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Reducing PAPR of OFDM Systems using Cyclic Prefix Shifting Algorithm

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Abstract

Objectives: In this paper, a novel scheme was introduced to mitigate the PAPR problem of OFDM system with reduced computational complexity. **Methods/Statistical Analysis**: The literature brought out many algorithms to overcome the PAPR problem, such as the amplitude clipping, which is simplest but degrades the BER performance, and the selected mapping, which is probabilistic (no BER degradation) but suffers from the increased computational complexity. Thus, by cyclically shifting the frequency-domain OFDM symbol to change the order of the samples, the PAPR will be reduced significantly. **Findings:** Simulation results show that the PAPR has been reduced, without introducing BER degradation, due to the probabilistic nature of the proposed method. The computational complexity of the suggested algorithm, if compared with the SLM scheme, is reduced dramatically, say 68%. Hence, the approach can be implemented without increasing the system's size or hurt the battery life of the end user equipment. **Application/Improvements:** The OFDM system employed in wide ranges of data rates systems, however, the computational complexity plays an important role in the fabrication, thus, using our proposed method, the system will not be expensive, hard to implement and still provides high data rates.

Keywords: Cyclic Prefix Algorithm for PAPR Mitigation, OFDM, PAPR, Selected Mapping

1. Introduction

During the last decade, there was a real revolution in terms of wireless communication systems. This revolution makes up big demand for high data rates, where the internet services becomes wider and supports large populations. Hence, the demand for video streaming, torrent downloads and files upload/download, etc. made up the requirements for new technologies to cover all the needs. However, the single carrier modulation systems cannot support all requirements and then the multicarrier modulation techniques started to grew, such as Frequency Division Multiplexing (FDM). But FDM did not utilize the frequency spectrum efficiently. In 1966 Chang introduced the first method for an overlapping subcarriers[1]. However, at that time, the technology was not capable to support this technology efficiently using cheap components. In 1980s, the Digital Signal Processors (DSPs) revolution started to arise, thus, it becomes as simple fabrication as possible of efficient algorithms using these DSP processors. Then, the time come to use the technique of Chang as an alternate for the FDM, therefore, the Discrete Fourier Transform (DFT) and the Inverse Discrete Fourier Transform (IDFT) played a crucial role in the communication systems[2], where an efficient Fast Fourier Transform (FFT) and efficient Inverse Fast Fourier Transform (IFFT) becomes as simple implemented on little DSP processors as possible. Accordingly, the Orthogonal Frequency Division Multiplexing (OFDM) can be implemented easily.

Because of the allowance of frequency overlap, OFDM becomes the most useful modulation technique for high data rates. Hence, OFDM becomes an essential part of modern telecommunication systems. In Europe the Digital Video Broadcast (DVB), IEEE802.11 series, Long Term Evolution (LTE) and LTE-Advanced (LTE-A).

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Furthermore, the channel will be divided into narrow-band subchannels, thus the OFDM signals are more immune to frequency selective and fading channels. Moreover, the employment of cyclic prefix made OFDM signals more resistive to Intersymbol Interference (ISI) and Intercarrier Interference (ICI). What is more is the efficient implementation of OFDM systems using I/FFT processors.

In spite of all advantages of OFDM systems, some major limitations are also possible in OFDM systems, such as the sensitivity to frequency offset and the high power peaks with respect to its average, which is addressed as Peak to Average Power Ratio (PAPR). High PAPR causes the system to works improperly, hence, intermodulation distortion will be seen, which leads to quite bad degradation in the BER performance. The literature is rich with PAPR problem mitigation algorithms like the amplitude limitation by clipping then filtering[3], which is reported as the simple method, but it produces non-linearity due to the discontinuity that produces in-band distortion and out of band radiation. Channel coding techniques can also be utilized to reduce the PAPR[4], but these techniques reduce the bit rate due to the required redundancy data. Multiple signal representations algorithms such as Partial Transmit Sequences (PTS)[5] and Selected Mapping (SLM)[6], however, PTS and SLM are probabilistic schemes, i.e. the BER will not be affected, but the computational complexity will be increased dramatically.

