

# Performance Analysis of FFT-OFDM and DWT-OFDM over AWGN Channel under the Effect of CFO

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## Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is a multi-carrier based system that is selected to be used in the broadband wireless communication system. However, the main drawback considered for OFDM system is the Carrier Frequency Offset (CFO) which causes the Inter-Carrier Interference (ICI) that degrades the performance of the OFDM system. This paper discusses the performance of the system based on Bit Error Rate (BER) in the presence of CFO. For this, the estimation of CFO is carried over in both Discrete Wavelet Transform (DWT) OFDM and FFT OFDM for five different values of CFO (CFO = 0, 0.05, 0.1, 0.15, 0.2) in AWGN (Additive White Gaussian Noise) channel using the same procedure and their BER performance are simulated using MATLAB software. A comparative analysis on FFT-OFDM and DWT-OFDM systems is offered that shows that the performance given by DWT-based OFDM system is better in comparison to the FFT based OFDM system. BER results are simulated for BPSK, QPSK, 8-PSK, 16-PSK, 32-PSK modulations for both DWT and FFT based OFDM models. The analysis of MATLAB simulation clearly shows that the DWT-OFDM offers an improvement of 5-10 dB for the diverse order of modulations over AWGN channel.

**Keywords:** CFO, DWT, FFT, IDWT, IFFT, OFDM

## 1. Introduction

OFDM system is widely used in multi-carrier modulation schemes. In this modulation, all sub-carriers are orthogonal to each other, which increases the bandwidth efficiency of the system. OFDM transmission frequency channel converts in the group of narrow band flat fading channel, one channel across each sub-channel. IEEE 802.11a standard has been given to the OFDM due to its wide usage across the wireless network (local area) standards<sup>1</sup>. It is an efficient technique to be used in multicarrier based communication system, in which a single stream of data is taken and transmitted over a sum of sub-carriers of the lower rates. In OFDM, bandwidth is divided into many narrow bands whose carriers are orthogonal to one another. Implementation of OFDM deals with the application of Fast Fourier Transform (FFT) to modulation and inverse of FFT for demodulation processes to generate carriers orthogonal to each

other. In conventional OFDM system, IFFT is used for transformation of signals at transmission section and FFT at the reception section. There is a need to append cyclic prefix at transmitter side to reduce the Inter carrier and symbol Interference and to increase spectral efficiency. Recently there is a proposition of Wavelet transform to generate sub channels with added advantage of the flexibility of transform and high suppression in side lobes. This paper provides an overview of the performance of DWT based OFDM system as compared to FFT based OFDM system to reduce Inter carrier Interference caused due to carrier frequency offset in AWGN<sup>2</sup>. Frequency offset in the signal is considered as a very severe drawback for the system. OFDM is sensitive to the synchronization errors caused due to frequency in the form of Carrier Frequency Offset which is nothing but the difference in the frequencies that exists between the local oscillators of the transmitter side with that of the receiver side. This difference may destroy the Orthogonality among the

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subcarriers and leads to the ICI<sup>3</sup>. In this paper, a relative analysis of two techniques is done. Both the techniques have their relative properties regarding the OFDM system. These two techniques are the FFT, which makes use of Fourier transform for the transformation of the signal in frequency domain only and in DWT which makes use of wavelets for the transformation of the signal in time and frequency domain<sup>4</sup>. The procedure using FFT makes the use of addition of cyclic prefix at the end of the transmitted symbol which protects the symbol from inter carrier interference which makes it complex in implementation while in case of DWT which makes use of wavelets does not require any use of addition of cyclic prefix at the transmitted symbol which makes the DWT procedure easy to implement in comparison to FFT procedure and also it gives better performance in comparison to the FFT even when the system suffers from the CFO conditions. The simulations are performed by considering different modulation schemes<sup>5</sup>. This paper discusses the effect of CFO in OFDM and makes a comparative investigation on the performance of FFT-OFDM and DWT-OFDM in the presence of CFO for different modulations. For the OFDM, the paper is organized as follows. The succeeding second section deals with the system model of both FFT OFDM and DWT-OFDM. The third section briefs about the effect of CFO over OFDM. The fourth section describes the simulation of both the OFDM systems in the presence of CFO for different modulations. The 5th Section is for the conclusion.

