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Modelling and Performance Analysis of Cellular Mobile Networks

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Abstract

Background/Objective: The Quality of Service (QoS) accustomed by individual customers is an important factor to determine the system performance. The present paper deals with the performance analysis of cellular mobile networks. **Analysis/Methods:** A cell of mobile communication network is considered and number of customers (m) generating the calls are taken as finite. Two channels are declared as guard channels whose access has been prohibited for the fresh calls. Moreover, a 'b' factor has been introduced, considering the case of spam calls and then analysis has been done on it. For the model, we have obtained steady state equations and later solved them by the matrix recursive method. **Findings**: As the number of customer's increase, making the network more complex, it is untrue that the customer behavior will have a neglectable impact on the network performance. In this paper, we focus on the effect of various parameters on the QoS in a cellular mobile network. Also, the novelty in this paper is of the spam calls wherein it was found that if there is an incoming spam call, then that call will not reach the server and hence will be blocked. By using matrix method and MATLAB, the steady state equations are solved. The different state probabilities are obtained and various performance measures have been formulated analytically and thus numerical simulation is done. Like, the effect of retrial rate on various measures like probability of fresh call being successful, handover call being successful is studied. Various graphs and tables are constructed to demonstrate the data depicting sensitivity analysis of various factors on performance measures. **Applications**: The present work in itself is an application based study. This can be considered as an application of retrial queueing theory to mobile and communication networks. Moreover, in its original form it is applicable to telecommunication and mobile systems.

Keywords: Abandonment, Cellular Mobile Networks, Guard Channels, Handoff Calls, Spam Calls

1. Introduction

Advances in the technology of telecommunications have proved an unambiguous threat to the totalitarian regimes everywhere. The telecom industry has been growing enormously since the past years. From having standard telephones to walkie talkies to pagers and now the most useful device of the 21st century- the mobile phones. Nowadays, cell phones have become an integral part of our lives. Be it about talking to our near and dear ones or doing online shopping for groceries, these small handsets are the handiest of all and help in carrying out such activities making our lives much easier.

These wireless devices popularly known as mobile phones have proven to be one of the most outstanding technological and commercial successes of the last decade. Wireless is the hotspot of telecom. Research on wireless communication has been developing gradually,

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in particular on cellular mobile networks. Cellular networks primarily provide voice services and have been witnessing the most successful story in infrastructure networking for the last two decades. Typically, the base stations are deployed to provide ubiquitous coverage to all mobile nodes at allocations in the network, the base stations being so close so as to provide seamless coverage to all areas served by the network.

In contemporary times, providing services by means of telephones has been a common method adopted by most of the companies. In order to manage any service operation effectively, it is significant to achieve a proper balance between operational efficiency and service quality. Call centers are regarded as service networks where the agents provide services – online based or telephonic. They are considered as one stop services where the agents answer the queries and solve the complaints of the customers. Millions of agents are employed worldwide to serve as a primary customer-facing channel for firms in many industries. Many firms use call centers to communicate with their clients. A large volume of calls is answered daily, solving the problems of the customers.

The onset of cellular radio systems has generated remarkable growth in mobile communications. Abundant research has been done in this field and sufficient literature is available on the performance analysis of cellular mobile networks. There are abundant applications of mobile communications, working of call center being one of them. A standardized format for creating and maintaining call center databases has been provided which is independent of the data source, and subsequently stores information at the individual call level[1]. Brown along with others[2] gave a broad analysis of operational data from a bank call center wherein they made large efforts to use different tools for bettering the performance of call centers. Usually the arrival processes of calls are associated with a Poisson process, but they illustrated non-Poisson arrival phenomenon using real data. Apart from these, queuing theory has also been applied to discourse traffic problems at a toll plaza[3].

In order to improve the quality of service, it is desirable to reduce the blocking probabilities of new and handoff calls. In [4], a new approach to the study of a multi-channel cutoff priority cellular radio system is examined in which the number of calls are infinite and the queued calls never renege. As an alternative to this system[5] investigated a new cutoff priority cellular radio system with finite queuing of both new and handoff calls.

