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On a Generalization of α-skew McCoy Rings

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Abstract

Objective: To generalize the α-skew McCoy rings. **Methods:** For a ring endomorphism α, we call a ring R Central α-skew McCoy if for each pair of nonzero polynomials $f(x)=\sum_{i=0}^{n} a_i x^i$ and $g(x)=\sum_{j=0}^{m} b_j x^j \in R[x;\alpha]$ satisfy $f(x)g(x) = 0$, then there exists a nonzero element $r \in R$ with $a_i\alpha^i(r) \in C(R)$. Findings: For a ring R, we show that if $\alpha(e) = e$ for each idempotent $e \in R$, then R is Central α-skew McCoy if and only if eR is Central α-skew McCoy if and only if $(1 - e)R$ is Central α-skew McCoy. Also, we prove that if $\alpha^t = I_R$ for some positive integer t, R is Central α-skew McCoy if and only if the polynomial ring $R[x]$ is Central α-skew McCoy if and only if the Laurent polynomial ring $R[x, x^{-1}]$ is Central α-skew McCoy. Moreover, we give some examples to show that if R is Central α-skew McCoy, then $T_n(R)$ is not necessary Central α-skew McCoy, but $D_n(R)$ and $V_n(R)$ are Central α-skew McCoy, where $D_n(R)$ and $V_n(R)$ are the subrings of the triangular matrices with constant main diagonal and constant main diagonals, respectively.

Keywords: Central α-skew McCoy Ring, McCoy Ring, Ore Extension,-skew McCoy ring, Triangular Matrix Ring, Skew Polynomial Ring

1. Introduction

Throughout this paper, R denotes an associative ring with identity and α is a ring endomorphism. We denote $R[x;\alpha]$ the Ore extension whose elements are the polynomials $\sum_{i=0}^{n} a_i x^i$, $a_i \in R$, where the addition is defined as usual and the multiplication subject to the relation $xa = \alpha(a)x$ for any $a \in R$. For notation $T_n(R)$, $S_n(R)$, $R[x]$, $C[R]$ and e_{ij} denote, its upper triangular matrix ring, its diagonal matrix ring, polynomial ring over R, the center of a ring R and the matrix with (i, j)-entry 1 and elsewhere 0, respectively. A ring R is Armendariz[1] if whenever polynomials $f(x)=\sum_{i=0}^{n} a_i x^i$ and $g(x)=\sum_{j=0}^{m} b_j x^j \in R[x]$ satisfy $f(x)g(x) = 0$ then $a_ib_j = 0$ for all i and j. *Agayev*[2] called a ring R Central Armendariz if whenever polynomials $f(x)=\sum_{i=0}^{n} a_i x^i$ and $g(x)=\sum_{j=0}^{m} b_j x^j \in R[x]$ satisfy $f(x)g(x) = 0$, then $a_ib_j \in C(R)$ for all i and j. They showed that the class of central Armendariz rings lies precisely between classes of Armendariz rings and abelian rings (that is, its idempotents belong to $C(R)$. Let α be an endomorphism of a ring R. According to[3], a ring R is called α-skew Armendariz if $f(x)g(x) = 0$, such that $f(x)=\sum_{i=0}^{n} a_i x^i$ and j $g(x)=\sum_{j=0}^{m} b_j x^j \in R[x;\alpha]$ implies that $a_i\alpha^i(b_j) = 0$ for all i, j. *Rege -Chhawchharia*[1] called a noncommutative ring R right McCoy if whenever nonzero polynomials $f(x)=\sum_{i=0}^{n} a_i x^i$ and $g(x)=\sum_{j=0}^{m} b_j x^j \in R[x]$ satisfy $f(x)g(x) = 0$, there exists nonzero elements $r \in R$ such that $a_ir = 0$. Left McCoy rings are defined similarly. A ring is McCoy if it is both left and right McCoy. Clearly, Armendariz rings are McCoy. So far McCoy rings are generalized in several forms[4-10]. In[5] *Basser, Kwak* and *Lee* called a ring R, α-skew McCoy ring with respect to α if for any nonzero polynomials $f(x)=\sum_{i=0}^{n} a_i x^i$ and $g(x)=\sum_{j=0}^{m} b_j x^j \in R[x;\alpha]$ satisfy $f(x)g(x) = 0$ implies $a_i\alpha^i(s) = 0$ for some nonzero $s \in R$.

