iteration by the most eligible candidate. The eligibility of these candidates is tested through a fitness function. The knowledge base existing within the belief space is categorized based on the domain which it represents. Accordingly, the belief has been classified into five basic categories^{13,14}:

(a) Normative Knowledge: This is a set of desirable value ranges which are expected to reside within the population space e.g. acceptable behaviour for the agents in the population.

(b) Domain Specific Knowledge: Called *prior* in the Bayesian statistics, it reflects some knowledge pertaining to the problem being optimized.

(c) Situational Knowledge: This domain refers to the knowledge pertaining to the vital incidents in the search space e.g. successful/unsuccessful solutions.

(d) Historical/Temporal Knowledge: The knowledge residing in the history of the search space. e.g. temporal patterns of the search space, is factored here.

(e) **Spatial Knowledge:** The information on the landscape or topography of the search space is factored under this head.

In this paper we have considered only two components viz. situational and normative knowledge, and represented the belief space as a tuple.

$$\mathcal{B}(t) = (\mathcal{S}(t), \mathcal{N}(t)) \tag{8}$$

Here S(t) represents the situational knowledge component whereas $\mathcal{N}(t)$ represents the normative knowledge component in the belief space. The set of best

solutions is encapsulated within the situational component and normative component as under:

$$\mathcal{S}(t) = \left\{ \widehat{\beta}_l(t) : l = 1, 2, \dots, n_s \right\}$$
(9)

$$\mathcal{N}(t) = \{\chi_1(t), \chi_2(t), \dots, \chi_{n_x}(t)\}$$
(10)

For each dimension in equation (10) following information is stored.

$$\chi_j(t) = \left(\mathbb{I}_j(t), \mathbb{L}_j(t), \mathbb{U}_j(t) \right)$$
(11)

Where \mathbb{I}_i denotes a closed interval,

$$\mathbb{I}_{j}(t) = \left[\alpha_{\min,j}(t), \alpha_{\max,j}(t)\right] = \left\{\alpha; \alpha_{\min,j} \le \alpha \le \alpha_{\max,j}\right\}$$
(12)

And \mathbb{L}_{j} , \mathbb{U}_{j} represents the lower and the upper bounds respectively $\chi_{j}(t)$ represents the scope of the *j*th dimensional normative knowledge.

3.2 Acceptance Function

Acceptance function selects those individuals from the population space who help shaping the belief space in a favourable manner. A variety of selection techniques may be employed e.g. elitism, tournament selection or roulette-wheel selection, given that the number of individuals remains the same. The number of individuals is determined as:

$$\eta_{\mathcal{B}}(t) = \left[\frac{\eta_{\mathcal{S}}\theta}{t}\right] \tag{13}$$

With $\vartheta \in [0,1]$ using this approach, the large initial belief space decreases exponentially with time. The acceptance function selects top 40% best positions which can directly influence the belief space.

3.3 Adjusting Belief Space

The individuals selected through the acceptance function defined by equation (13) above. The normative and situational components can thus be updated as under, the function being minimized is assumed to be continuous and unconstrained:

(a) Situational Knowledge: We have assumed that only one element has been kept in the situational knowledge component¹⁵.

$$\mathcal{S}(t) = \left\{ \hat{\beta}(t+1) \right\} \tag{14}$$

Where

$$\hat{\beta}(t+1) = \begin{cases} \min_{l=1,2,\dots,\eta_{\mathcal{D}(t)}} \{\alpha_l(t)\} iff\left(\min_{l=1,2,\dots,\eta_{\mathcal{D}(t)}} \{\alpha_l(t)\}\right) < f\left(\hat{\beta}(t)\right) \\ \hat{\beta}(t) otherwise \end{cases}$$
(15)

(b) Normative Knowledge: The interval update is as follows

$$\alpha_{\min,j}(t+1) = \begin{cases} \alpha_{lj}(t)if\alpha_{lj}(t) \le \alpha_{\min,j}(t)orf(\alpha_l(t)) < \mathbb{L}_j(t) \\ \alpha_{\min,j}(t)otherwise \end{cases}$$
(16)

 $\alpha_{min,j}(t + 1)$ if the lower bound of jth dimensional normative knowledge for tth updating.

$$\alpha_{max,j}(t+1) = \begin{cases} \alpha_{lj}(t)if\alpha_{lj}(t) \ge \alpha_{max,j}(t)orf(\alpha_l(t)) < \mathbb{U}_j(t) \\ \alpha_{max,j}(t)otherwise \end{cases}$$
(17)

