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Semigraph Folding Approach for Generalization of Planar Triangulation

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Abstract

We applied triangulation in the cycle graphs C_n, n ≥ 3 and generalized to n – transformation, also we observed that on splicing and folding introduced by Tom Head and E. El-Kholy & co. respectively in C_{2m}, $m \geq 2$ and it's generalization leads to the resultant graph is $P_2, G_3^0, G_3^1, \ldots, G_3^{n-1}$ whereas on splicing and semigraph folding introduced by S. Jeyabharathi & Co. in C_{2m-1}, $m \geq 2$ and its generalization leads to the resultant semigraph with and an edge and one semi edge.

Keywords: Folding and Semigraph Folding, Planar Trinagulation, Splicing, Semigraph

1. Introduction

The concept of splicing system introduced by Tom Head has become a new interesting area on DNA molecule[6] and the graph splicing scheme of Rudolf Freund with semigraphs[9] introduced by E. Sampath kumar representing the spliced semigraph[3] which is more powerful than graph. This presents communication volume accurately. Semigraph model will eventually replace graph partitioning in scientific computing. Here we apply the theory of splicing system and folding techniques in the planar triangulation[4] where the planar triangulations on the cycle graph is plane graphs in which every face is a triangle. Triangulation today is used for many purposes, including surveying, navigation, metrology, astrometry, binocular vision, model rocketry and gun direction of weapons. In computational geometry, polygon triangulation is the decomposition of a polygonal area (simple polygon) into a set of triangles, i.e., finding a set of triangles with pair wise non-intersecting interiors whose union is polygonal area.

2. Preliminaries

2.1 Splicing System

Splicing is a model of the recombinant behaviour of double stranded molecules of DNA under the action of restriction enzymes and ligases. A single stranded of DNA is an oriented sequence of nucleotides A, C, G & T but since A can bind to T & G to C, two strands of DNA bind together to form a double stranded DNA molecule, if they have matching pairs of nucleotides when reading the second one along the reverse orientation.

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2.2 Graph

A graph over V is a triple (N, E, L) where N is the set of nodes, E is the set of edges of the form (n, m) with n, m $\in$ N, n $\neq$ m and L is the function from N to V assigning a label from V to each node of N. The set of all graphs over V is denoted by γ (V).

2.3 SemiGraph

A semigraph G is a pair (V, X) where V is a non - empty set whose elements are called vertices of G and X is a set of n - tuples called edges of G of distinct vertices for various n $\geq$ 2, satisfying the following conditions:

S.G-1 Any two edges have at most one vertex in common.

S.G-2 Two edges $(u_1, u_2, u_3, \ldots, u_n)$ and $(v_1, v_2, v_3, \ldots, v_m)$ are considered to be equal if and only if (i) m = n and (ii) either $u_i = v_i$ or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$. Thus the edges $(u_1, u_2, u_3 \ldots u_n)$ are the same as the edge $(u_n, u_{n-1}, \ldots, u_1)$.

2.4 Semi Vertices

Let G be a graph, when splicing G, we obtain new vertices which are called as semi vertices denoted by V', where |V'|=p'.

2.5 Semi Edges

Let G be a graph when splicing G, we obtain new edges by decomposition of edges which are called as semi edges denoted by E', where |E' | = q'.

2.6 Spliced SemiGraph map

Let $SSG_1 = (V_1, E_1)$ and $SSG_2 = (V_2, E_2)$ be two Spliced Semigraphs and a map f : $SSG_1 \rightarrow SSG_2$ is said to be a **spliced semigraph map**, if

i. for each vertex $v \in V_1$, f(v) is a vertex in V_2.
ii. for each semi vertex $v' \in V_1$, f(v') is a vertex in V_2.
iii. for each edge $e \in E_1$, $\dim(f(e)) \leq \dim(e)$.
iv. for each semi edge $e' \in E_1$, $\dim(f(e')) \leq \dim(e')$.

where V_1 and V_2 are the set of vertices and semi vertices of the Spliced Semigraphs SSG_1 and SSG_2 respectively. E_1 and E_2 are the set of edges and semi edges of the Spliced Semigraphs SSG_1 and SSG_2 respectively.