Motivated by the above results, for an endomorphism α of a ring R, we call a ring R Central α-skew McCoy if for each pair of nonzero polynomials $f(x)=\sum_{i=0}^{n} a_i x^i$ and $g(x)=\sum_{j=0}^{m} b_j x^j \in R[x;\alpha]$ satisfy $f(x)g(x) = 0$, then there exists a nonzero element $r \in R$ with $a_i\alpha^i(r) \in C(R)$. Clearly, all commutative rings, α-skew McCoy rings and α -skew Armendariz rings are Central α-skew McCoy.

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2. Central α-skew McCoy Rings

We start this section by the following definition:

Definition 2.1. Let α be an endomorphism of a ring R. The ring R is called a Central α-skew McCoy ring if for each pair of nonzero polynomials $f(x)=\sum_{i=0}^{n} a_i x^i$ and $g(x)=\sum_{j=0}^{m} b_j x^j \in R[x;\alpha]$ satisfy $f(x)g(x) = 0$, then there exists a nonzero element $r \in R$ with $a_i\alpha^i(r) \in C(R)$.

It is clear that α-skew McCoy rings are Central α-skew McCoy, but the converse is not always true by the following example.

Example 2.2. Let R_1 and R_2 be any commutative rings. Suppose $R = R_1 \oplus R_2$ with the usual addition and multiplication. Let $\alpha : R \to R$ be an endomorphism defined by $\alpha((a, b)) = (b, a)$, then for $f(x) = (1,0) - (1,0)x$ and $g(x) = (0,1) + (0,1)x$ in $R[x; \alpha]$, $f(x)g(x) = 0$, but if there exists $(r_1, r_2) \in R$ \$ such that $(0,1)(r_1,r_2) = 0$ and $(0,1)x(r_1,r_2) = 0$, then we have $r_1 = r_2 = 0$. Therefore R is not α-skew McCoy. But R is Central α-skew McCoy, since R is commutative.

Example 2.3. Let K be a field and $K\langle x, y, z\rangle$ be the free algebra with noncommuting indeterminates x, y, z over K. Set R be the factor ring of $K\langle x, y, z\rangle$ with relations

$$x^2 = yx = x,\ y^2 = xy = y,\ z^2 = 0,\ zx = xz = yz = zy = z$$

We coincide $\{x, y, z\}$ with their images in R, for simplicity. Consider the subring of R generated by $\{a, x, y, z | a \in K\}$, say S. Then every element of S is of the form, $a_1 + a_2x + a_3y + a_4z$ with a_i's in the field K. Let α:$S \to S$ be an endomorphism defined by $\alpha(a_1 + a_2x + a_3y + a_4z) = a_1 + a_2y + a_3x + a_4z$. By the construction of S, we have $(x + yt^2)((1 - x) + (1 - y)t) = 0$ while $(x + yt^2)$ and $(1 - x) + (1 - y)t$ are nonzero polynomials over S. Assume there exists $a_1 + a_2x + a_3y + a_4z \in R$ such that $x(a_1 + a_2x + a_3y + a_4z) = 0$ and $y\alpha^2(a_1 + a_2x) = 0$. Then $a_1 = a_2 = a_3 = a_4 = 0$. Thus S is not α-skew McCoy.

Next we show that S is Central α-skew McCoy. Let $f(t)=\sum_{i=0}^{n}(a_{1i}+a_{2i}x+a_{3i}y+a_{4i}z)t^i$ and $g(t)=\sum_{j=0}^{m}(b_{1j}+b_{2j}x+b_{3j}y+b_{4j}z)t^j$ be nonzero in $S[t]$ with $f(t)g(t) = 0$. Then

$$(a_{1i}+a_{2i}x+a_{3i}y+a_{4i}z)\alpha(z)=(a_{1i}+a_{2i}+a_{3i})z \in C(S)$$

since

$$\begin{aligned}(a_{1i}+a_{2i}+a_{3i})z(a_1+a_2x+a_3y+a_4z) &= (a_{1i}+a_{2i}+a_{3i})(a_1+a_2+a_3)z\\ &= (a_1+a_2x+a_3y+a_4z)(a_{1i}+a_{2i}+a_{3i})z\end{aligned}$$

for each $(a_1 + a_2x + a_3y + a_4z) \in S$.

