Method of Analytical Calculation of Critical Stress Intensity Factor and its Application in CAE System

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Abstract

Background/Objectives: Current importance of the study is stipulated by the high costs associated with ASTM E-399 field trials. The aim is to develop the new method to avoid or reduce them. **Methods:** The principle methods applied to the investigation of this problem are as follows: calculation of the critical value of stress intensity factor (SIF) employing analytical method and making use of the phySIFal and mechanical properties of the materials; calculation of SIF critical value experimentally, according to standard ASTM E-399 requirements. The study is accomplished with the verification tasks that prove the workability of the method and with the results of its implementation within the prototype software (PS). The PS belongs to Computer-Aided Engineering (CAE) systems and is applied to solve the issues of fracture strength and cracking resistance. **Findings:** The study presents the new method founded on the application of the modified Murakami formula to calculate the critical value of SIF. There is a new algorithm that explains the functions of the PS. The algorithm consists of two parts. The first part considers the structure without cracks; the second part describes the structure with a crack. The second part of the algorithm has a block that includes the modified formula for calculating the critical value of SIF. The new method of analytical calculation of the critical SIF holds for quasi-brittle materials (plasticity zone at the top of the crack is no larger than 20 %), and it takes into account the cracks in continual three-dimensional environment. It is used for the 1st type crack. This method in combination with the relevant PS is an innovation in the sphere of strength and cracking resistance analysis, insofar as it helps either reduce or avoid the costs associated with the field tests. **Applications/Improvements:** The materials of the study are of practical importance for industrial companies, educational and scientific institutions that study the issues of fracture strength and cracking resistance.

Keywords: Fracture Mechanics, Critical Stress Intensity Factor, Finite Elements Method, Crack, Computer-Aided Engineering Systems, LEFM

1. Introduction

Linear Elastic Fracture Mechanics (LEFM) (plasticity zone at the top of the crack is no larger than 20 % of the size of the crack)[1] has failed to acquire wider application in the cracked structure strength calculations. This can be largely explained by the fact that the application of the existing methods of strength analysis beyond the yield point do not ensure the required precision of the obtained results. Suggesting different fracture models to analyze crack resistance the investigators used to exemplify them by simple models and only to determine some certain ranges of the parameters under investigation. To develop the relevant PS, the results of the earlier multiple investigations have to be generalized for the common case of stress and for any types of the initial conditions.

It should be noted that the strength analysis beyond the yield point was founded on the linear and non-linear strength analyses undertaken by Research and Software Development Company APM for different units and parts of equipment. These solutions make the basis for manufacturing the new product that solves the tasks of fracture mechanics.

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Calculations in fracture mechanics are of lesser precision; thus, to obtain the authentic and reliable results, special investigations have to be undertaken in the sphere of validation and verification. Therefore, the study pursues these objectives.

Fracture mechanics describes the conditions of the material located close to the top of the crack using such parameters as stress intensity factor (SIF) Ki, (strength criterion), energy release rate Gi (energy criterion) and J – integral. Index i describes the type of the crack. These parameters predetermine further propagation of the crack that can be either stable or unstable.[1,2] Fracture mechanics distinguishes three types of cracks: 1st type – cleavage crack, 2nd type – shear crack, and 3rd type – tearing mode crack relative to the front of the crack (Figure 1).
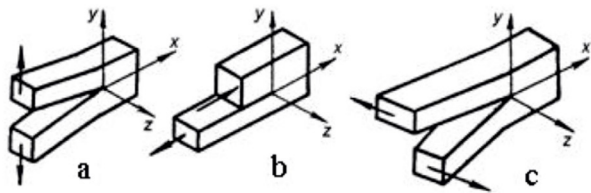

Figure 1. Types of cracks. a) cleavage crack, b) shear crack, c) tearing mode crack.

The parameters are calculated using asymptotical formulae[1,2] of LEFM that describe the strain-stress state at the top of the crack. These formulae (with Irwin plastic zone correction)[1,2] are applicable for most of the real materials. Such materials are called quasi-brittle materials[1,2].