Many researchers suggested improvements for the above techniques. For instance, in[7] the authors proposed to mitigate the drawback of the clipping operation by introducing a pre-defined function, at both the transmitter and the receiver sides and then the clipped sample's indices will be sent to the receiver as side information. This approach works in the time-domain, in other words, the BER may degrade. The probabilistic methods are efficient with respect to BER performance and PAPR reduction gain, but the computational complexity will be the main problem,[8] introduced an approach, which is a modification to the conventional SLM scheme, such that the computational complexity can be reduced. Furthermore, in[9] the related SLM's computational complexity has been reduced significantly, but at the expense of the BER performance degradation, since the proposed approach translated the conventional SLM to the time-domain.

Inspired by[8,9], n this paper, a novel methodology will be introduced. The suggested method is based on the cyclically shifting the frequency domain OFDM samples in order to change the order of the data, such that the PAPR can be altered. It may be not good approach without checking the PAPR at each shift, thus, there will be multiple blocks of the IFFT operations and accordingly, the computational complexity will arise again. Therefore, the cyclically shifted samples will be increased, in other words the OFDM signal will be divided into disjoint blocks, but not exceeds the number of candidates of the traditional SLM scheme, to reduce the computational complexity compared with the SLM scheme. The rest of the papers are organized as follows: Section 2 introduces the OFDM system and the SLM method for PAPR reduction, Section 3 suggests the cyclically shifting approach for PAPR reduction. In Section 5, the results and their discussions will be explained and then Section 6 concludes this work.

1.1 Clarifications

In this paper, bold font capital letter stand for matrix or vector in frequency-domain, bold font small letter is a matrix or vector in time-domain, italic capital letter represents a scalar in frequency-domain, and finally, small letter italic represents the time-domain scalar.

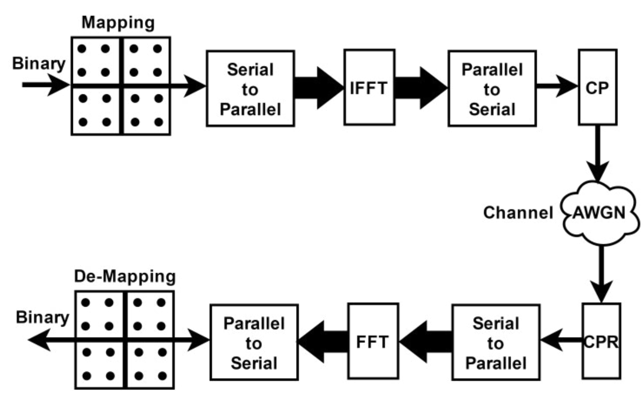

Figure 1. Orthogonal frequency division multiplexing system.

2. OFDM and SLM Modeling

OFDM consists mainly from the IDFT with some other data manipulation blocks. Figure 1 states the simple OFDM structure, which will be adopted in this paper. Firstly, data passed to the mapping block, which is either Multi-level Quadrature Amplitude Modulation (M-QAM) or Phase Shift Keying (M-PSK). That is, the data will be mapped on either constellation type, then serial-to-parallel (de-multiplexing) converted to feed them to the amplitude modulation using orthogonal sub-carriers, which is done by the IDFT or its fast version, IFFT. After modulating the data, the signal will undergo parallel-to-serial (multiplexing) operation, to convert the data back to the serial form. Then the cyclic prefix interval will be added, to eliminate the ISI and the ICI. Since the paper scope is a baseband OFDM system, only Additive White Gaussian Noise (AWGN) channel was employed, as shown in Figure 1.

The OFDM signal that generated using Figure 1 can be represented mathematically as:

$$s[n] = \frac{1}{\sqrt{N}} \sum_{k-0}^{N-1} S[k] e^{j2\pi \frac{kn}{N}} \quad (1)$$

Where n and k = 0, 1,..., N-1, N is the size of the OFDM symbol, $S[k]$ are randomly drawn symbols from either M-QAM or M-PSK constellation mapping families. Equation (1) tells that the output signal $s[n]$ is a summation of frequencies with different phases and magnitudes, then there will be either destructive addition or constructive addition. The last one may produces high power peaks with respect to the average that is make the system to work in the non-linear region. In other words, the power amplifier will be derived to the non-linear region, which in this case the intermodulation distortion will happen. However, these phenomena can be formulated mathematically as follows:

$$PAPR = \frac{\max[|s[n]|]^2}{average\{|s[n]|\}} \quad (2)$$

In the last expression, the discrete time-domain OFDM symbols are represented by s[n]. Conventionally, the PAPR expressed in decibels (dB), therefore, it will be measured in dBs in this paper:

$$PAPR_{dB} = 10\log_{10}\left(\frac{\max[|s[n]|]^2}{average\{|s[n]|\}}\right) \quad (3)$$