Now we turn our attention to study some extensions of Central α-skew McCoy rings.

Let R_k be a ring, where $k \in Z$, α_k an endomorphism of R_k. Let $R=\prod_{k\in Z} R_k$ and $\langle \oplus_{k\in Z}R_k, 1_R\rangle$ be the subring of R generated by $\oplus_{k\in Z}R_k$ and $\{1_R\}$. Then the map α: $R \to R$ defined by $\alpha((a_k))=(\alpha_k(a_k))$ is an endomorphism of R.

Proposition 2.4. Let R_k be a ring with a endomorphism α_k, where $k \in Z$. Then R_k is Central α_k -skew McCoy for each $k \in Z$ if and only if $R=\prod_{k\in Z} R_k$ is Central α-skew McCoy if and only if $\langle \oplus_{k\in Z}R_k, 1_R\rangle$ is Central α-skew McCoy.

Proof. Let each R_k be Central α_k-skew McCoy and $f(x)=\sum_{i=0}^{n} a_i x^i$, $g(x)=\sum_{j=0}^{m} b_j x^j \in R[x;\alpha]\setminus\{0\}$ such that $f(x)g(x) = 0$, where $a_i = (a_i^{(k)})$, $b_j = (b_j^{(k)})$. If there exists $t \in Z$ such that $a_i^{(t)} = 0$ for each $0 \le i \le m$, then we have $a_i\alpha^i(c) = 0 \in C(R)$ where $c = (0,0, \ldots, 1_R, 0, \ldots,0)$. Now, suppose for each $k \in Z$, there exists $0 \le i_k \le m$ such that $a_{i_k}^{(k)} \neq 0$. Since $g(x) \neq 0$, there exists $t \in Z$ and $0 \le j_t \le n$ such that $b_{j_t}^{(t)} \neq 0$. Consider $f_t(x)=\sum_{i=0}^{m} a_i^{(t)} x^i$ and $g_t(x)=\sum_{j=0}^{n} b_j^{(t)} x^j \in R_t[x;\alpha_t]\setminus\{0\}$. We have $f_t(x)g_t(x) = 0$. Thus there exists nonzero $c_t \in R_t$ such that $a_i^{(t)}\alpha^i(c_t) \in C(R_t)$, for each $0 \le i \le m$, since R_t is Central α_k-skew McCoy ring. Therefore $a_i\alpha^i(c) \in C(R)$, for each $0 \le i \le m$, where $c = (0, \ldots, 0, c_t, 0, \ldots,0)$ and so R is Central α-skew McCoy. Conversely, suppose R is Central α-skew McCoy and $t \in Z$. Let and $f(x)=\sum_{i=0}^{m} a_i x^i$ and $g(x)=\sum_{j=0}^{n} b_j x^j$ be nonzero polynomials in $R_t[x;\alpha_t]$ such that $f(x)g(x) = 0$. Let

$$F(x)=\sum_{i=0}^{m}(0,\ldots,0,a_i,0,\ldots,0)x^i,$$

$$G(x)=\sum_{j=0}^{n}(0,\ldots,0,b_j,0,\ldots,0)x^j \in R[x;\alpha]$$

Hence $F(x)G(x) = 0$ and so there exists $0 \neq c = (c_1, c_2, \ldots, c_{t-1}, c_t, c_{t+1}, \ldots, C_{m-1}, c_m)$ suchthat $(0, \ldots, 0, a_i, 0, \ldots, 0)\alpha^i(c) \in C(R)$. Therefore, $\alpha^i a_t^i(c_t) \in C(R_t)$ and so R_t is Central α_k-skew McCoy. By a similar way, one can prove that $\langle \oplus_{k\in Z}R_k, 1_R\rangle$ is Central α-skew McCoy if and only if each R_i is Central α_i-skew McCoy.