Various call handling schemes in cellular mobile networks are considered in order to improve the efficiency of calls. A few call handling/call management schemes[6] have also been studied based on comprehensive sharing of guard channels so as to improve the quality of service (QoS)[7]. It is believed that the network performance of cells can be improved by efficient call handling/call management mechanisms.

To augment the services provided via telephones in various companies, abandonment is one of the major factors which should be taken into consideration. A simple abandonment model has been analysed[8], in which customers' patience is exponentially distributed and the system's waiting capacity is unlimited. Whereas Zohar along with others[9] did modeling and analysis of abandonments from a queue that is covert to the occupiers i.e. the customers are unaware about any information regarding their queue size before entering for service. Apart from abandonment, balking is another factor to be taken into consideration[10].

The fact that the retrial rate depends on the number of retried calls waiting in the system makes the model intractable and so various numerical procedures have been used to compute the performance of the system[11]. Apart from retrials, work on call back option[12] and after-call work[13] has been done for the call centers wherein the system performance measures have been evaluated.

Handoff is one of the important factors of cellular and mobile communication. Analysis of the performance of handoff techniques is done and the various performance measures have been found out[14]. In contrast to this, [15] has assumed that both the handover as well as new calls have the same distribution but with different mean rates, and then found out an effective method for calculating the loss probabilities. Analysis of effective queuing system giving priority to handover calls is also taken into account[16]. For providing priorities to handover calls over fresh calls, different policies are discussed. The well-known policy is the so-called Guard Channel Policy[17]. A model with one guard channel in which only handover calls and retried fresh calls are allowed to enter has been considered[18]. No reattempts of handover calls are allowed.

In order to access various services, improved handover process is required. For this a VHO decision algorithm is designed that ensures that a mobile user is transferred to the desired network in order to improve the quality of service[19]. Apart from all of this, advance work in related fields have also been done[20–22].

This motivates us to consider a new model with two guard channels for which we explore new analytical and numerical results. From the modeling point of view, the novelty is the consideration of two guard channels and introduction of a 'b' factor called the spam factor which decides the entering and baring of calls to the handset. The rest of the paper is organized as follows. Section 2 deals with the narration of the model with several assumptions and notations. The governing equations are established in Section 3. The steady state probabilities of the system have been solved using Matrix Recursive Approach method in Section 4. Various performance measures have been obtained in Section 5. Numerical simulation has been done in Section 6. The conclusions are drawn in Section 7.

2. Model Description

A cell of mobile communication network is considered with two guard channels in which the arrivals of fresh calls are prohibited. The basic assumptions of the model are described as:

i. A model with guard channels is considered which gives emphasis to handover calls as compared to fresh calls.
ii. The number of customers generating calls in the cell are taken to be finite with m customers in total. There are n channels available for a customer where m > n. Out of these n channels two are declared as guard channels whose access is prohibited for the fresh calls.
iii. The arrival process of the customers is assumed to follow a poison process. It consists of two streams–fresh calls with rate λ_F and handoff calls with rate λ_H.
iv. An arriving call may find the server in one of the following states:
   - Idle state – Initially when a customer calls, it is in the idle state.
   - Active state – If a fresh call becomes successful, it enters the active state with the probability α. After completion of the service, it returns back to its idle state with rate μ.
   - Wait-for-reattempt state – If a customer initiates a call and all the channels are occupied, the customer enters the wait-for-reattempt state and waits till the call becomes active
v. A fresh call will retry with a probability θ. No reattempts of handoff calls are considered.
vi. Keeping in view the applications in smartphones these days, a factor 'b' say spam factor is taken which basically decided the entering and baring of calls to the handset. The 'b' factor is considered to take values only 0 and 1. If b = 1, then the incoming call will reach the user and it will be considered either as a fresh call. If b = 0, then the incoming call will not reach the user and will be regarded as a spam call.
vii. Two random variables are used to describe the present state of the model. The number of active customers Y (i.e. those customers which are actually occupying a channel) and the number of customers waiting for reattempt Z. The state probability is defined as:

   P (i, j) = Pr {Y=i, Z=j}

   Y = 0, 1… n; Z = 0, 1… q

   The different notations used to define the model are outlined below:

   λ_F : Arrival rate of fresh calls

   λ_H : Arrival rate of handover calls

   λ: Arrival rate of both fresh calls as well as handover call ($\lambda_H + \lambda_F$)

   θ: Retrial probability of fresh calls

   μ : Service rate for the customers

   b: Spam factor

The state transition diagram of the Cellular Mobile network has been represented in Figure 1.

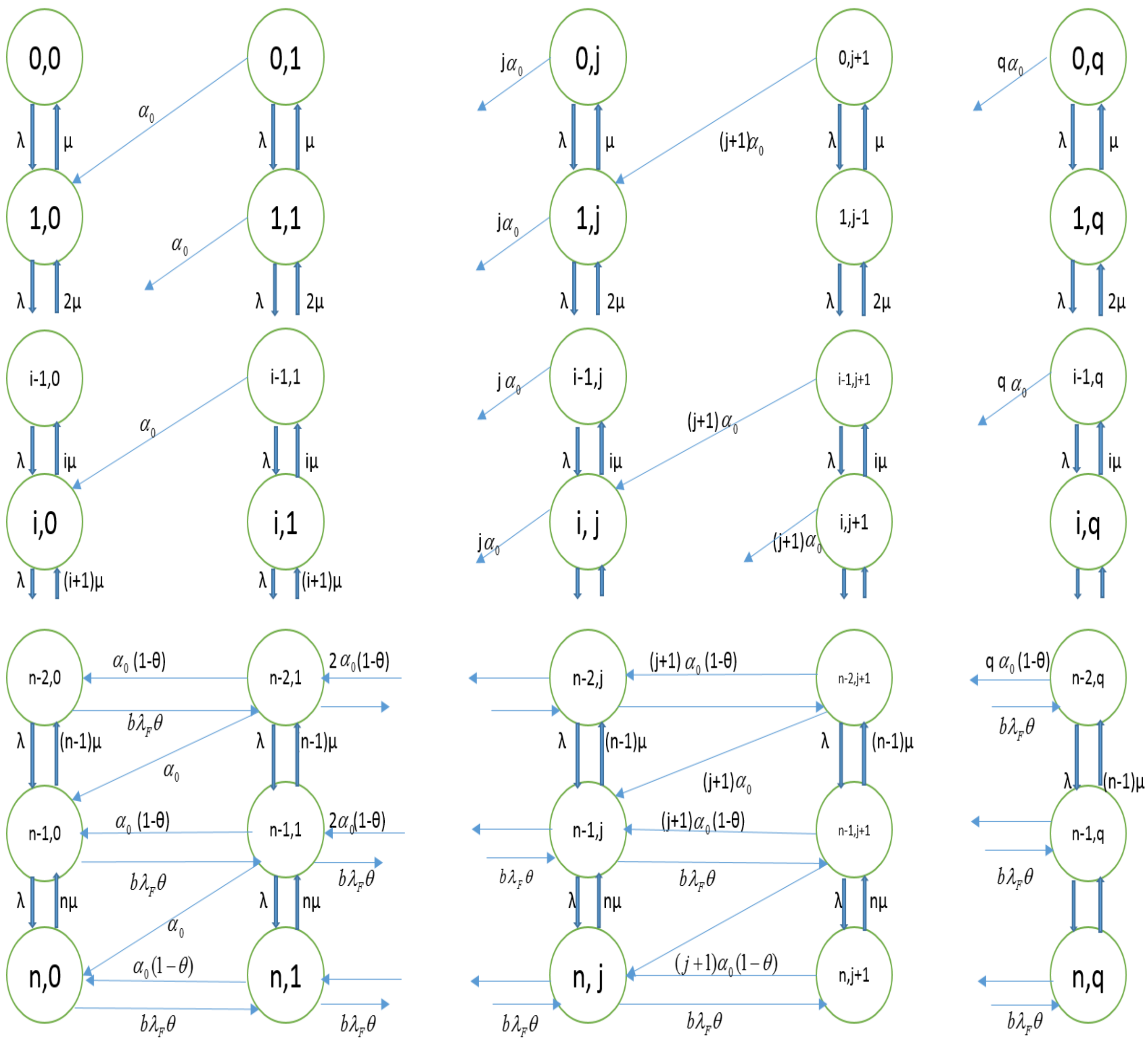

Figure 1. State transition diagram.

3. Governing Equations

The Chapman-Kolmogorov equations are constructed as written below.

At (0, 0)

$$\mu P_{1,0} = \lambda P_{0,0} \quad (3.1)$$

At (i, 0)

$$\lambda P_{i-1,0} + (i+1)\mu P_{i+1,0} + (\alpha_0)P_{i-1,1} = (i\mu)P_{i,0} + \lambda P_{i,0} \tag{3.2}$$

At (n-2, 0)

$$\lambda P_{n-3,0} + (n-1)\mu P_{n-1,0} + \alpha_0(1-\theta)P_{n-2,1} + (\alpha_0)P_{n-1,1} = (n-2)\mu P_{n-2,0} + \lambda_H P_{n-2,0} + (b\lambda_F\theta)P_{n-2,0} \tag{3.3}$$

At (n-1, 0)

$$\lambda_H P_{n-2,0} + (n\mu)P_{n,0} + \alpha_0 P_{n-2,1} + \alpha_0(1-\theta)P_{n-1,1} = (n-1)\mu P_{n-1,0} + \lambda_H P_{n-1,0} + (b\lambda_F\theta)P_{n-1,0} \tag{3.4}$$