2.7 Semigraph Folding

A Spliced Semigraph map f: $SSG_1 \rightarrow SSG_2$ a semigraph folding, if and only if f maps vertices to vertices, semi vertices to semi vertices, edges to edges and semi edges to semi edges.

Example for semigraph folding:[2]

The graph G [fig 1] represents 1-Cut splicing (u_{c_1}) with the semivertices {1', 2', 3', 4', 5', 6'} and the semiedges {(1, 1'), (2, 2'), (2. 3'), (5, 5'), (5, 4'), (4, 6')}. Here |V ' | = 6 and |E' | = 6.

On splicing, the graph G forms 2 bipartite semigraphs G_1 & H_1 [Fig 2]. Further on applying sequence of semigraph folding on either G_1 or H_1, the resultant semigraph with one edge and one semiedge. [fig 3]

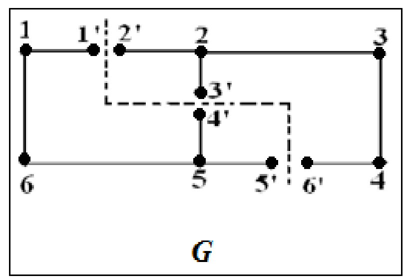


Figure 1. Graph G with 1-Cut splicing (u_{c_1}).

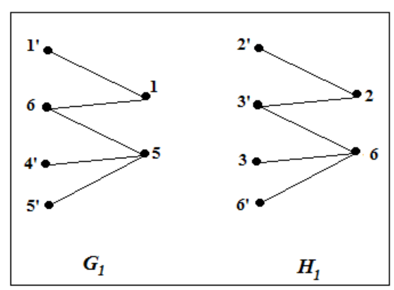


Figure 2. Bipartite semigraphs.

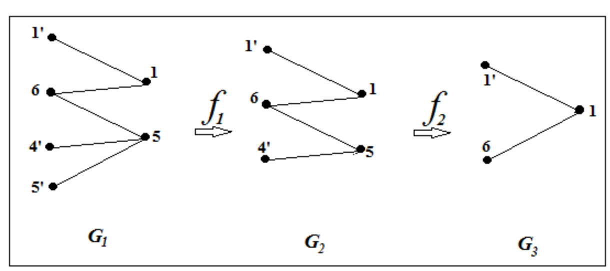


Figure 3. Semigraph folding on G_1 results in G_3.

2.8 SemiGraph (SG) notation

The semigraph SG is denoted by ***quadruple SG = (V, E, V', E').***

where *V* denotes the set of vertices in the semigraph SG
E denotes the set of edges in SG
V' denotes the set of semi vertices in SG and
E' denotes the set of semi edges in SG.

Example for Semigraph (SG) Notation:
Form Fig. 3, the semigraph G_3 is denoted by quadruple G_3 = ({1, 6}, {(1, 6)}, {1' },{(1,1')})

2.9 η(SG)

The number of vertices, edges, semi vertices and semi edges in a semigraph SG is denoted by ***quadruple η(SG) = (η(V), η(E), η(V'), η(E'))***

where *η(V)* denotes the number of vertices in SG.
η(E) denotes the number of edges in SG
η(V›) denotes the number of semi vertices in SG and
η(E›) denotes the number of semi edges in SG.

Example for η(SG):
Form Fig. 3, the number of vertices, edges, semi vertices and semi edges in a semigraph G_3 is denoted by quadruple $\eta(G_3)$ = (2, 1, 1, 1).

3. Generating triangulation in C_n

3.1 Algorithm for generating triangulation in C_n graph

Step 1: Take a cycle graph '*G*' with 'n' vertices (v_1, v_2, v_3,…., v_n), n ≥ 3. Let it be denoted by G_n^0

Step 2: Introduce a vertex 'vn+1' inside G_n^0 and connect the vertex 'vn+1' to all other vertices by an edge in G_n^0. The resultant graph is denoted by G_n^1. And T1 is the transformation of generating triangulation from the graph G_n^0 to G_n^1.

Step 3: Introduce vertices v_{n+2}, v_{n+3}, v_{n+4},…., v_{2n+1} in each C_3 embedded in G_n^1 and connect the corresponding vertices v_{n+2}, v_{n+3}, v_{n+4},…., v_{2n+1} to all 3 vertices in the embedded C_3 in which it lies in that region. The resultant graph is denoted by G_n^2. And T_2 is the transformation of generating triangulation from the graph G_n^1 to G_n^2.