The signal in Equation 1 is sampled using Nyquist rate, in other words, the discrete-time signal may not behave as the continuous-time signal, therefore and some events of large peaks may not be captured. For that reason, the discrete time-domain signal has to be up-sampled, to make its attitude similar to its continuous time-domain version, thus, and according to Tellambura[10], if the signal up-sampled four times, the discrete-time signal will produce almost the same properties of its continuous-time version, hence, in this paper, the signal will be up-sampled by a factor $L = 4$, and then, the signal will be checked for high peaks using Equation 3. As a consequence, all collected PAPR values will be used to determine the Complementary Cumulative Distribution Function (CCDF) values using the following equation:

$$CCDF(PAPR) = \Pr(PAPR \geq \lambda) \quad (4)$$

Where λ is the clipping threshold of the PAPR. As aforementioned previously, SLM is one of the multiple signal representations algorithms, where the original signal will be represented multiply C times, where all these copies have the same original data, **S**, without modifications. Consequently, each copy will be phase rotated by multiplying each copy by its corresponding phase rotation vector, **v**. Then, all resultant phase rotated copies will be fed to the corresponding IFFT function, this operation of rotating the original data can be formulated as follows:

$$S^c = v^c \times S \quad (5)$$

where c = 1, 2,..., C, $\mathbf{v}^c$ is the c^{th} phase rotation vector, $\mathbf{S} = [S[0], S[1],..., S[N\text{-}1]]^T$, where the superscript T stand for the transpose operator and:

$$S^c = \begin{bmatrix} S^c[0] \\ \vdots \\ S^c[N-1] \end{bmatrix} \quad (6)$$

The phase rotation vector, $\mathbf{v}^c$, could be:

$$\mathrm{v}^c = \begin{bmatrix} v^c[0] & \cdots & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & v^c[N-1] \end{bmatrix} \quad (7)$$

Now, the candidate with lower PAPR will be used for transmission, sending alongside with it the index of the phase rotation vector, which showed lower PAPR:

$$s^{\tilde{c}} = F_N \times [v_{\tilde{c}} \times S] \quad (8)$$

Where stands for the branch index of lowest PAPR, $\mathbf{F}_N$ is the twiddle factor matrix of the IFFT function:

$$\sqrt{} \begin{bmatrix} F_N^{-0.0} & F_N^{-0.1} & F_N^{-0.2} & & F_N^{-0.(\ \ -1)} \\ F_N^{-1.0} & F_N^{-1} & F_N^{-2} & & F_N^{-(\ \ -1)} \\ F_N^{-2.0} & F_N^{-2} & F_N^{-4} & & F_N^{-2(\ \ -1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ F_N^{-(N-1).0} & F_N^{-(N-1)} & F_N^{-2(N-1)} & & F_N^{-(N-1)(N-1)} \end{bmatrix} \quad (9)$$

The size of the side information can be determined as the number of bits, which represents the index of the branch with lower PAPR. According to the above scenario, the computational complexity of the conventional SLM scheme can be formulated according to the number of multiplications operations[11]:

$$m_{SLM} = \frac{CN}{2} \log_2 N \tag{10}$$

And the number of mathematical operations of additions[11]:

$$a_{SLM} = CN \log_2 N \tag{11}$$

It is worth mention here that the above two expression, 10 and 11, are for complex mathematical operations, and that the phase rotation vectors will be selected from Hadamard matrix since they are reported to be the optimum vectors[12], furthermore, the side information can be determined as $\log_2 C$, where C is the number of phase rotation vectors employed in the SLM method.

3. Cyclic Prefix Shifting (CPS) Algorithm

This section presents the proposed algorithm, it is called Cyclic Prefix Shifting (CPS) method to reduce the PAPR of OFDM systems. The CPS fashion can be reported as a multiple signal representation method. Hence, the frequency domain samples will be represented D times, where $D < C$ to keep the lower complexity. The intent of shifting the samples, which are produced by the mapper, is to change the order of the phases in order to reduce the correlation between the data, accordingly the PAPR will be altered. CPS can be described as follows; given $\mathbf{S} = [S[0], S[1],..., S[N-1]]^T$, then $\mathbf{S}$ has to be partitioned into D disjoint parts:

$$S = [S^1 \quad S^2 \quad \cdots \quad S^\beta]^T \tag{12}$$

where β stands for the size of the partition, which is $\beta = N/D$. Suppose, for instance, that the number of shifts $D = 4$, then $\beta = N/4$, in other words, the size of each partition will be $0.25N$. If $D = C$ then the computational complexity of CPS scheme is equivalent to that of the conventional SLM method:

$$m_{CPS} = \frac{DN}{2} \log_2 N \tag{13}$$

And the number of mathematical operations of additions:

$$a_{CPS} = DN \log_2 N \tag{14}$$

Furthermore, the number of bits as side information will be represented by the index D as:

$$si_{CPS} = \log_2 D \tag{15}$$

Hence, if for example, the samples are partitioned into four disjoint parts, then $\mathbf{S} = [\mathbf{S}^1, \mathbf{S}^2, \mathbf{S}^3, \mathbf{S}^4]^T$, then there will be maximum of four shifts, $d = 0, 1, 2, 3$, i.e., $D = 4$, where $d = 0$ represents no shifting operation at all, to keep the original data as they are to compare the PAPR of the other shifted versions with it. According to the above scenario, the four-shifted versions can be expressed as follows:

$$\begin{aligned} \tilde{S}^0 &= [S^1 \quad S^2 \quad S^3 \quad S^4]^T \\ \tilde{S}^1 &= [S^4 \quad S^1 \quad S^2 \quad S^3]^T \\ \tilde{S}^2 &= [S^3 \quad S^4 \quad S^1 \quad S^2]^T \\ \tilde{S}^1 &= [S^2 \quad S^3 \quad S^4 \quad S^1]^T \end{aligned} \tag{16}$$

note that there will be no need for the phase rotation vector as in the SLM method. For each shifted version of $\mathbf{S}$, $\tilde{S}^d$, the PAPR will be computed. This scenario of CPS algorithm can be more clear in Figure 2. Consequently, the shifted version that shows better PAPR performance will be adopted for transmission alongside with the side information, which is the shifting index.

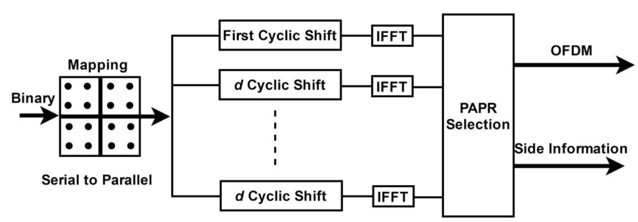

Figure 2. Cyclic prefix shifting algorithm mechanism.

4. Results and Discussion

The evaluation of the proposed method will be shown in this section. A comparison with the traditional SLM algorithms will be conducted in terms of the computational complexity, side information, and PAPR reduction gain.

Approximately 50,000 OFDM symbols will be generated randomly in this simulation, to confirm the strengthens of the suggested scheme, CPS. The size of the OFDM symbol, N, will be conducted in two scenarios, first, $N = 1024$, and the second scenario $N = 2048$. In both scenarios the constellation family fixed to 16-QAM. Regarding the SLM approach, the number of phase rotation vectors will be 16 vectors drawn from the Hadamard matrix. As mentioned previously, the oversampling factor $L = 4$.

Figure 3 explains the PAPR comparison of the original OFDM signals, without PAPR reduction being used, the traditional SLM and the PAPR of the CPS algorithm. It is shown that with 16 phase rotation vectors in SLM algorithm, the PAPR was reduced significantly, from 12.2 dB to 7.8 dB. That is, a reduction of 4.4 dB was achieved using SLM method. While CPS method has reduced the PAPR from 12.2 dB to, approximately, 8 dB, in other words, the PAPR has been reduced by 4.2 dB, although there is a slight loss in the PAPR reduction gain, which is around 4.5%, which it can be ignorable, but the computational complexity has been reduced dramatically, as will be shown later.

When the size of the OFDM symbol has been increased from 1024 to 2048 subcarriers, the PAPR of the traditional OFDM signals was increased, 12.6 dB. Thus, Figure 4 shows that the PAPR of the traditional OFDM symbols was reduced to 9 dB using SLM-scheme with 16 phase rotation vectors, i.e., a reduction of 3.6 dB has been obtained using the SLM algorithm, while the CPS method reduced the PAPR from 12.6 dB to 9.3 dB, or there is a reduction of 3.3 dB if the CPS scheme being employed for PAPR reduction. As mentioned in the previous scenario, when the size of the OFDM symbol was 1024 subcarriers, the CPS algorithm has less PAPR reduction gain, compared with the SLM method, hence, there are 8% losses in the reduction gain of the PAPR compared with that of the SLM method. However, the computational complexity reduction was the other hand achievement in the CPS-scheme, as will be stated in the next subsequent subsection.