One of the principal problems of LEFM is represented by the calculation of critical value of SIF for the 1st type crack (K_{IC}) under the plain-strain conditions[3]. Critical value of SIF (K_{IC}) found according to standard ASTM E-399 (equivalent to GOST 25.506-85) holds for ideally brittle materials that are seldom applied in practice. For quasi-brittle materials the critical value of SIF is not constant and depends on the size of the crack[1,2]. It is neither economically feasible nor technically practicable to carry out multiple field tests in order to calculate the critical value of SIF (K_{IC}) with the Specimens featuring cracks of different sizes. One of the options to solve this problem is represented by analytical calculation of the critical value of SIF ($K_{ICsolve}$) making use of physical and mechanical properties of the material.

2. Concept Headings

This study presents the new method to obtain the critical value of SIF analytically ($K_{ICsolve}$) using physical and mechanical properties of the material. The method is founded on the Murakami formula[4].

The study also represents the upgraded algorithm for solving LEFM tasks that was already introduced in some earlier studies[5]. The improvements were introduced to the first part of the algorithm, namely, to the block called "Determining the critical value of SIF ($K_{ICsolve}$)", and they primarily imply the introduction of the new method.

Analytical calculations of fracture mechanics parameters are complex tasks and thus they are applied only to primitive models. Numerical methods, such as finite elements method (FEM)[6] employed by the advanced CAE systems[7,8] make it possible to calculate the parameters of fracture mechanics in the complex tree-dimensional deformed environment.

Along with the modified formula for calculating the critical value of SIF in continual three-dimension environment, this algorithm has been implemented at Bauman Moscow State Technical University in the sphere of the finite elements analysis.
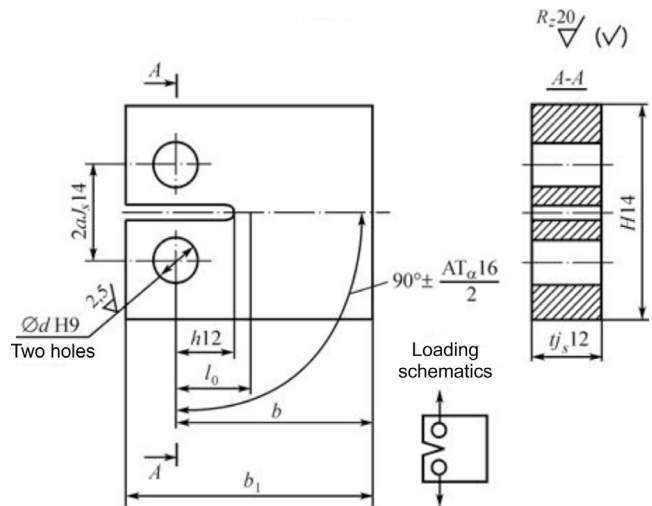

Figure 2. Compact specimen (DCB) with eccentric tension.

The study has been structured as follows. The first section provides the list of the requirements for the Specimen prerequisite for obtaining the authentic critical value of SIF (K_{IC}) experimentally. The second section presents the new method founded on the Murakami formula to

calculate critical SIF ($K_{ICsolve}$) analytically. The requirements listed in the first section are used to solve the tasks of verification to confirm the workability of the modified formula. The verification issues are considered within the third section. Section four provides the assessment of the precision of the applied formula and the recommendations on its application. Section five describes the algorithm for solving the tasks of fracture mechanics (LEFM) and shows the results of its implementation within the PS.

2.1 Requirements to Shape and Dimensions of the Specimen According to Standard ASTM E-399

To obtain the authentic value of critical SIF of the material under investigation, the shape and dimensions of the Specimen should meet the requirements listed in ASTM E-399 (equivalent to GOST 25.506-85). The shape and size of the standard Specimen (double cantilever beam, DCB) with the eccentric tension crack are shown in Figure 2.

Requirements to overall dimensions of the Specimen:

- thickness t of the Specimen should meet the following requirements:

$$t \geq 20 \tag{1}$$

- ratio b/t should meet the following requirements:

$$2 \leq b/t \leq 4 \tag{2}$$

- ratio l_0/b should meet the following requirements:

$$0.45 \leq l_0/b \leq 0.55 \tag{3}$$

The difference l_0-h of the dimensions at the top of the crack (Figure 3) should meet the requirement as follows:

$$l_0 - h \geq 1.5 \tag{4}$$

If the requirements (1) – (4) are met, then the obtained value SIF (K_Q) is to be checked with the inequality as follows:

$$t, l_0 \geq 2.5 \left(\frac{K_Q}{\sigma_{0.2}}\right)^2 \tag{5}$$

where, t – thickness of the Specimen (mm), l_0 – distance between the top of the crack and the fixture of DCB Specimen (mm) (Figure 2), K_Q – calculated value of SIF (MPa√mm), $\sigma_{0.2}$ – yield point (MPa).