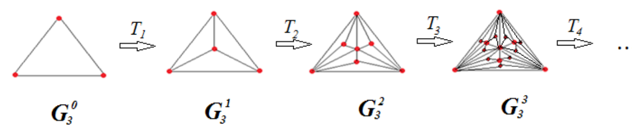
Step 4: Repeat Step 3 to required number of times.

Note : On repeating step 4 further to infinite and by introducing of new vertex (v) in a each C_3 (a triangle) embedded will lie on the edges at some point.

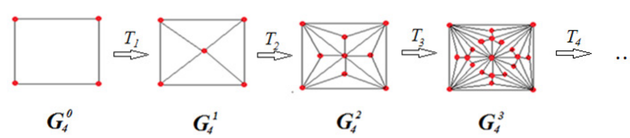
4. Working model of the proposed algorithm

Let G_j^k, j ≥ 3, k ≥ 0 be the denotion of the Cycle graph (C_n) and its generalization, j indicates the number of vertices in the cycle graph and k indicates the generalization of each stages which takes the value from 0,1, 2, 3,… .

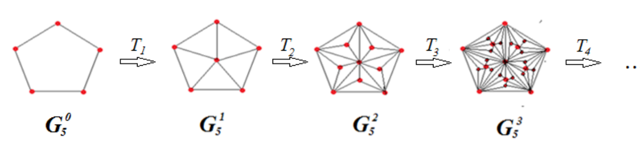
C_3 (Triangle) and its generalization:



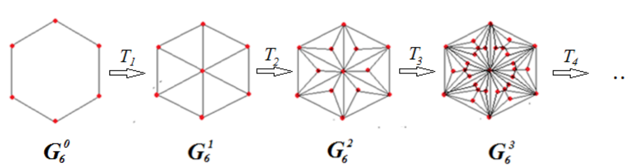
C_4 (Square) and its generalization:



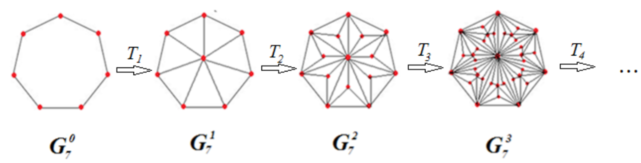
C_5 (Pentagon) and its generalization:



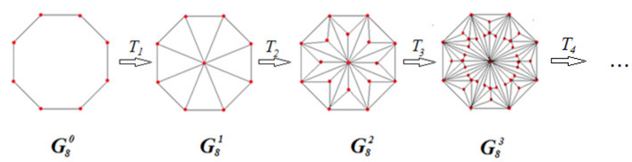
C_6 (Hexagon) and its generalization:



C_7 (Heptagon) and its generalization:



C_8 (Octagon) and its generalization:



4.1 Cycle graph (C_n), n ≥ 3 Versus number of vertices

The number of vertices in each triangulation of C_n, $n \geq 3$ is given in Table 1.

Table 1. C_n Vs Number of vertices

Cycle graph(C_n), n ≥ 3 Vs number of vertices	G_n^0	G_n^1	G_n^2	G_n^3	G_n^4	
	n	**n_1**	**n_2**	**n_3**	**n_4**	**…**
C_3	3	4	7	16	43	…
C_4	4	5	9	21	57	…
C_5	5	6	11	26	71	…
C_6	6	7	13	31	85	…
C_7	7	8	15	36	99	…
…	…	…	…	…	…	…

Note: In Table I, the denotion of n, n_1, n_2, n_3 ,…. indicates the number of vertices in each cycle graph and its generalization.

On applying triangulation for cycle graph with 'n' vertices and in its generalization in each triangulation is observed as recurrence relation

$$n_k = n_{k-1} + (n \times 3^{k-2}),\ k = 2, 3, \ldots.\ for\ n_1 = n+1.$$

4.2 Cycle graph (C_n) Versus number of C_3

The number of C_3 embedded in each triangulation is given in Table 2.