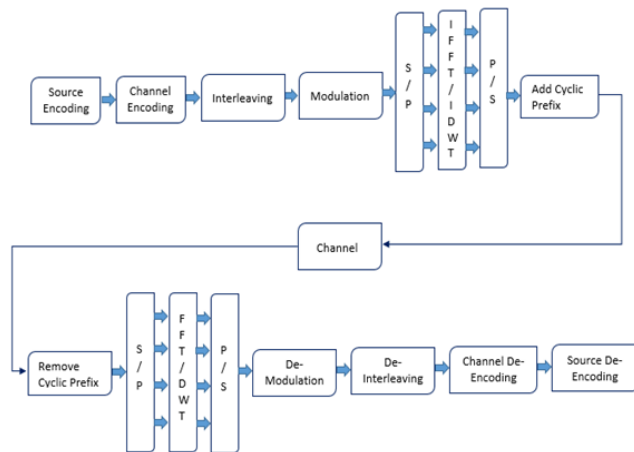


Figure 1. Block diagram of OFDM.

## 2. OFDM Model

A multi-carrier based modulation technique, Orthogonal

FDM is defined as a kind of modulation scheme used to achieve higher data rates by transmitting some overlapped orthogonal sub-carriers<sup>3,6,7</sup>. The sub-carriers should be orthogonal to each other to improve spectral efficiency. At the receiver side, it is easy to recover data in each sub-carrier as long as carriers are orthogonal to each other as shown in Figure 1. The main concept in the OFDM is the Orthogonality to be maintained between the sub-carriers of the system.

### 2.1 FFT-based OFDM

Fourier transform procedures help in maintaining the Orthogonality factor among the carriers by defining the OFDM signal. In OFDM systems, a large amount of narrow band closely spaced carriers were transmitted, provided no crosstalk from other sub channels at the centrally defined frequency of each given sub-channel. Initially, the input bit stream that is to be transmitted is being multiplexed into the number of symbol streams which is denoted by N, each one having some particular symbol period denoted by T<sub>s</sub>. The each one multiplexed stream of the symbol was further used for modulation of the parallel and the synchronized sub-carriers. The spacing between the sub-carriers was defined in frequency as 1/NT<sub>s</sub> and in this way they become orthogonal to each other over a defined [0,T<sub>s</sub>] interval. In a discrete-time OFDM transceiver system, first of all a serial-to-parallel conversion is done by the S/P converter which groups the input bit streams available from source encoder to the form of log2M bits, in which M defines the size of the scheme used for digital modulation that is implemented on each of the sub carriers. A sum of N such symbols denoted by X<sub>m</sub> were formed. Further, the mapping of N symbols to bins is done of the form of the IFFT. The bins formed by IFFT relates to the sub-carriers of the symbol that are orthogonal to each other. The Cyclic prefix is added to the IFFT output to avoid ICI (Inter-Carrier Interference)<sup>8</sup>. At last, the demonstrated OFDM symbol is shown as:

$$x(n) = \frac{1}{N \sum_{m=0}^{N-1} \left( X_m e^{\frac{j2\pi mn}{N}} \right)}; 0 \leq n \leq N-1 \quad (1)$$

Here, X<sub>m</sub> defines the baseband data available on each of the sub-carrier. The digital-to-analog conversion process creates a signal in time-domain that is considered to be analog in nature and is passed from the AWGN. The signal

relating to each of the sub-carrier is further taken back for conversion in the form of a discrete N-point sequence that is denoted by  $y(n)$ . At the receiver side, the discrete signal received was further taken for demodulation process that is done by the usage of the N-point FFT operation. The symbol stream that is demodulated expressed in the form:

$$y(m) = \sum_{n=0}^{N-1} y(n) e^{-\frac{j2\pi mn}{N}} + W(n); 0 \leq m \leq N-1 \quad (2)$$

The Fast Fourier Transform for the  $w(n)$  samples is denoted by  $W(m)$ , which is considered to be the Gaussian Noise added in the AWGN channel. The computational complexity for the FFT-based system and the cost for implementing this system is less which makes it advantageous. Its limitations are its wasted bandwidth and hence spectral efficiency due to the addition of the cyclic prefix. This motivates the use of DWT based OFDM; that does not use cyclic prefix<sup>9,10</sup>.