The calculated value of SIF (K_Q) holds for the plane-strain condition. The works of Weiss and Zessler[9] show that the plane-strain state is the predetermining factor in the central part across the thickness of the Specimen, if:

$$\frac{t}{\rho} \geq 10 \tag{6}$$

where, t – thickness of the Specimen (mm), ρ – radius of incision curvature (mm).
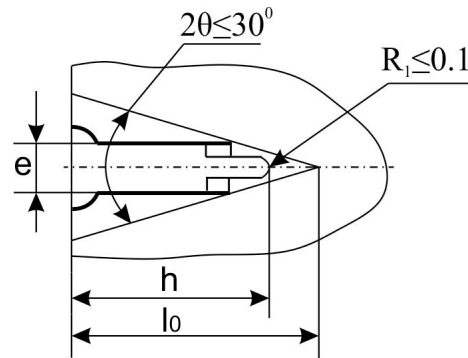

Figure 3. Shape and dimensions of the top of the crack

Thus, if the inequality (5) and the condition (6) are met, then $K_Q = K_{IC}$. Otherwise, the thickness t of the Specimen should be increased.

According to GOST 25.506-85, approximate thickness t of plane Specimens is determined using the modulus of elasticity E and the yield point $\sigma_{0.2}$ (Table 1)

Table 1. Approximate thickness t of flat specimens

$\dfrac{\sigma_{0.2}}{E}$	t, mm	$\dfrac{\sigma_{0.2}}{E}$	t, mm
Up to and including 0.0050	100	0.0071 – 0.0075	32
0.0050 – 0.0057	75	0.0075 – 0.0080	25
0.0057 – 0.0062	63	0.0080 – 0.0085	20
0.0062 – 0.0065	50	0.0085 – 0.0100	12
0.0065 – 0.0068	44	0.0100 – and more	6
0.0068 – 0.0071	38		

2.2 Analytical Calculation of Critical Stress Intensity Factor

Murakami,[4] a Japanese expert in LEFM, has developed the formula for calculating SIF of the random shape of the 1st type three-dimensional crack.

$$K_I \cong 0.65\sigma_0 \sqrt{\pi\sqrt{area}} \tag{7}$$

where, σ_0 – external tensile stress (MPa), $area$ – the surface of the crack in plane x – y (mm²) (Figure 4).
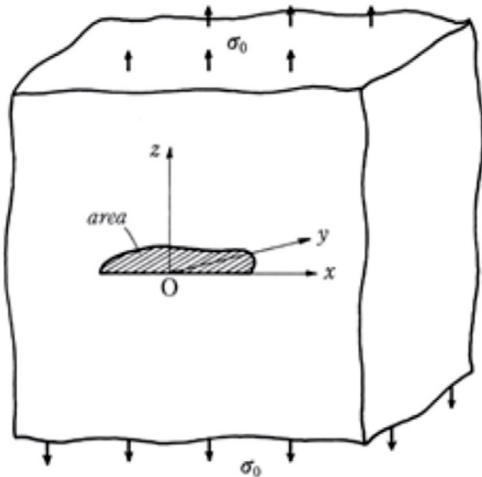

Figure 4. Plane x – y in random crack.

Before starting to develop the modified formula to calculate the critical value of SIF based on the abovementioned formula (7), consider the principal postulate of LEFM.

One of the studies[2] mentions the general asymptotical formula for calculating the value of SIF:

$$K_I = \sigma\sqrt{\pi r}Y \tag{8}$$

where, σ – external tensile stress (MPa), r – length of the crack (mm), Y – coefficient that depends on the ratio of the length of the crack to the dimensions of the body.

Formula (8), in somewhat transformed representation is used to find the plasticity zone at the top of the crack[2]:

$$r_y = \frac{K_I^2}{2\pi\sigma_m^2} \tag{9}$$

where, σ_T – yield point (MPa).