Thus, the PAPR can be reduced significantly using CPS-algorithm, but with slight loss in the reduction gain, which can be ignored, if the computational complexity was reduced dramatically compared with the SLM method. In fact, the computational complexity will be calculated using Equation 10 and Equation 11 for the number of multiplications and number of additions, respectively, and Equations 13 and 14, for the number of multiplications and number of additions, respectively, when using CPS-method. Hence, there are 81920, 40960 operations of multiplications and additions, respectively, when employing SLM method, while there are 25600 and 12800 operations of multiplications and additions, respectively, if the CPS has been used for PAPR reduction. In other words, the number of multiplications and additions operations has been reduced by 68.75% with respect to the SLM method, hence, the suggested method has better performance compared with SLM method in terms of computational complexity and the required side information.

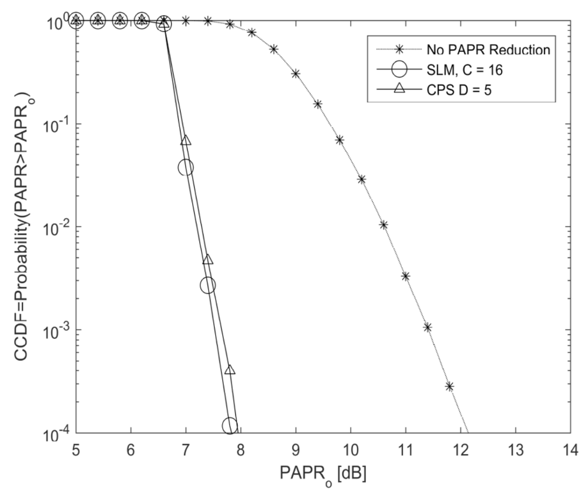

Figure 3. PAPR comparison between traditional 1024-OFDM, SLM-OFDM and CPS-OFDM systems, when number of phase rotation vectors is 16 and number of shifting is 5, all for 16-QAM mapping.

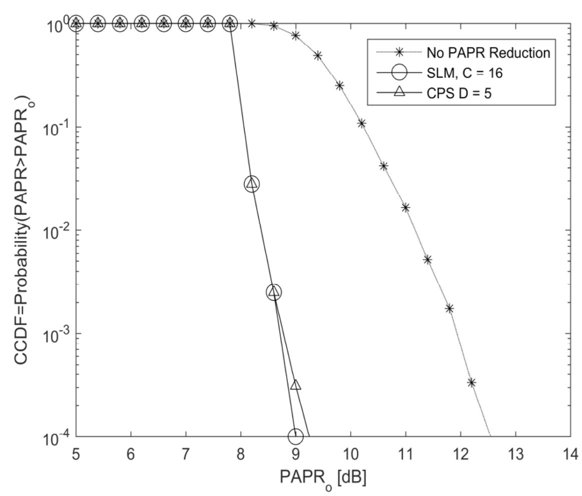

Figure 4. PAPR comparison between traditional 2048-OFDM, SLM-OFDM and CPS-OFDM systems, when number of phase rotation vectors is 16 and number of shifting is 5, all for 16-QAM mapping.

Since the number of bits as side information depends on the number of phase rotation vectors and number of shifts for the SLM and CPS algorithms, respectively, the required number of bits in the SLM approach is four bits, while for the CPS the required number of bits as side information is, approximately, 3 bits, where in the above results, the number of phase rotation vectors was 16 and the number of cyclic shifting was 5, as aforementioned previously.

5. Conclusions

A novel PAPR reduction for OFDM systems has been introduced. The suggested algorithm employed a smaller number of cyclic shifting than the number of phase rotation vectors of the SLM method. It was shown that the CPS algorithm can reduce the PAPR significantly and can be comparable with the SLM PAPR reduction performance. The computational complexity was lower than the SLM, dramatically. The proposed method is a probabilistic approach, which it does not degrade the BER performance as in the SLM method. Furthermore, there is no restriction on the constellation family or even the size of the OFDM symbol.

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