Table 2. C_n Vs Number of C_3

Cycle graph (C_n) Vs number of C_3	*n*	Number of C_3 in each cycle graph and it's generalization						
		G_n^0	G_n^1	G_n^2	G_n^3	G_n^4	G_n^5	
		I_0	I_1	I_2	I_3	I_4	I_5	…
G_3^0	3	1	3	9	27	81	243	…
G_4^0	4	0	4	12	36	108	324	…
G_5^0	5	0	5	15	45	135	405	…
G_6^0	6	0	6	18	54	162	486	…
G_7^0	7	0	7	21	63	189	567	…
G_8^0	8	0	8	24	72	216	648	…
G_9^0	9	0	9	27	81	243	729	…
G_0^0	10	0	10	30	90	270	810	…
…	…	…	…	…	…	…	…	…

From Table 2, I_i, i = 0, 1, 2, 3,… denotes the number of triangles in each cycle graph and it's generalization.

Thus, for any cycle graph with 'n' vertices ∀ n ≥ 3, the number of C_3 embedded in each triangulation is separated as Case 1 and Case 2.

Case 1 : When n = 3, the number of C_3 embedded in each triangulation is 1, 3^0, 3^1, 3^2, 3^3,….

Case 2 : When n ≥ 4 , the number of C_3 embedded in each triangulation is 0, 3^0n, 3^1n, 3^2n, 3^3n, ….

4.3 Folding on C_{2m} , m ≥ 2 and its Generalization

Let f_i ,i ≥1 be the folding on the graphs.

Table III shows the folding on C_{2m} , m ≥ 2 and its generalization.

4.3.1 Proposition

For C_4, the number of folding is 2.

Table 3. Folding on C_{2m}, m ≥ 2 and its generalization

C_{2m}, $m \geq 2$ its generalization	m	n	Graph	Applying folding techniques on graph	Resultant graph after folding
G_{2m}^{n}	2	0	G_4^0	f_1 , f_2	P_2 (Path of length 1)
		1	G_4^1	f_1 , f_2	C_3 (which is G_3^0)
		2	G_4^2	f_1 , f_2	G_3^1
		3	G_4^3	f_1 , f_2	G_3^2
		…..	….	f_1 , f_2	…
		n	G_4^n	f_1 , f_2	G_3^{n-1}
	3	0	G_6^0	f_1 , f_2 , f_3	P_2 (Path of length 1)
		1	G_6^1	f_1 , f_2 , f_3	C_3 (which is G_3^0)
		2	G_6^2	f_1 , f_2 , f_3	G_3^1
		3	G_6^3	f_1 , f_2 , f_3	G_3^2
		…..	…..	f_1 , f_2 , f_3	…
		n	G_6^n	f_1 , f_2 , f_3	G_3^{n-1}
	4	0	G_8^0	f_1 , f_2 , f_3	P_2 (Path of length 1)
		1	G_8^1	f_1 , f_2 , f_3	C_3 (which is G_3^0)
		2	G_8^2	f_1 , f_2 , f_3	G_3^1
		3	G_8^3	f_1 , f_2 , f_3	G_3^2
		…..	…..	f_1 , f_2 , f_3	…
		n	G_8^n	f_1 , f_2 , f_3	G_3^{n-1}
	…	…	…	…	…

Proof:
From Appendix 5.1 , It is evident that the proposition holds true.

4.3.2 Proposition

For every C_{2n}, n ≥ 3, the number of folding is 3.

Proof:
From Appendix 5.2, It is evident that the proposition holds true.

4.4 Folding on C_{2m-1}, m ≥ 2 and its Generalization

Let f_i ,i ≥1 be the folding on the graphs and S_i , i ≥1 be the semigraph splicing.

Table 4 shows the folding on C_{2m+1} , m ≥ 2 and its generalization.

Note: From Table 4, the resultant semigraph after splicing and folding η(SG) is (2,1,1,1) which is equivalent to Fig. i [From Appendix 5.3]

4.4.1 Proposition

The number of folding on C_{2m-1} , m ≥ 2 and its generalization is increased by 2.

Proof :
From Appendix 5.4, it is evident that the folding f_1 is used in Fig.j whereas f_1, f_2 and f_3 in Fig.k.

Also from Appendix 5.5, the folding f_1 and f_2 is used in Fig.l and the foldings f_1, f_2, f_3 & f_4 in Fig.m .