Applying the same technique (9), replace σ_0 with σ_T in formula (7). This will make it possible to take into account the dangerous conditions at the top of the crack and to analytically determine $K_{ICsolve}$. The modified formula (7) for calculating critical SIF will be presented as follows:

$$K_{ICsolve} \cong 0.65\sigma_T \sqrt{\pi\sqrt{area}} \tag{10}$$

Formula (10) provides authentic results for the small size cracks. The recommended dimensions of the crack should meet the condition as follows[4]:

$$\frac{l}{c} \geq 10 \tag{11}$$

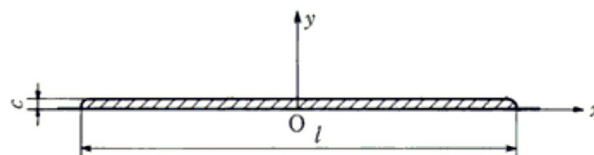where, l – width of the crack (mm), c – length of the crack (mm) (Figure 5).

Figure 5. Crack dimensions.

Applying formula (11), the area of the crack will be calculated as follows:

$$area = lc \tag{12}$$

The authenticity of the results of applying the modified formula (10) for calculating critical SIF for the 1st type crack is described in the following section.

3. Results

3.1 Calculations

The tools to confirm the workability of the modified formula (10) were represented by the mechanical properties of the metals and their alloys (standard ASTM E-399 holds for this category of materials). Principal mechanical properties are represented by conventional yield point ($\sigma_{0.2}$) and by the critical value of SIF for the 1st type crack (K_{IC}) obtained experimentally (the values are known beforehand)[10,11]. In the end of the calculations, the critical value of SIF (K_{IC}) obtained experimentally is correlated with the critical value of SIF obtained analytically ($K_{ICsolve}$) according to formula (10).

3.1.1 Specimen No. 1

Specimen No.1 is made of Aluminum alloy (356.0 – T7). Mechanical properties of 356.0 – T7[10] and the critical value of SIF (K_{IC})[10] are shown in Table 2.

Dimensions of Specimen No.1, according to the handbook[10] and according to formula (4), are shown in Table 3.

To determine the area of the crack (12), parameter $l=l_0$, and $c=l_0-h$. Hence, the area of the crack of Specimen No.1 will be as follows:

$$area = 1.5 \cdot 9.525 = 14.287 mm^2 \tag{13}$$

Inserting (13) into (10), obtain the following:

$$K_{ICsolve} = 0.65 \cdot 232 \cdot \sqrt{\pi\sqrt{14.287}} = 519.65\ MPa\sqrt{mm} = 16.43\ MPa\sqrt{m} \tag{14}$$

Table 2. Mechanical properties 356.0 – T7

T, ⁰C	$\sigma_{0.2}$, MPa	$K_{IC}, MPa\sqrt{m}$
20	232	16.9

Table 3. Dimensions. Specimen No. 1

l_0,mm	t, mm	b, mm	l_0-h, mm
12.7	9.52	38.1	1.5

3.1.2 Specimen No. 2

Specimen No.2 is made of Titanium alloy (Ti-6Al-4V). Mechanical properties of Ti-6Al-4V[11] and the critical value of SIF (K_{IC})[11] are shown in Table 4.

Dimensions of Specimen No.2, according to the study[11] and according to formula (4), are shown in Table 5.

To determine the area of the crack (12), parameter $l=l_0$, and $c=l_0-h$. Hence, the area of the crack of Specimen No.2 will be as follows:

$$area = 1.5 \cdot 19.8 = 29.7\ mm^2 \tag{15}$$

Inserting (15) into (10), obtain the following:

$$K_{ICsolve} = 0.65 \cdot 916 \cdot \sqrt{\pi\sqrt{29.7}} = 2463.61\ MPa\sqrt{mm} = 77.90\ MPa\sqrt{m} \tag{16}$$

Table 4. Mechanical properties Ti-6Al-4V

T, ⁰C	$\sigma_{0.2}$, MPa	$K_{IC}, MPa\sqrt{m}$
21	916	82

Table 5. Dimensions. Specimen No. 2

l_0,mm	t, mm	b, mm	l_0-h, mm
7.6	19.8	38.1	1.5

3.1.3 Specimen No. 3

Specimen No.3 is made of Steel alloy (4140). Mechanical properties of 4140[11] and the critical value of SIF (K_{IC})[11] are shown in Table 6.