Corollary 2.5. Let D be a ring and C a subring of $D$with $1_D \in C$. Let

$$R(C,D)=\left\{(d_1,\ldots,d_n,c,c,\ldots)\ \middle|\ d_i \in D, c \in C, n \ge 1\right\}$$

with addition and multiplication defined componentwise, $R(D, C)$ is a ring. Let α be an endomorphism of

Dsuch that $\alpha(C) \subseteq C$. Then the map $\overline{\alpha}: R \to R$ defined by $\overline{\alpha}((C,D)) = (\alpha(C), \alpha(D))$ is an endomorphism of R. Then D is Central α-skew McCoy if and only if $R(D, C)$ is Central $\overline{\alpha}$-skew McCoy.

Proposition 2.6. Let α be an endomorphism of a ring R. Let S be a ring and φ:$R \to S$ an isomorphism. Then R is central α-skew McCoy if and only if S is central $\varphi\alpha\varphi^{-1}$-skew McCoy.

Proof . Let $\alpha' = \varphi\alpha\varphi^{-1}$. Clearly, α' is an endomorphism of S. Suppose that $\alpha' = \varphi(\alpha)$, for $a \in R$. Note that $\varphi(a\alpha^i(b) = a'\varphi(\alpha^i(b)$ for all a, $b \in R$. Also and $f(x) = \sum_{i=0}^{n} a_i x^i$, $g(x) = \sum_{j=0}^{m} b_j x^j$ be nonzero polynomials in $R[x{:}\alpha]$ if and only if $f'(x) = \sum_{i=0}^{n} a_i' x^i$, $g'(x) = \sum_{j=0}^{m} b_j' x^j$ be nonzero polynomials in $S[x;\alpha']$. On the other hand $f(x)g(x) = 0$ in $R[x;\alpha]$ if and only if $f'(x)g'(x) = 0$ in $S[x;\alpha']$. Also since φ is an isomorphism, $a_i'(\phi\alpha\phi)^i c' = a_i'\phi\alpha^i\phi^{-1}(c') = \phi(a_i)\phi\alpha^i(c) = \phi(a_i\alpha^i(c)) \in C(S)$ if and only if $a_i\alpha^i(c) \in C(R)$. Thus R is Central α-skew McCoy if and only if S is Central $\varphi\alpha\varphi^{-1}$-skew McCoy.

The following example shows that, if R is Central α-skew McCoy, then $T_2(R)$ is not necessary Central $\overline{\alpha}$-skew McCoy.

Example 2.7. Let R be a commutative ring and $\overline{\alpha}: T_2(R) \to T_2(R)$ be an endomorphism defined by $\overline{\alpha}(P_{11}e_{11} + P_{12}e_{12} + P_{22}e_{22}) = P_{11}e_{11} - P_{12}e_{12} + P_{22}e_{22}$. Let $f(x) = e_{11} + (e_{11} + e_{12})x$, $g(x) = -e_{22} + (e_{12} + e_{22})x \in (T_2(R))[x;\alpha]$. Then $f(x)g(x) = 0$. If $T_2(R)$ is central $\overline{\alpha}$-skew McCoy, then there exists $P = P_{11}e_{11} + P_{12}e_{12} + P_{22}e_{22} \in T_2(R)$ such that $e_{11}P$, $(e_{11} + e_{12})\overline{\alpha}(P) \in C(T_2(R))$. Therefore $(P_{11}e_{11} + P_{12}e_{12})e_{11} = e_{11}(P_{11}e_{11} + P_{12}e_{12})$ and so $P_{12} = 0$.

Also $(P_{11}e_{11} + P_{12}e_{12})e_{12} = e_{12}(P_{11}e_{11} + P_{12}e_{12})$ and so $P_{11} = 0$. On the other hand, $(e_{11} + e_{12})\overline{\alpha}(P) = P_{22}e_{12} \in C(T_2(R))$ implies $(P_{22}e_{12})e_{22} = e_{22}(P_{22}e_{12})$ and so $P_{22} = 0$. Therefore $T_2(R)$ is not Central $\overline{\alpha}$-skew McCoy. But R is Central α-skew McCoy (for any endomorphism α:$R \to R$), since R is commutative.