At (n, 0)

$$\lambda_H P_{n-1,0} + \alpha_0(1-\theta)P_{n,1} + \alpha_0 P_{n-1,1} = (n\mu)P_{n,0} + (b\lambda_F\theta)P_{n,0} \tag{3.5}$$

At (i, j)

$$\lambda P_{i-1,j} + (i+1)\mu P_{i+1,} + (j+1)\alpha_0 P_{i-1,j+1} = (i\mu)P_{i,j} + \lambda P_{i,j} + (j\alpha_0)P_{i,j} \tag{3.6}$$

At (n-2, j)

$$\begin{aligned} &\lambda P_{n-3,j} + (n-1)\mu P_{n-1,j} + (b\lambda_F\theta)P_{n-2,j-1} + (j+1)\alpha_0(1-\theta)P_{n-2,j+1} + (j+1)\alpha_0 P_{n-3,j+1} = \\ &(n-2)\mu P_{n-2,j} + (b\lambda_F\theta)P_{n-2,j} + \lambda_H P_{n-2,j} + j\alpha_0(1-\theta)P_{n-2,j} + (j\alpha_0)P_{n-2,j} \end{aligned} \tag{3.7}$$

At (0, j)

$$\mu P_{1,j} = \lambda P_{0,j} + (j\alpha_0)P_{0,j} \tag{3.8}$$

At (n-1, j)

$$\lambda_H P_{n-2,j} + (j+1)\alpha_0(1-\theta)P_{n-1,j+1} + n\mu P_{n,j} + (b\lambda_F\theta)P_{n-1,j-1} + (j+1)\alpha_0 P_{n-2,j+1} = (n-1)\mu P_{n-1,j} + (b\lambda_F\theta)P_{n-1,j} + \lambda_H P_{n-1,j} + (j\alpha_0)P_{n-1,j} + j\alpha_0(1-\theta)P_{n-1,j} \tag{3.9}$$

At (n, j)

$$\lambda_H P_{n-1,j} + (j+1)\alpha_0 P_{n-1,j+1} + (j+1)\alpha_0(1-\theta)P_{n,j+1} + (b\lambda_F\theta)P_{n,j-1} = n\mu P_{n,j} + (b\lambda_F\theta)P_{n,j} + j\alpha_0(1-\theta)P_{n,j} \tag{3.10}$$