Dimensions of Specimen No.3, according to the study[11] and according to formula (4), are shown in Table 7.

To determine the area of the crack (12), parameter $l=l_0$, and $c=l_0-h$. Hence, the area of the crack of Specimen No.3 will be as follows:

$$area = 1.5 \cdot 102 = 153\ mm^2 \tag{17}$$

Inserting (17) into (10), obtain the following:

$$K_{ICsolve} = 0.65 \cdot 455 \cdot \sqrt{\pi\sqrt{153}} = 1843.6\ MPa\sqrt{mm} = 58.3\ MPa\sqrt{m} \tag{18}$$

Table 6. Mechanical properties 4140

T, ⁰C	$\sigma_{0.2}$, MPa	$K_{IC}, MPa\sqrt{m}$
24	455	60

Table 7. Dimensions. Specimen No. 3

l_0,mm	t, mm	b, mm	l_0-h, mm
102	102	260	1.5

3.1.4 Specimen No. 4

Specimen No.4 is made of Aluminum alloy (6061). Mechanical properties of 6061[11] and the critical value of SIF (K_{IC})[11] are shown in Table 8.

Dimensions of Specimen No.4, according to the study[11] and according to formula (4), are shown in Table 9.

To determine the area of the crack (12), parameter $l=l_0$, and $c=l_0-h$. Hence, the area of the crack of Specimen No.4 will be as follows:

$$area = 1.5 \cdot 38.1 = 57.15\ mm^2 \tag{19}$$

Inserting (19) into (10), obtain the following:

$$K_{ICsolve} = 0.65 \cdot 296 \cdot \sqrt{\pi\sqrt{57.15}} = 937.63\ MPa\sqrt{mm} = 29.65\ MPa\sqrt{m} \tag{20}$$

Table 8. Mechanical properties 6061

T, ⁰C	$\sigma_{0.2}$, MPa	$K_{IC}, MPa\sqrt{m}$
21	296	28

Table 9. Dimensions. Specimen No. 4

l_0,mm	t, mm	b, mm	l_0-h, mm
38.1	38.1	76.2	1.5

3.1.5 Specimen No. 5

Specimen No.5 is made of Cast iron (EN-GJS-900-2). Mechanical properties of EN-GJS-900-2[11] and the critical

value of SIF (K_{IC})[11] are shown in Table 10.

Dimensions of Specimen No.5, according to formulae (2), (4) and (5), are shown in Table 11.

To determine the area of the crack (12), parameter $l=l_0$, and $c=l_0-h$. Hence, the area of the crack of Specimen No.5 will be as follows:

$$area = 1.5 \cdot 56 = 84 \ mm^2 \tag{21}$$

Inserting (21) into (10), obtain the following:

$$K_{ICsolve} = 0.65 \cdot 600 \cdot \sqrt{\pi\sqrt{84}} = 2092.71 \ MPa\sqrt{mm} = 66.17 \ MPa\sqrt{m} \tag{22}$$

Table 10. Mechanical properties EN-GJS-900-2

T, ⁰C	$\sigma_{0.2}$, MPa	K_{IC}, MPa$\sqrt{m}$
20	600	72

Table 11. Dimensions. Specimen No. 5

l_0, mm	t, mm	b, mm	l_0-h, mm
56	56	112	1.5

4. Discussion

4.1 Analysis of the Results

The results of the calculations for each specimen are presented in Table 12.

The accuracy δ in determining the critical value of SIF according to ASTM E-399 amounts to circa 9%. The benchmark critical value of SIF (K_{IC}) was assumed to be the value obtained in line with standard ASTM E-399 in earlier studies.[10,12] The results of the calculations show that the relative error Δ does not exceed δ. Consequently, formula (10) can be considered workable.

4.2 The Modified Formula Precision Assessment

The verification tests carried out in Section 3 showed the authenticity of formula (10) for the specimens under

investigation. The graph in Figure 6 shows the dependency of critical SIF (K_{IC}) upon the yield point $(\sigma_{0.2})$. According to the graph, the value of SIF (K_{IC}) increases in proportion to the yield point $(\sigma_{0.2})$ which is characteristic for many modern materials.[2] The trend line (with the accuracy of the approximation of 0.9567) describes the change of the time series. This change is of linear nature.