Let α be endomorphism of a ring R. The endomorphism α of R is extended to the endomorphism $\overline{\alpha}: T_n(R) \to T_n(R)$ defined by $\overline{\alpha}((a_{ij})) = (\alpha(a_{ij}))$.

Theorem 2.8. Let α be an endomorphism of a ring R. Then R is Central α-skew McCoy if and only if one of the following holds:

(1) $D_n(R) = \left\{ \begin{pmatrix} a & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & a & a_{23} & \cdots & a_{2n} \\ 0 & 0 & a & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & a \end{pmatrix} \middle| \ a, a_{ij} \in R \right\}$ is Central $\overline{\alpha}$-skew McCoy for any n ≥ 1.

(2) $V_n(R) = \left\{ \begin{pmatrix} a_1 & a_2 & a_3 & a_4 & \cdots & a_n \\ 0 & a_1 & a_2 & a_3 & \cdots & a_{n-1} \\ 0 & 0 & a_1 & a_2 & \cdots & a_{n-2} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & a_2 \\ 0 & 0 & 0 & 0 & \cdots & a_1 \end{pmatrix} \middle| \ a_1, a_2, \cdots, a_n \in R \right\} \cong \frac{R[x;\alpha}{(x^n)}$ is Central $\overline{\alpha}$-skew McCoy for any n ≥ 1, where (x^n) is a two- sided ideal of $R[x;\alpha]$ generated by x^n.

Proof . (1) Let $F(x) = \sum_{i=0}^{p} A_i x^i$, $G(x) = \sum_{j=0}^{q} B_j x^j \in D_n(R)[x;\alpha]$ where,

$$A_i = \begin{pmatrix} a_i & a_{12}^i & \cdots & a_{1n}^i \\ 0 & a_i & \cdots & a_{2n}^i \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_i \end{pmatrix}, \quad B_j = \begin{pmatrix} b_j & b_{12}^j & \cdots & b_{1n}^j \\ 0 & b_j & \cdots & b_{2n}^j \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_j \end{pmatrix}$$

Then

$$F(x) = \begin{pmatrix} f(x) & f_{12}(x) & \cdots & f_{1n}(x) \\ 0 & f(x) & \cdots & f_{2n}(x) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & f(x) \end{pmatrix}, \quad G(x) = \begin{pmatrix} g(x) & g_{12}(x) & \cdots & g_{1n}(x) \\ 0 & g(x) & \cdots & g_{2n}(x) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g(x) \end{pmatrix}$$

Where $f(x) = \sum_{i=0}^{p} a_i x^i$, $f_{kl}(x) = \sum_{i=0}^{p} a_{kl}^i x^i$, $g(x) = \sum_{j=0}^{q} b_j x^j$ and $g_{kl}(x) = \sum_{j=0}^{q} b_{kl}^j x^j$ for any $k = 1, 2, \ldots, n$, $l = 2, 3, \ldots, n$ and $k < l$. Suppose $F(x)G(x) = 0$, and $F(x)G(x) \neq 0$. Set $H(x) = F(x)G(x) = (h_{pq}(x))$ for $p, q = 1, 2, \ldots, n$.

CASE 1. If $f(x) \neq 0$, $g(x) \neq 0$, then $h_{11}(x) = f(x)g(x) = 0$. Since R is Central α-skew McCoy there exists $r \in R\backslash\{0\}$ such that $a_i\alpha^i(r) \in C(R)$. Let $A = rE_{1n}$. Then $A_i\overline{\alpha}^i(A) \in C(D_n(R))$.

CASE 2. If $f(x) \neq 0$ and $g(x) \neq 0$, then there exists $g_{kl}(x) \neq 0$, such that $g_{(k+u)l}(x) = 0$ for some k, l, and $1 \leq u \leq n - k$, since $G(x) \neq 0$. So $h_{kl}(x) = f(x)g_{kl}(x) = 0$. Hence there exists $r \in R\backslash\{0\}$ such that $a_i\alpha^i(r) \in C(R)$. Let $A = rE_{1n}$. Then $A_i\overline{\alpha}^i(A) \in C(D_n(R))$.