Thus the number of folding is increased by 2 on C_{2m-1} , m ≥ 2 and its generalization.

4.4.2 Proposition

The number of folding techniques applied on each C_{2m-1} , m ≥ 2 and its generalization is increased by 1 from one cycle graph to another.

Proof:
The folding f_1 is used in Fig.j of Appendix 5.4 whereas the folding f_1 and f_2 in Fig.l of Appendix 5.5.

Also the foldings f_1, f_2 and f_3 is used in Fig.k of Appendix 5.4 whereas the folding f_1, f_2, f_3 & f_4 in Fig.m of Appendix 5.5.Thus the number of folding is increased by 1 from one cycle graph and it's generalization to another.

5. Appendix

5.1 Folding on C_4

On applying folding technique f_1 & f_2 on each $G_4^0, G_4^1, G_4^2, G_4^3, \ldots$ the corresponding resultant graph is P_2 (*Path of length* 1) $G_3^0, G_3^1, G_3^2, \ldots$ [From Fig. a, Fig. b, Fig. c, Fig. d]

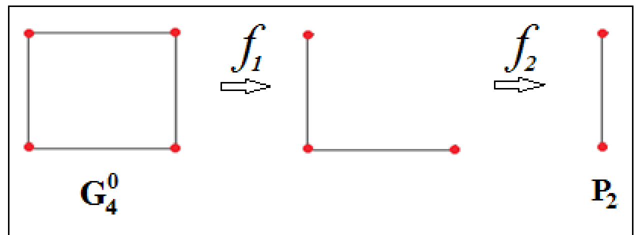


Fig. a

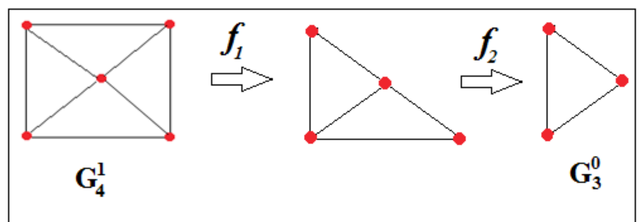


Fig. b

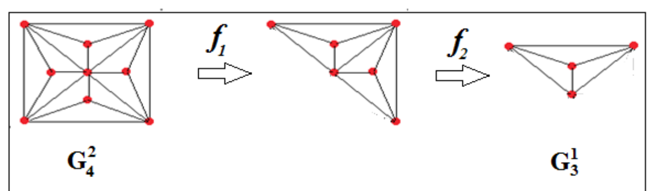


Fig. c

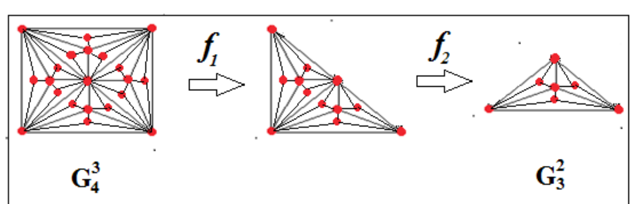


Fig. d

Table 4. Folding on C_{2m+1}, m ≥ 2 and its generalization

C_{2m-1}, m ≥ 2 and its generalization	*n*	*Graph*	*Applying splicing*	*Applying folding*	*Sequence of applying splicing and folding in graph*	*Resultant semigraph after splicing and folding η(SG)*
G_3^n	0	G_3^0	S_1	f_1	$S_1 - f_1$	(2,1,1,1)
	1	G_3^1	S_1, S_2	f_1, f_2, f_3	$S_1 - f_1 - S_2 - f_2 - f_3$	(2,1,1,1)
	2	G_3^2	S_1, S_2, S_3	f_1, f_2, f_3, f_4, f_5	$S_1 - f_1 - S_2 - f_2 - f_3 - S_3 - f_4 - f_5$	(2,1,1,1)
	3	G_3^3	S_1, S_2, S_3, S_4	$f_1, f_2, f_3, f_4, f_5, f_6, f_7$	$S_1 - f_1 - S_2 - f_2 - f_3 - S_3 - f_4 - f_5 - S_4 - f_6 - f_7$	(2,1,1,1)
	…	…	…	…	…	(2,1,1,1)