Inasmuch as the study considers LEFM that holds for quasi-brittle materials, the precision of the results obtained with formula (10) improves when the mechanical properties of the material under consideration are in close proximity to the trend line. This has to be taken into account to avoid the incorrect results.
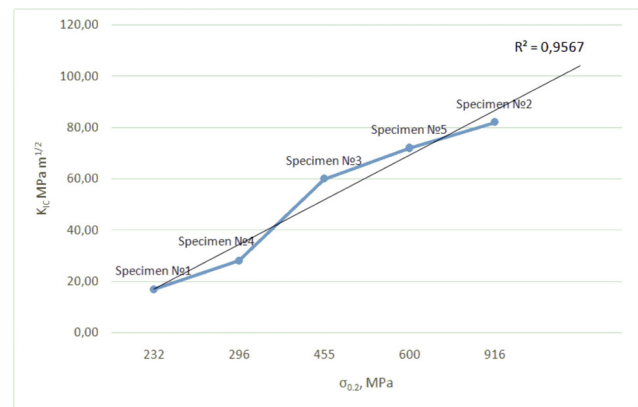

Figure 6. Dependency of critical SIF (K_{IC}) on yield point $(\sigma_{0.2})$.

4.3 Using LEFM in CAE System

The application of the tools of LEFM in the PS has been represented as an algorithm elsewhere.[5] The algorithm has been implemented within the PS. The PS belongs to the class of CAE systems that make it possible to carry out the strength and crack resistance analysis of a structural element or its part affected by mechanical and non-mechanical forces.

The algorithm of the PS operation consists of two stages described in earlier studies.[5] The flow chart of the first stage of the algorithm is shown in Figure 7. The flow

Table 12. Calculation results

	Specimen No. 1	Specimen No. 2	Specimen No. 3	Specimen No. 4	Specimen No. 5
K_{IC}, MPa$\sqrt{mm}$	16.9	82	60	28	72
$K_{ICsolve}$, MPa$\sqrt{mm}$	16.43	77.9	58.3	29.65	66.17
Percentage error, Δ (%)	2.8	5	2.83	5.9	8.1

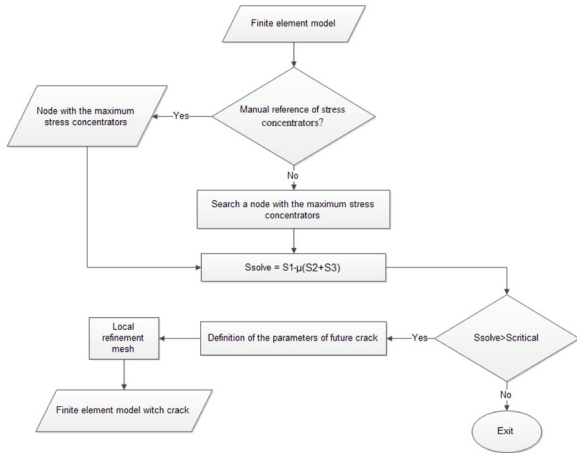chart of the second stage of the algorithm is shown in Figure 8.

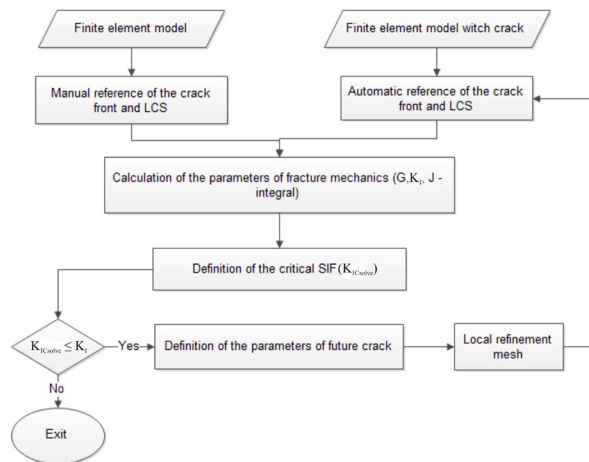**Figure 7.** Flow chart of the first part of the algorithm.

Figure 8. Flow chart of the second part of the algorithm.