CASE 3. If $f(x) = 0$, then clearly $A_i\overline{\alpha}^i(A) = 0$, where $A = E_{1n}$.

Thus $D_n(R)$ is Central $\overline{\alpha}$-skew McCoy.

Conversely, assume that $f(x)g(x) = 0$, where $f(x)=\sum_{i=0}^{n} a_i x^i,\ g(x)=\sum_{j=0}^{m} b_j x^j$ are nonzero polynomial of $R[x;\alpha]$. Let $F(x)=\sum_{i=0}^{n} A_i x^i,\ G(x)=\sum_{j=0}^{m} B_j x^j$, where

$$A_i=\begin{pmatrix} a_i & a_i & \cdots & a_i\\ 0 & a_i & \cdots & a_i\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & a_i\end{pmatrix},\quad B_j=\begin{pmatrix} b_j & b_j & \cdots & b_j\\ 0 & b_j & \cdots & b_j\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & b_j\end{pmatrix}$$

for any $i = 0, 1, 2, \ldots, n$, $j = 0, 1, 2, \ldots, m$. Then

$$F(x)G(x)=\begin{pmatrix} f(x) & f(x) & \cdots & f(x)\\ 0 & f(x) & \cdots & f(x)\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & f(x)\end{pmatrix}\begin{pmatrix} g(x) & g(x) & \cdots & g(x)\\ 0 & g(x) & \cdots & g(x)\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & g(x)\end{pmatrix}=0.$$

Hence there exists $A=\begin{pmatrix} s & s_{12} & \cdots & s_{1n}\\ 0 & s & \cdots & s_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & s\end{pmatrix}\in D_n(R)\backslash\{0\}$ such that $A_i\overline{\alpha}^i(A) \in C(D_n(R))$, since $D_n(R)$ is Central $\overline{\alpha}$-skew McCoy. If $S \neq 0$, then $a_i\alpha^i(s) \in C(R)$. If $S = 0$, then there exists $S_{ij} \neq 0$ for some i, j, such that $S_{i+v,j} = 0$ for any $1 \leq v \leq n-i$. We also have $a_i\alpha^i(s_{ij}) \in C(R)$. Thus, R is Central α-skew McCoy.

(2) The proof is similar to (1).

Theorem 2.8. Let α be an endomorphism of a ring R and $\alpha^t = I_R$ for some positive integer t. Then R is Central α-skew McCoy if and only if $R[x]$ is Central α-skew McCoy.

Proof. Assume that R is Central α-skew McCoy. Let $R[x][y;\alpha]$ denote the polynomial ring with an indeterminate y over $R[x]$. Suppose that $p(y) = f_0 + f_1y + \ldots + f_my^m$, $q(y) = g_0 + g_1y + \ldots + g_ny^n \in R[x][y;\alpha]$ and $p(y)q(y) = 0$. We also let $f_i = a_{i0} + a_{i1}x + \cdots + a_{iw_i}x^{w_i}$ and $g_i = b_{j0} + b_{j1}x + \cdots + b_{jv_j}x^{v_j}$ for each $0 \leq i \leq m$ and $0 \leq j \leq m$, where $a_{i0},\cdots,a_{iw_i},b_{j0},\cdots,b_{jv_j} \in R$. We claim that there exists $c \in R[x]$ such that $f_i\alpha^i(c) \in C(R[x])$, for all $0 \leq i \leq m$, $0 \leq j \leq n$. Take a positive integer k such that $k \geq \deg(f_0(x)) + \ldots + \deg(f_m(x)) + \deg(g_0(x)) + \ldots + \deg(g_n(x))$, where the degree is as polynomials in $R[x]$ and the degree of zero polynomial is taken to be 0. Since $p(y)q(y) = 0$ in $R[x][y;\alpha]$, we have

$$(1)\qquad \begin{cases} f_0(x)g_0(x)=0\\ f_0(x)g_1(x)+f_1(x)\alpha(g_0(x))=0\\ \vdots\\ f_m(x)\alpha^m(g_n(x))=0\end{cases}$$

in $R[x]$. Now put

$$(2)\qquad \begin{aligned} f(x)&=f_0(x^t)+f_1(x^t)x^{tk+1}+f_2(x^t)x^{2tk+2}+\cdots+f_m(x^t)x^{mtk+m}\\ g(x)&=g_0(x^t)+g_1(x^t)x^{tk+1}+g_2(x^t)x^{2tk+2}+\cdots+g_n(x^t)x^{ntk+n}.\end{aligned}$$