At (0, q)

$$\mu P_{1,q} = (q\alpha_0)P_{0,q} + \lambda P_{0,q} \tag{3.11}$$

At (i, q)

$$\lambda P_{i-1,q} + (i+1)\mu P_{i+1,q} = (i\mu)P_{i,q} + \lambda P_{i,q} + (q\alpha_0)P_{i,q} \tag{3.12}$$

At (n-2, q)

$$\lambda P_{n-3,q} + (n-1)\mu P_{n-1,q} + (b\lambda_F\theta)P_{n-2,q-1} = (n-2)\mu P_{n-2,q} + \lambda_H P_{n-2,q} + q\alpha_0(1-\theta)P_{n-2,q} + (q\alpha_0)P_{n-2,q} \tag{3.13}$$

At (n-1, q)

$$\lambda_H P_{n-2,q} + (b\lambda_F\theta)P_{n-1,q-1} + (n\mu)P_{i-1,q} = q\alpha_0(1-\theta)P_{n-1,q} + (n-1)\mu P_{n-1,q} + \lambda_H P_{n-1,q} + (q\alpha_0)P_{n-1,q} \tag{3.14}$$

At (n, q)

$$\lambda_H P_{n-1,q} + (b\lambda_F\theta)P_{n,q-1} = (n\mu)P_{n,q} + q\alpha_0(1-\theta)P_{n,q} \tag{3.15}$$

Normalizing condition is

$$\sum_{j=0}^{q}\sum_{i=0}^{n} P(i,j) = 1 \tag{3.16}$$

4. Matrix Recursive Approach

To obtain the steady state probabilities of the system, we use the "Matrix Recursive Approach". The corresponding transition rate matrix Q of this Markov chain has the blocking tridiagonal structure. The set of equations (3.1) -(3.15) can be written in matrix form as:

$$Q\prod = \prod(0) \qquad (4.1)$$

$$\begin{bmatrix}
A & G & 0 & \cdots & & & \cdots & & & \cdots & & & 0 \\
B & C_1 & D_1 & & & & & & & & & & \vdots \\
0 & B & C_2 & D_2 & & & & & & & & & \\
\vdots & & B & C_3 & D_3 & & & & & & & & \\
 & & & B & C_4 & D_4 & & & & & & & \vdots \\
 & & & & & \vdots & \ddots & & & & & & \\
 & & & & & B & C_{i-1} & D_{i-1} & & & & & \\
\vdots & & & & & & B & C_i & D_i & & & & \\
 & & & & & & & B & C_{i+1} & D_{i+1} & & & \vdots \\
 & & & & & & & & & \vdots & \ddots & & \\
 & & & & & & & & & F & E_{n-2} & D_{n-2} & 0 \\
 & & & & & & & & & & F & E_{n-1} & D_{n-1} \\
0 & \cdots & & & & & \cdots & & & & 0 & F & L
\end{bmatrix}
\begin{bmatrix}
\prod_1 \\ \prod_2 \\ \prod_3 \\ \prod_4 \\ \prod_5 \\ \vdots \\ \prod_i \\ \prod_{i+1} \\ \prod_{i+2} \\ \vdots \\ \prod_{n-1} \\ \prod_n \\ \prod_{n+1}
\end{bmatrix}
\begin{bmatrix}
\mathbf{O} \\ \mathbf{O} \\ \mathbf{O} \\ \mathbf{O} \\ \mathbf{O} \\ \mathbf{O} \\ \mathbf{O} \\ \mathbf{O} \\ \mathbf{O} \\ \mathbf{O} \\ \mathbf{O} \\ \mathbf{O} \\ \mathbf{O}
\end{bmatrix}$$