4.3.1 The First Part of the Algorithm

At the first stage the Finite Elements (FE) model without cracks is analyzed to determine the zone of the dangerous conditions of the material based on one of the strength criteria. Given the data on this zone, the dimensions of the future (germ) crack are determined[12]. If the dangerous conditions of the material exceed the critical value,[12] then in this zone the local change of the grid occurs automatically taking into account the size of the crack. The geometry of the grid adapts automatically to the front of the crack in order to obtain the authentic values of the parameter of fracture mechanics. An example of the calculated model of a structural element represented by a plate with the crack is shown in Figure 9.
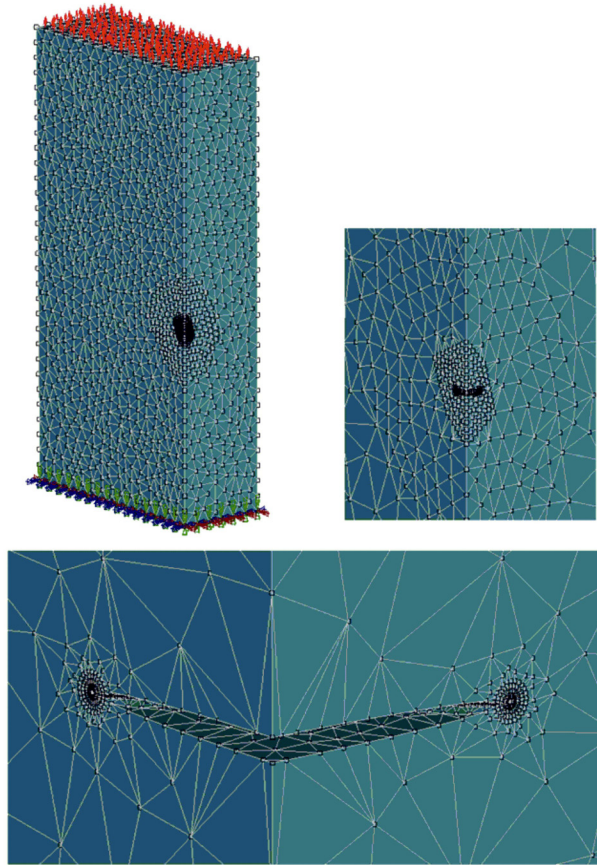

Figure 9. Finite element model with crack.

4.3.2 The Second Part of the Algorithm

The second part analyzes the crack resistance of the calculated model with a crack. This algorithm includes the block "Determining the critical value of SIF (K_{IC})" that includes the modified formula (10) which helps determine the critical value of SIF for the crack of the current dimensions (taking into account the area of the crack propagation).

The value of SIF (K_I) that presently exists at the front of the crack is determined through the stress shifts at the top of the crack[2,13–17].

In LEFM, the crack grows, if the following condition is met:

$$K_{ICsolve} \leq K_I \tag{23}$$

The calculation of the current value of SIF (K_I) applying the PS has already been carried out earlier[5], and for the Specimen under consideration (Figure 10) it was as follows:

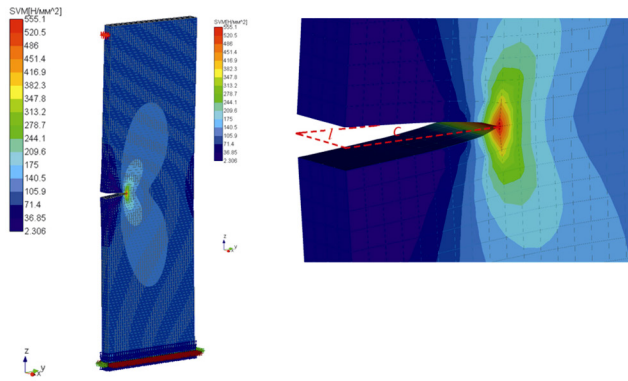$$K_I = 2030.62\, MPa\sqrt{mm} = 64.21\, MPa\sqrt{m} \tag{24}$$

Figure 10. Strain-stress state of FE model with crack.

Crack dimensions (Figure 10): l = 100 mm; c = 225 mm; $area$ = 225·100 = 22500 mm^2.

Physical and mechanical properties: $\sigma_{0.2}$ = 235 MPa.

According to formula (10):

$$K_{ICsolve} = 0.65 \cdot 235 \cdot \sqrt{p\sqrt{22500}} = 3315.9 \ MPa\sqrt{mm} = 104.85 \ MPa\sqrt{m} \quad (25)$$

The data of the calculations (24) and (25) show that the condition of the crack growth (23) is not met.