Note that $\alpha^t = I_R$. Then

$$\begin{aligned} f(x)g(x)&=f_0(x^t)g_0(x^t)+(f_0(x^t)g_1(x^t)+f_1(x^t)\alpha(g_0(x^t)))x^{tk+1}\\ &\quad+\cdots+f_m(x^t)\alpha^m(g_n(x^t))x^{m+n(tk+1)}\end{aligned}$$

in $R[x;\alpha]$. Using (1) and $\alpha^t = I_R$, we have $f(x)g(x) = 0$ in $R[x;\alpha]$. In the other hand, from (2) we have

$$\begin{aligned} f(x)g(x)=(&a_{00}+a_{01}x^t+\cdots+a_{0w_0}x^{w_0t}+a_{10}x^{tk+1}+a_{11}x^{tk+t+1}+\cdots+\\ &a_{1w_1}x^{tk+w_1t+1}+\cdots+a_{m0}x^{mtk+m}+a_{m1}x^{mtk+t+m}+\cdots+a_{mw_m}x^{mtk+w_mt+m})\\ (&b_{00}+b_{01}x^t+\cdots+b_{0v_0}x^{v_0t}+b_{10}x^{tk+1}+b_{11}x^{tk+t+1}+\cdots+\\ &b_{1v_1}x^{tk+v_1t+1}+\cdots+b_{n0}x^{ntk+n}+b_{n1}x^{ntk+t+n}+\cdots+b_{nv_n}x^{ntk+v_nt+n})=0\end{aligned}$$

Since R is Central α-skew McCoy and $\alpha^t = I_R$, so there exists $c \in R$ such that $a_{iu}\alpha^i(c) \in C(R)$ for each $0 \leq i \leq m$ and $u \in \{0, 1, \ldots, w_i\}$. Since $C(R)$ is closed under addition, we have $f_i(x^t)\alpha^i(c) \in C(R[x])$ for every $0 \leq i \leq m$. Now it is easy to see that $f_i(x)\alpha^i(c) \in C(R[x])$, and hence $R[x]$ is Central α-skew McCoy.

Conversely, suppose that $R[x]$ is Central α-skew McCoy. Let $f(x)g(x) = 0$ for nonzero polynomial $f(x)=\sum_{i=1}^{n} a_i x^i$ and $g(x)=\sum_{j=1}^{m} b_j x^j \in R[x;\alpha]$. Suppose $f(t)$ and $g(t)$ is constant polynomial in $R[x][t;\alpha]$ such that $f(t)g(t) = 0$. Then there exists $0 \neq c(x) = c_0 + c_1x + \ldots + c_kx^k$ $R[x]$ such that $f(t)\alpha^i(c(x)) \in C(R[x])$. So $a_i\alpha^i(c(x)) \in C(R[x])$. Since $c(x)$ is a nonzero element, then at least one of the c_i's is nonzero element, for example c_t. So $a_i\alpha^i(c_t) \in C(R)$. Therefore R is Central α-skew McCoy.

Proposition 2.10. Let R be a ring and e a central idempotent element of R and α be an endomorphism of a ring R with $\alpha(e) = e$\$. Then R is Central α-skew McCoy ring if and only if eR is Central α-skew McCoy ring if and only if $(1 - e)R$ is Central α-skew McCoy.

Proof. Assume that R is Central α-skew McCoy ring and consider $f(x)=\sum_{i=0}^{n} ea_i x^i$, $g(x)=\sum_{j=0}^{m} eb_j x^j$ $\in eR[x;\alpha]\setminus\{0\}\subseteq R$ such that $f(x)g(x) = 0$. Since R is Central α-skew McCoy ring, there exists $s \in R$ such that $(ea_i)\alpha^i(s) \in C(R)$. So $(ea_i)\alpha^i(s)r = r(ea_i)\alpha^i(s)$, for any $r \in R$. Therefore $((ea_i)\alpha^i(es))(er)=(er)((ea_i)\alpha^i(es))$ .So $(ea_i)\alpha^i(es)\in C(eR)$. Hence eR is Central α-skew McCoy. Similarly, we prove that $(1 - e)R$ is Central α-skew McCoy ring.