where,

$$Q = \begin{bmatrix}
A & G & 0 & \cdots & & & \cdots & & & \cdots & & & 0 \\
B & C_1 & D_1 & & & & & & & & & & \vdots \\
0 & B & C_2 & D_2 & & & & & & & & & \\
\vdots & & B & C_3 & D_3 & & & & & & & & \\
 & & & B & C_4 & D_4 & & & & & & & \vdots \\
 & & & & & \vdots & \ddots & & & & & & \\
 & & & & & B & C_{i-1} & D_{i-1} & & & & & \\
\vdots & & & & & & B & C_i & D_i & & & & \\
 & & & & & & & B & C_{i+1} & D_{i+1} & & & \vdots \\
 & & & & & & & & & \vdots & \ddots & & \\
 & & & & & & & & & F & E_{n-2} & D_{n-2} & 0 \\
 & & & & & & & & & & F & E_{n-1} & D_{n-1} \\
0 & \cdots & & & & & \cdots & & & & 0 & F & L
\end{bmatrix}$$

$$A = \begin{bmatrix} -\lambda & 0 & 0 & \cdots & \cdots & 0 \\ 0 & -\lambda-\alpha_0 & 0 & & & \\ 0 & 0 & \ddots & & & \vdots \\ & & -\lambda-(j-1)\alpha_0 & & & \\ \vdots & & 0 & -\lambda-j\alpha_0 & & 0 \\ & & & 0 & -\lambda-(j+1)\alpha_0 & \\ \vdots & & & & \ddots & \vdots \\ & & & & -\lambda-(q-1)\alpha_0 & 0 \\ 0 & 0 & \cdots & \cdots & & -\lambda-q\alpha_0 \end{bmatrix}$$

$$B = \begin{bmatrix} \lambda & \alpha_0 & 0 & \cdots & \cdots & 0 \\ 0 & \lambda & 2\alpha_0 & & & 0 \\ & \ddots & \ddots & & & \\ \vdots & & \lambda & (j-1)\alpha_0 & & \\ & & & \lambda & j\alpha_0 & \vdots \\ & & & & \lambda \quad (j+1)\alpha_0 & \\ \vdots & & & \ddots & \ddots & \vdots \\ & & & & \lambda \quad (q-1)\alpha_0 & 0 \\ 0 & & & & \lambda & q\alpha_0 \end{bmatrix}$$

$$C_i = \begin{bmatrix} i\mu-\lambda & 0 & \cdots & & \cdots & 0 \\ 0 & i\mu-\lambda-\alpha_0 & & & & \\ 0 & & \ddots & & & \vdots \\ \vdots & & i\mu-\lambda-(j-1)\alpha_0 & & & \\ & & & i\mu-\lambda-j\alpha_0 & & \vdots \\ & & & & i\mu-\lambda-(j+1)\alpha_0 & \\ \vdots & & & & \ddots & 0 \\ & & & & i\mu-\lambda-(q-1)\alpha_0 & 0 \\ 0 & \cdots & & \cdots & & i\mu-\lambda-q\alpha_0 \end{bmatrix} \quad (1 \le i \le n-3)$$

$$D_i = \begin{bmatrix} i\mu & 0 & \cdots & & & & 0 \\ 0 & i\mu & & & & & \\ 0 & & \ddots & & & & \vdots \\ \vdots & & & i\mu & & & \\ & & & & i\mu & & \vdots \\ \vdots & & & & & i\mu & \\ & & & & & \ddots & 0 \\ 0 & \cdots & & & \cdots & & i\mu \end{bmatrix} \qquad (1<= i<=n-1)$$

$$F= \begin{bmatrix} \lambda_H & \alpha_0 & 0 & \cdots & & \cdots & & 0 \\ 0 & \lambda_H & 2\alpha_0 & & & & & 0 \\ & \ddots & & \ddots & & & & \\ \vdots & & \lambda_H & & j\alpha_0 & & & \\ & & & \lambda_H & & (j+1)\alpha_0 & & \vdots \\ & & & & \lambda_H & & & \\ \vdots & & & & & \ddots & \ddots & \vdots \\ & & & & & & \lambda_H & q\alpha_0 \\ 0 & & \cdots & & & \cdots & & \lambda_H \end{bmatrix}$$