5. Conclusion

In order to confirm the workability of the modified formula (10) for analytical calculation of the critical value of SIF ($K_{ICsolve}$), five specimens of the materials featuring different physical and mechanical properties (wide ranges of the critical values of SIF (K_{IC}) and yield points ($\sigma_{0.2}$)) have been analyzed. The quality of the results improves as the requirements of LEFM are met and as physical and mechanical properties of the materials get closer to the trend line.

The implementation of the new method associated with the modified formula (10) makes it possible to calculate the critical value of SIF ($K_{ICsolve}$) for the cracks of different sizes in the continual three-dimension environment without any expensive tests according to ASTM E-399 (equivalent to GOST 25.506 - 85) taking into account both mechanical and non-mechanical forces affecting the structure.

The updated algorithm for solving the tasks of LEFM employing the relevant PS will make it possible to bring the process of simulation and crack modeling up to the new level. The strength analysis operates with the structures made of the modern materials affected by mechanical and non-mechanical forces. Also, the introduction of the PS

will help reduce the costs for the implementation of the field tests and expedite the "design-to-manufacturing" cycle of safe structural elements simultaneously providing the possibility to assess the reliability and durability of the structural elements that already possess such defects as cracks.

6. Acknowledgement

7. References

1. Zehnder AT. Fracture mechanics. Springer Science & Business Media; 2012. DOI 10.1007/978-94-007-2595-9.
2. Pestrikov VM. Fracture mechanics: Lectures. Profession; 2012.
3. Carroll J, Daly S. Fracture, Fatigue, Failure, and Damage Evolution, 5: Proceedings of the 2014 Annual Conference on Experimental and Applied Mechanics. Springer. 2015. DOI 10.1007/978-3-319-06977-7.
4. Shirakil N, Watanabe T, Kanno T. Relationship between fatigue limit and defect size in spheroidal graphite cast iron with different graphite spheroidization ratios and microstructures. materials transactions. The Japan Institute of Metals and Materials; 2015. DOI: 10.2320/matertrans.F-M2015826.
5. Renev SA. Development of an algorithm for solving problems of fracture mechanics. Materials Physics and Mechanics, Institute of Problems of Mechanical Engineering; 2016.
6. Verzhbitskiy VM. Numerical methods (mathematical analysis and common differential equations). Textbook. Direct-Media; 2013.
7. Gubich L. Industrial implementation of product life-cycle support information technologies. Litre; 2012.
8. Davim JP. Statistical and computational techniques in manufacturing. Springer Science and Business Media; 2012. DOI: 10.1007/978-3-642-25859-6.
9. Carroll J. Fracture and fatigue. 7: Proceedings of the 2013 Annual Conference on Experimental and Applied Mechanics. Springer Science & Business Media; 2013. DOI: 10.1007/978-3-319-00765-6.
10. Kutz M. Mechanical engineers' handbook. 1: Materials and Engineering Mechanics. John Wiley & Sons; 2015.

11. Total Materia. Key to Metals AG [Internet]. 1999–2016. [cited 2016 Nov 22]. Available from: http://www.totalmateria.com.

12. Renev SA. Development of mathematical model for detection of conditional sizes primordial cracks in LEFM and its implementation in Russian Cax system. Procedia Engineering. 150, 2nd International Conference on Industrial Engineering (ICIE-2016); 2016. p. 683–8. DOI: 10.1016/j.proeng.2016.07.078.

13. ANSYS Release 14.5 Documentation; 2012.

14. Morozov EM, Muyzemnek AYu, Shadskiy AS. ANSYS in the hands of an engineer: Fracture Mechanics. Stereotype. Moscow: LENAND; 2014.

15. Kuna M. Finite elements in fracture mechanics: Theory - Numerics - Applications. Springer Science & Business Media; 2013. DOI: 10.1007/978-94-007-6680-8.

16. Banihashemi MR, Mirzagoltabar AR, Tavakoli HR. The effects of yield mechanism selection on the performance based plastic design of steel moment frame. Indian Journal of Science and Technology. 2015 May; 8(9).

17. Dhas LAK, Raguraman D, Muruganandam D, Senthilkumar B. Temperature prediction using finite element modeling on friction stir welding of AA6061-AZ61. Indian Journal of Science and Technology. 2015 Nov; 8(31).