Conversely, assume that eR is Central α-skew McCoy ring. Consider $f(x)=\sum_{i=0}^{n} a_i x^i$, $g(x)=\sum_{j=0}^{m} b_j x^j$ $\in R[x;\alpha]\setminus\{0\}$ such that $f(x)g(x) = 0$. Clearly, $ef(x), eg(x)\in eR[x;\alpha]$ and $(ef(x))(eg(x)) = e^2 f(x)g(x) = ef(x)g(x) = 0$, since e is a central idempotent element of R. Then there exists $s \in R$ such that $(ea_i)\alpha^i(s)\in C(eR)$. So $(ea_i)\alpha^i(s)er = er(ea_i)\alpha^i(s)$, for any $r \in R$. Therefore $(a_i\alpha^i(s))r = r(a_i\alpha^i(s))$. So $a_i\alpha^i(s)\in C(R)$. Hence, R is Central α-skew McCoy. Similarly, this fact is satisfied if $(1 - e)R$ is Central α-skew McCoy ring.

Let S denote a multiplicatively closed subset of a ring R consisting of central regular elements. Let RS^{-1} be the localization of R at S. Then we have:

Proposition 2.11. Let α be an automorphism of a ring R. If R is Central α-skew McCoy, then RS^{-1} is Central α-skew McCoy.

Proof. Suppose that R is Central α-skew McCoy ring. Let $f(x)=\sum_{i=0}^{n}(a_i/s_i)x^i$, $g(x)=\sum_{j=0}^{m}(b_j/d_j)x^j$ $\in RS^{-1}[x;\alpha]$ and $f(x)g(x) = 0$. Let $a_i s_i^{-1} = c^{-1}a_i'$ and $b_j d_j^{-1} = d^{-1}b_j'$ with c, d regular elements in R. Then we have $(a_0' + \cdots + a_n' x^n)d^{-1}(b_0' + \cdots + b_m' x^m) = 0$. We know that for each element $f(x)\in RS^{-1}[x;\overline{\alpha}]$ there exists a regular element $C \in R$ such that $f(x) = h(x)c^{-1}$, for some $h(x) \in R[x;\alpha]$, or equivalently, $f(x)c \in R[x;\alpha]$. Therefore there exist a regular element e in R and $(b_0'' + \cdots + b_t'' x^t)\in R[x;\alpha]$, such that $d^{-1}(b_0' + \cdots + b_m' x^m) = (b_0'' + \cdots + b_t'' x^t)e^{-1}$. Hence $(a_0' + \cdots + a_n' x^n)(b_0'' + \cdots + b_t'' x^t) = 0$. Since R is Central α-skew McCoy, then there exists $r \in R$ such that $a_i'\alpha^i(r)\in C(R)$ for all i. Therefore $ca_i s_i^{-1}\alpha^i(r)\in C(R)$. Since c is regular element of R, $a_i s_i^{-1}\alpha(r)$ are central in RS^{-1} for all i.

Corollary 2.12. Let R be a ring and α an automorphism of R, such that $\alpha^t = I_R$ for some positive integer t, then the following are equivalent:

1. R is Central α-skew McCoy.
2. $R[x]$ is Central α-skew McCoy.
3. $R[x, x^{-1}]$ is Central α-skew McCoy.

Proof. Let $S = \{1, x, x^2, x^3, x^4, \ldots\}$. Then S is a multiplicatively closed subset of $R[x]$ consisting of central regular elements. Then the proof follows from Proposition 2.11.

3. Conclusions

In this paper, we have defined a new class of rings called Central α-skew McCoy and illustrated with some examples. Central α-skew McCoy rings are proper extension of α-skew McCoy rings. Also, some related results have been studied and proved. This particular class of ring is not Morita Invariant.

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