## 2.2 DWT-based OFDM

Wavelets are defined as a class of mathematical functions that shows oscillations with a particular amplitude that starts at zero, rises and then sets back again at its starting point to zero. These wave-like oscillations are used for localizing a function that is given in both spaces. The filters used by wavelets are recognized by the wavelet bases. The techniques for signal transmission allows a considerable increase in the capacity of the wireless system without making any increase in its bandwidth. The FFT-based system uses multi-carrier based modulation in which sub-carriers considered to be orthogonal. The window that is rectangular to be used for implementation leads to the creation of large lobes on the sides. However, the DWT applied OFDM shows larger basis function that provides much larger suppression at the side lobes<sup>6,11,12</sup>. Practically, OFDM is implemented by the use of Discrete Fourier Transform (DFT) technique. When applying the DFT, a basis set is formed which is orthogonal. Also, the signal in vector space of DFT form a linear combination of the sinusoids that are also orthogonal. One vision of the DFT is that the transformation of DFT correlates the signal that is taken as input with respect to each one of the defined sinusoidal basis functions. The type of transformation used at the transmitter side for mapping the input signal onto the set of orthogonally placed sub-carriers that are the orthogonal basis functions defined by the DFT. Similarly, the same transformation was applied another

time at the reception side of OFDM for the processing of received sub carriers. The signals that are further derived from all of the sub carriers were then collected to make a complete estimation of the source signal received from the sender. The basic functions obtained in the DFT were uncorrelated which makes the specified particular sub-carrier notice energy for that equivalent defined sub-carrier only. Due to the un-correlation of sub carriers, the energy obtained from them causes no interference this separation in signal energy becomes the reason behind the overlapping in the spectrums of sub-carriers without any interference.

In case of DWT-OFDM, the IFFT block is replaced with IDWT block and similarly FFT with DWT. Because of the nature of the DWT without the (additional extension of the signal provided by) cyclic prefix the ICI can be avoided. So the addition and deletion of cyclic prefix blocks are not needed in this case. The cyclic prefix is the redundant data which restricts the useful data rate. By avoiding this, data rate and hence the efficiency can be improved<sup>3</sup>. The wavelet low-pass filter presents approximation coefficients, where the high-pass filter presents detail coefficients. Different wavelet families have different filters satisfying Orthogonality principle<sup>4</sup>. The output of the (IDWT) can be expressed as follows:

$$x_i(n) = \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} D_k^m 2^{\frac{k}{2}} \varphi(2^k n - m) \quad (3)$$

Where  $D_k^m$  are the wavelet coefficients and  $\psi(t)$  is the wavelet basis function with compression factor  $k$  and shifted  $m$  for each subcarrier. The output of DWT is given by<sup>5</sup>,

$$D_k^m = \sum_{n=0}^{N-1} x_i(k) 2^{\frac{k}{2}} [\varphi(2^k n - m)] \quad (4)$$

## 3. Carrier Frequency Offset (CFO)

Orthogonal Frequency Division Multiplexing (OFDM) is a popular method for high data rate wireless transmission. OFDM is very sensitive to frequency offset and time synchronization. The sensitivity to Carrier Frequency Offset (CFO) effects its performance by introducing Inter-Carrier-Interference (ICI)<sup>9</sup>. CFO refers to the difference in carrier frequency at transmitter and receiver. The carrier modulation helps in converting the baseband transmit

signal up to the pass-band and then helps in converting it down to the level of baseband by the taking the local signal as carrier with the equivalent carrier frequency at the reception side. Two types of distortions are related with the signal used as carrier, first was the phase offset which causes due to the occurrence of instability in the carrier signal generators which are being applied at the Tx and Rx section, which could be further modelled as a Wiener random process as zero-mean. The other one considered was the Carrier Frequency Offset (CFO) that is caused by Doppler shift in frequency and a mismatch between the local oscillator at the transmitter and receiver. CFO destroys the orthogonality between the sub-carriers. Therefore, the CFO synchronization is essential to OFDM system. The CFO normalized is being divided into two parts that were defined as integral CFO (IFO)  $\xi_i$  and the other was the fractional CFO (FFO)  $\xi_r$ . IFO produce a cyclic shift by  $\xi_i$  in receiver side to the corresponding sub-carrier, and it does not destroy orthogonality among the sub-carrier frequency component and FFO destroy the orthogonality between the sub carriers.