	n	G_3^n	$S_1, S_2, S_3, \ldots. S_{n+1}$	$f_1, f_2, f_3, \ldots, f_{2n-1}, f_{2n}, f_{2n+1}$	$S_1 - f_1 - S_2 - f_2 - f_3 - \ldots - S_{n+1} - f_{2n} - f_{2n+1}$	(2,1,1,1)
G_5^n	0	G_5^0	S_1	f_1, f_2	$S_1 - f_1 - f_2$	(2,1,1,1)
	1	G_5^1	S_1, S_2	f_1, f_2, f_3, f_4	$S_1 - f_1 - f_2 - S_2 - f_3 - f_4$	(2,1,1,1)
	2	G_5^2	S_1, S_2, S_3	$f_1, f_2, f_3, f_4, f_5, f_6$	$S_1 - f_1 - f_2 - S_2 - f_3 - f_4 - S_3 - f_5 - f_6$	(2,1,1,1)
	3	G_5^3	S_1, S_2, S_3, S_4	$f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8$	$S_1 - f_1 - f_2 - S_2 - f_3 - f_4 - S_3 - f_5 - f_6 - S_4 - f_7 - f_8$	(2,1,1,1)
	…	…	…	…	…	(2,1,1,1)
	n	G_5^n	$S_1, S_2, S_3, \ldots. S_{n+1}$	$f_1, f_2, f_3, \ldots, f_{2n}, f_{2n+1}, f_{2n+2}$	$S_1 - f_1 - f_2 - S_2 - .. - S_{n+1} - f_{2n+1} - f_{2n+2}$	(2,1,1,1)
G_7^n	0	G_7^0	S_1	f_1, f_2, f_3	$S_1 - f_1 - f_2 - f_3$	(2,1,1,1)
	1	G_7^1	S_1, S_2	f_1, f_2, f_3, f_4, f_5	$S_1 - f_1 - f_2 - f_3 - S_2 - f_4 - f_5$	(2,1,1,1)
	2	G_7^2	S_1, S_2, S_3	$f_1, f_2, f_3, f_4, f_5, f_6, f_7$	$S_1 - f_1 - f_2 - f_3 - S_2 - f_4 - f_5 - S_3 - f_6 - f_7$	(2,1,1,1)
	3	G_7^3	S_1, S_2, S_3, S_4	$f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9$	$S_1 - f_1 - f_2 - f_3 - S_2 - f_4 - f_5 - S_3 - f_6 - f_7 - S_4 - f_8 - f_9$	(2,1,1,1)
	…	…	…	…	…	(2,1,1,1)
	n	G_7^n	$S_1, S_2, S_3, \ldots. S_{n+1}$	$f_1, f_2, f_3, \ldots, f_{2n+1}, f_{2n+2}, f_{2n+3}$	$S_1 - f_1 - f_2 - f_3 - S_2 - f_4 - f_5 - S_{n+1} - f_{2n+2} - f_{2n+3}$	(2,1,1,1)
	…	…	…	…	…	(2,1,1,1)

5.2 Folding on C_6 :

On applying folding technique f_1, f_2 & f_3 on G_6^0, G_6^1, G_6^2, G_6^3, …. , the corresponding resultant graph is P_2 (Path of length 1), G_3^0, G_3^1, G_3^2, …. [From Fig. e, Fig. f, Fig. g, Fig. h]