 $\alpha_{max,j}(t+1)$ if the upper bound of jth dimensional normative knowledge for tth updating.

$$\mathbb{L}_{j}(t+1) = \begin{cases} f(\alpha_{l}(t)) \ if \alpha_{lj}(t) \leq \alpha_{min,j}(t) orf(\alpha_{l}(t)) < \mathbb{L}_{j}(t) \\ \mathbb{L}_{j}(t) otherwise \end{cases}$$
(18)

 $\mathbb{L}_{j}(t+1)$ denotes overall performance score for Lower Bound.

$$\mathbb{U}_{j}(t+1) = \begin{cases} f(\alpha_{lj}(t)) \ if \alpha_{lj}(t) \ge \alpha_{max,j}(t) orf(\alpha_{l}(t)) < \mathbb{U}_{j}(t) \\ \mathbb{U}_{j}(t) otherwise \end{cases}$$
(19)

 $\mathbb{U}_j(t+1)$ Denotes overall performance score for Upper Bound. Lower bound α_{\min} and upper bound α_{\max} are the boundary values which are updated by the acceptance function. The initial optimal position is stored as the initial situational knowledge. $\mathbb{L}_j(t+1)$, $\mathbb{U}_j(t+1)$ may be initialized to $+\infty$ to obtain optimized minimum value.

3.4 Influence Function

The individuals in the population are adjusted using beliefs to conform closer to the global beliefs. These adjustments are realized via influence functions.

4. Cooperative Spectrum Sensing based on Cultural Algorithm

In this section an endeavour has been made to find the optimal weight vector w_0 so as to maximize P_d .

$$P_{max} = P_d = Q\left(\frac{Q^{-1}(P_{fa})\sqrt{w^T A w} - E_s h^T w}{\sqrt{w^T B w}}\right)$$
(20)

As the *Q* function in equation (20) is monotonically decreasing, thus maximizing P_{d} . This is achieved by minimizing the following function:

$$f(w) = \frac{Q^{-1}(P_{fa})\sqrt{w^T A w} - E_{gh}^T w}{\sqrt{w^T B w}}$$
(21)

If w_0 is chosen as an optimal solution which minimizes f(w), then Λw_0 will also be an optimal solution minimizing f(w) where Λ is any positive real number. The optimal values of w_i which minimize the overall fitness function f(w)can be expressed as $w_i = (w_1, w_2, \dots, w_N)$ where $1 \le i \le N$. Values of w, are bounded through various environmental parameters, like signal power, noise power, delay spread, spectrum information etc, and transmission parameters like transmit power, modulation type, bandwidth, symbol rate etc. By changing the search size and direction, the value of respective w, is also adjusted accordingly with the help of a fitness function, which optimizes the search result. The fitness function evaluates the status of each weight vector; here our objective is to enhance the spectrum sensing by maximizing the detection probability P_{d} , which is achieved by minimising the fitness function. Thus our optimisation problem transforms into:

$$\begin{split} \min_{w} f(w), \ st. \ \sum_{l=1}^{N} w_{l} &= 1, \ 0 \leq w_{l} \leq 1 \\ \end{split}$$

Situational Knowledge _{candidate} ← Accept Situational Knowledge (Population)

Update Situational Knowledge (KnowledgeBase, Situational Knowledge _{candidate})

 $Children \leftarrow ReproduceWithInfluence (Population, Knowledgebase)$

Population - Select (Children, Population)

 $NormativeKnowledge_{candidate} \leftarrow Accept \qquad NormativeKnowledge_{candidate} \leftarrow Accept \qquad NormativeKnowledge_{candidate} \qquad NormativeKnow$

UpdateNormativeKnowledge(Knowledge Base, Normative Knowledge _{candidate})

End

Return(Knowledgebase)

5. Simulation and Results

The efficacy of CA in spectrum sensing is evaluated by comparing the results obtained with some of the commonly used EAs like GA and PSO. The algorithm has been simulated for two scenarios N=6 and N=8 i.e. for 6 (Figure 3) and 8 (Figure 4) CR users respectively. The

results reveal that CA offers better detection probability $P_{\rm d}$. In Figure 3, $\sigma = \delta = [1, 1, 1, 1, 1, 1]^{\rm T}$ and the received SNR are [-3.94, -3.68, -3.05, -3.37, -3.35, -3.79]. In Figure 4, $\sigma = \delta = [1,1,1,1,1,1]^{\rm T}$ and the received SNR are [-3.33, -3.58, -3.05, -3.56, -3.93, -3.83,-3.10, 3.34]. It is clearly evident from the results that as the probability of false alarm increases, the detection probability also increases.