- **For CFO estimation in the time domain:** Two techniques that are the Cyclic Prefix (CP) and the training sequence are used. CP based estimation is analyzed assuming negligible channel effect<sup>9</sup>. CFO can be found from the phase offset between CP and the consequent part of the symbol and in training estimation training sequence is added in front of OFDM symbols to facilitate normalized CFO estimation.
- **For CFO estimation in the frequency domain:** This techniques involves the comparison of the phase of each sub carrier to the successive symbol, the phase shift in the symbol is due to CFO. Two different modes were given for CFO estimation in pilot based method that are acquisition mode and the tracking. In acquisition, estimation is done on a large range of CFO and in tracking mode only the fine CFO was estimated.

### 3.1 Influence of CFO on OFDM System

As mentioned above, the main causes of CFO n communication systems are Doppler frequency shift  $f_D$  and frequency mismatch  $f_m$  between transmitter and receiver<sup>4</sup>. If normalized CFO  $\epsilon$  is defined as a ratio of a sum  $f_D + f_m$  to spacing in subcarrier  $\Delta f$ , the received OFDM signal  $r(n)$  for transmitted signal  $s(n)$  is shown as:

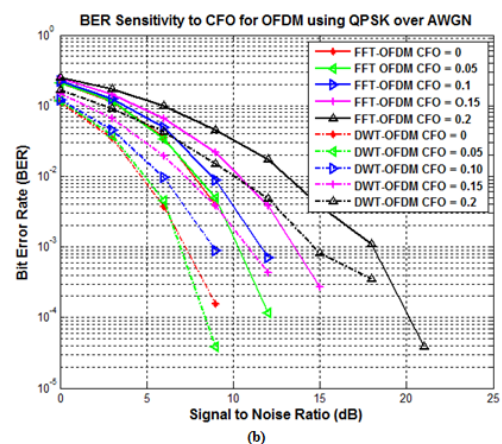
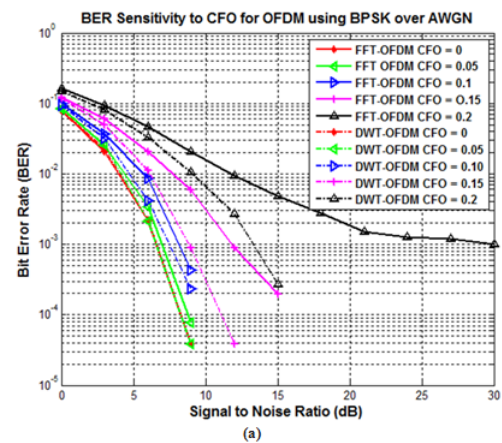
$$r(n) = e^{j\frac{2\pi N(f_D + f_m)}{B_s} n} s(n) = e^{j\frac{2\pi n \epsilon}{N_{FFT}}} s(n) \tag{5}$$

Where  $B_s$  is the OFDM signal's bandwidth and  $B_s = N_{FFT} \Delta f$ . The modulated signal  $R(k)$  can be deduced:

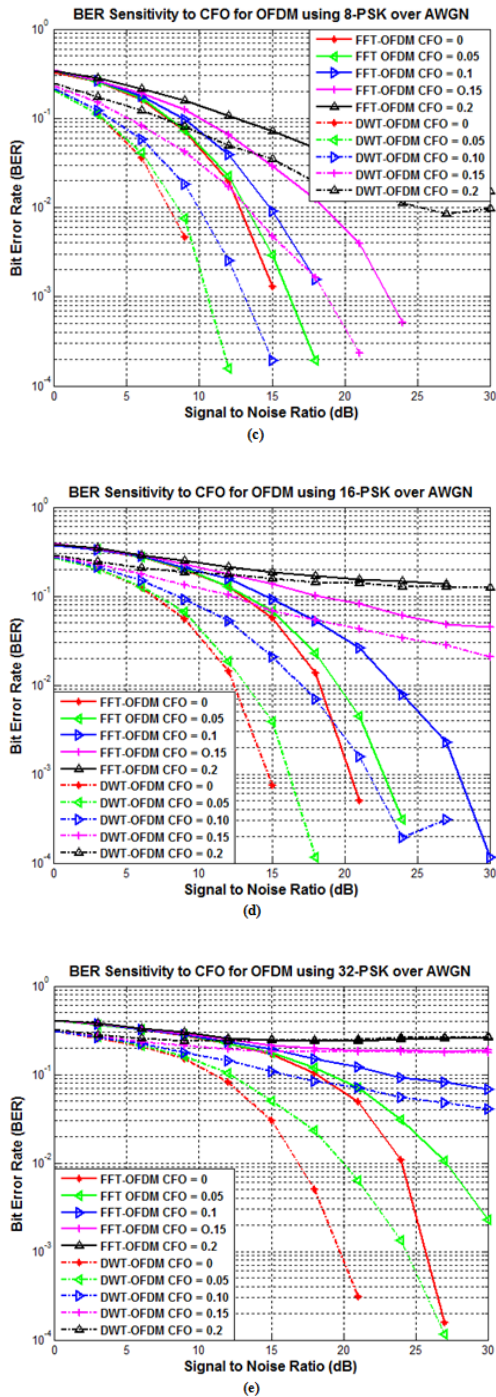
$$R_\epsilon(k) = FFT[e^{j\frac{2\pi n \epsilon}{N_{FFT}}} s(n)] = S(k - \epsilon) \tag{6}$$

## 4. Results Discussion

Simulated analysis for \*FFT and the DWT system is performed by using MATLAB. BER versus  $E_b/No$  simulation is carried out in the presence of CFO for  $\epsilon$  values in the range of 0 to 0.2 in steps of 0.05. A comparative analysis of both the systems is done in the presence of CFO for various modulations, i.e., for BPSK, QPSK, 8-PSK, 16-PSK, 32-PSK. As the CFO value increases, the bit error also increases. So, its mitigation is very much required for the system to perform effectively. The performance measure is done on both the systems and analyzed their performance with five practical values of CFO. The simulation is done in AWGN channel for different modulations.







**Figure 2.** (a) Performance comparison of DWT-OFDM and FFT-OFDM for BPSK. (b) Performance comparison of DWT-OFDM and FFT-OFDM for QPSK. (c) Performance comparison of DWT-OFDM and FFT-OFDM for 8-PSK. (d) Performance comparison of DWT-OFDM and FFT-OFDM for 16-PSK. (e) Performance comparison of DWT-OFDM and FFT-OFDM for 32-PSK.

In Figure 2(a-e) a performance analysis on the basis of bit error rate vs signal to noise ratio (dB) is done between the DWT-OFDM and FFT-OFDM in the presence of CFO lying in range from 0 to 0.2 for BPSK, QPSK, 8-PSK, 16-PSK, 32-PSK modulations for AWGN channel. Simulations show that the BER curve shows a great improvement in the bit error graph for DWT-OFDM systems by mitigating the CFO effect in OFDM for different modulation schemes employed. So after performing simulations by considering different parameters, we can recommend that the DWT-OFDM gives better performance over the conventional FFT-OFDM even in the presence of CFO by mitigating it. The BER performance for OFDM is improved in terms of dB for different modulations by using the DWT.

## 5. Conclusion

CFO is considered as the main drawback for the OFDM systems, which causes inter carrier interference in the symbols which degrades the performance and effecting its spectral efficiency. So, CFO estimation and compensation is required for the improved system's performance. As the CFO increases the error rate also increases. It is obvious from the plots that, in the rate of change there may be less difference between the DWT and FFT OFDM. In this paper, a comparison of these two techniques is done to check the most efficient method from these and the performance provided by them shows that the DWT-OFDM method performs better in mitigating the CFO effectively from the system and provides accuracy in the system by improving the BER

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