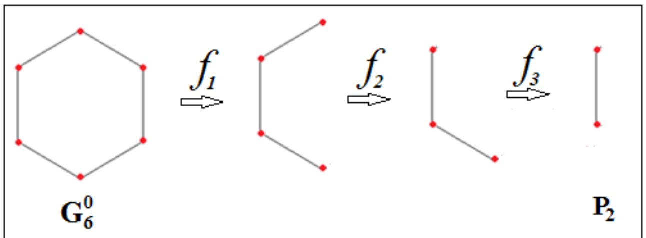


Fig. e

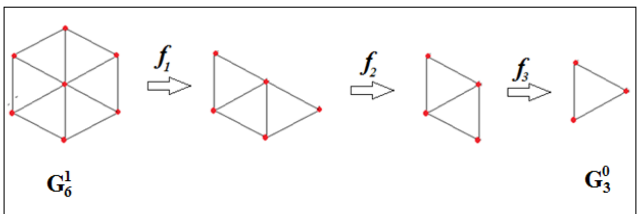


Fig. f

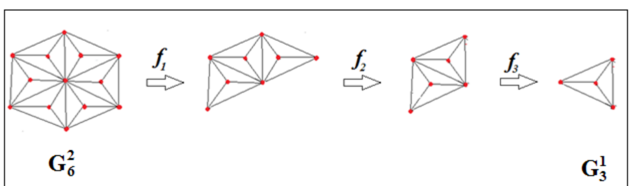


Fig. g

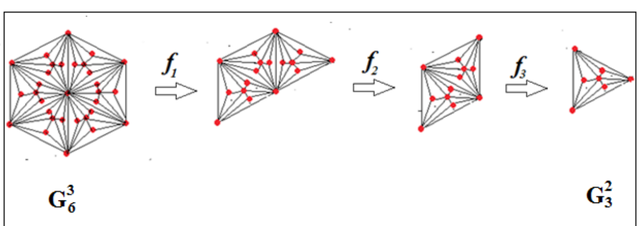


Fig. h

The semigraph [Fig. i] with one edge and one semiedge. Here V_1, V_2 are vertices and S_1 is the semi vertices. The edge joining the vertices V_1 and V_2 is an edge whereas the edge joining the vertices V_1 and S_1 is the semiedge. The semigraph notation **η(SG)** of the below graph is (2,1,1,1).

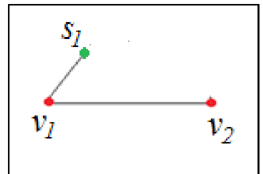


Fig. i

5.3 Folding on G_3^n , n = 0, 1, 2, 3, ...

In Fig. j & Fig.k, 's' indicates the splicing along the edge and ' r ' is the semivertices (r_1 & r_2) after applying splicing along the edge.

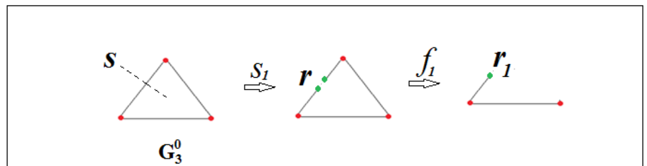


Fig. j

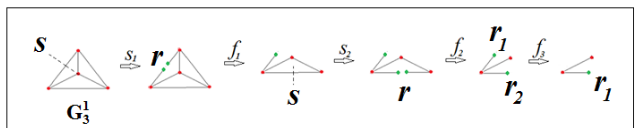


Fig. k

On applying splicing rule S_1 and folding technique f_1 on G_3^0 resulting to a semigraph $\eta(G_3^0)$ = (2,1,1,1). [From Fig. j]

On applying splicing rule S_1, S_2 and folding technique f_1, f_2 & f_3 on G_3^1 resulting to a semigraph $\eta(G_3^1)$ = (2,1,1,1). [From Fig. k]

5.4 Folding on G_5^n , n = 0, 1, 2, 3, ...

In Fig.l & Fig.m,'s' indicates the splicing along the edge and ' r ' is the semivertices (r_1 & r_2) after applying splicing along the edge.

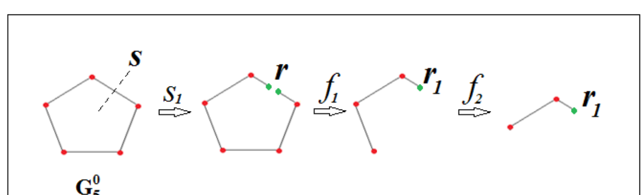


Fig. l

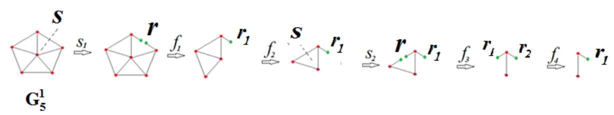


Fig. m

On applying splicing rule S_1 and folding technique f_1 & f_2 on G_5^0 resulting to a semigraph $\eta(G_5^0) = (2,1,1,1)$. [From Fig. l]

On applying splicing rule S_1, S_2 simultaneously and folding technique f_1, f_2, f_3 & f_4 on G_5^1 resulting to a semigraph $\eta(G_5^1) = (2,1,1,1)$. [From Fig.m]

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7. References

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