Figure 3(d). $P_{fa} = 0.5$.

Figure 3(a-d). Performance Comparison for CA, GA and PSO algorithm for M=6.

Further, the performance of CA is found to be superior to GA and PSO, while the performance of PSO is worst under given scenario. The results have also been tabulated in Tables 1,2 for M=6 and M=8 respectively. Figure 5 shows a comparative analysis between CA, GA and PSO in the form of a curve between P_d vs P_{fa} . It is observed that

the detection probability is best for CA followed by GA and then PSO, thereby re-establishing the fact that CA is a promising technique which can be employed at the FC to deduce better sensing results and improve the detection probability of the system.





Figure 4(d). $P_{fa} = 0.5$





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w	СА		GA		PSO		
	P _d	w	P _d	w	P _d	w	
P _{fa} = 0.01							
w1	0.615	0.272	0.595	0.219	0.615	0.177	
w2		0.169		0.190		0.177	
w3		0.221		0.155		0.177	
w4		0.078		0.010		0.177	
w5		0.139		0.199		0.117	
w6		0.122		0.228		0.177	
$P_{fa} = 0.3$							
w1		0.237		0.129	0.8	0.179	
w2		0.152		0.164		0.179	
w3	0.01	0.139	0.8	0.164		0.135	
w4	0.81	0.183		0.207		0.179	
w5		0.144		0.180		0.179	
w6		0.145		0.156		0.150	
$P_{c_{0}} = 0.4$							
w1		0.192	0.8	0.225	0.82	0.196	
w2		0.201		0.010		0.196	
w3	0.83	0.152		0.235		0.125	
w4		0.154		0.132		0.167	
w5		0.168		0.198		0.125	
w6		0.134		0.200		0.191	
P _{fa} = 0.5							
w1		0.211	0.92	0.159	0.84	0.179	
w2		0.219		0.094		0.179	
w3	0.845	0.139		0.144		0.179	
w4		0.154	0.02	0.236		0.125	
w5		0.161		0.159		0.179	
w6		0.206		0.210		0.158	

Table 1. Comparison of Results for M=6

Table 2. Comparison of Results for M=8

w	СА		GA		PSO	
	Pd	w	Pd	w	Pd	w
Pfa =0.01						
w1	0.69	0.160	0.655	0.096	0.700	0.140
w2		0.221		0.157		0.118
w3		0.112		0.133		0.093
w4		0.074		0.230		0.010
w5		0.121		0.096		0.131
w6		0.176		0.096		0.197
w7		0.100		0.096		0.170
w8		0.126		0.096		0.141

Pfa=0.3						
w1	0.875	0.133	0.82	0.096	0.84	0.171
w2		0.080		0.157		0.119
w3		0.196		0.133		0.017
w4		0.071		0.230		0.010
w5		0.149		0.096		0.171
w6		0.110		0.096		0.171
w7		0.146		0.096		0.171
w8		0.114		0.096		0.171
Pfa=0.4						
w1		0.107	0.8021	0.096	0.87	0.162
w2		0.119		0.157		0.086
w3		0.236		0.133		0.162
w4	0.895	0.089		0.230		0.010
w5	0.095	0.104		0.096		0.162
w6		0.106		0.096		0.162
w7		0.095		0.096		0.112
w8		0.144		0.096		0.146
Pfa=0.5						
w1		0.047	- 0.86	0.096	0.88	0.153
w2		0.101		0.157		0.073
w3	0.89	0.209		0.133		0.153
w4		0.206		0.230		0.010
w5		0.092		0.096		0.153
w6		0.112		0.096		0.153
w7		0.083		0.096		0.153
w8		0.150		0.096		0.153

6. Conclusion

It is evident from the results obtained that CA based cooperative sensing is a promising candidate for improving the overall detection probability of the system. The limitations offered by the conventional methods like HDF and SDF techniques have been greatly overcome by evolutionary techniques which help the SU to cognitively adapt to the surrounding prevailing situations. The results show that the detection probability is significantly improved using CA. Comparison with other EAs like PSO and GA, further corroborate the fact that CA can be utilized for optimising spectrum sensing problem. Scope of this study can further be analysed in a fast changing atmosphere like the modern day battlefield where a large number of wireless networks co-exists and jamming of various frequency spots/bands renders many radio sets out of operation. To the best of our knowledge, this the first endeavour using CA for analysing the cooperating spectrum sensing for improvement of detection probability.

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