$$L = \begin{bmatrix} -n\mu - b\lambda_F\theta & \alpha_0(1-\theta) & 0 & \cdots & \cdots & 0 \\ b\lambda_F\theta & \begin{pmatrix} -n\mu - b\lambda_F\theta \\ -\alpha_0(1-\theta) \end{pmatrix} & 2\alpha_0(1-\theta) & & & \vdots \\ 0 & \ddots & \ddots & \ddots & & \\ \vdots & b\lambda_F\theta & \begin{pmatrix} -n\mu - b\lambda_F\theta \\ (j-1)\alpha_0(1-\theta) \end{pmatrix} & j\alpha_0(1-\theta) & & \vdots \\ & & b\lambda_F\theta & \begin{pmatrix} -n\mu - b\lambda_F\theta - \\ j\alpha_0(1-\theta) \end{pmatrix} & (j+1)\alpha_0(1-\theta) & 0 \\ \vdots & & & \ddots & \ddots & \ddots \\ & & & b\lambda_F\theta & \begin{pmatrix} -n\mu - (q-1)\alpha_0(1-\theta) \\ -b\lambda_F\theta \end{pmatrix} & q\alpha_0(1-\theta) \\ 0 & \cdots & \cdots & & b\lambda_F\theta & \begin{pmatrix} -n\mu - \\ q\alpha_0(1-\theta) \end{pmatrix} \end{bmatrix}$$

$$G = \begin{bmatrix} \mu & 0 & \cdots & & & & 0 \\ 0 & \mu & & & & & \\ 0 & & \ddots & & & & \vdots \\ & & & \mu & & & \\ \vdots & & & & \mu & & \vdots \\ & & & & & \mu & \\ & & & & & \ddots & 0 \\ 0 & & & & & & \mu \end{bmatrix}$$

$$E_i = \begin{bmatrix} -i\mu - b\lambda_F\theta - \lambda_H & \alpha_0(1-\theta) & 0 & \cdots & \cdots & 0 \\ b\lambda_F\theta & \begin{pmatrix} -i\mu - b\lambda_F\theta - \lambda_H \\ -\alpha_0(1-\theta) - \alpha_0 \end{pmatrix} & 2\alpha_0(1-\theta) & & & \vdots \\ 0 & \ddots & \ddots & \ddots & & \\ \vdots & b\lambda_F\theta & \begin{pmatrix} -i\mu - b\lambda_F\theta - \lambda_H \\ (j-1)\alpha_0(1-\theta) - \\ (j-1)\alpha_0 \end{pmatrix} & j\alpha_0(1-\theta) & & \vdots \\ & & b\lambda_F\theta & \begin{pmatrix} -i\mu - b\lambda_F\theta - \lambda_H \\ -j\alpha_0(1-\theta) - j\alpha_0 \end{pmatrix} & (j+1)\alpha_0(1-\theta) & 0 \\ \vdots & & \ddots\; C_i = & \ddots & \ddots & \\ & & & b\lambda_F\theta & \begin{pmatrix} -i\mu - (q-1)\alpha_0(1-\theta) \\ -b\lambda_F\theta - \lambda_H - (q-1)\alpha_0 \end{pmatrix} & q\alpha_0(1-\theta) \\ 0 & \cdots & \cdots & & b\lambda_F\theta & \begin{pmatrix} -i\mu - \lambda_H - q\alpha_0 \\ -q\alpha_0(1-\theta) \end{pmatrix} \end{bmatrix}$$

where, (n-2 $\leq$ i $\leq$ n-1)

All the sub-matrices are of order (q+1,q+1)
The vector $\prod$ is defined as:

$$\prod\nolimits_1 = \left(P_{0,0}, P_{0,1}, \cdots\cdots, P_{0,q}\right)$$

$$\prod\nolimits_2 = \left(P_{1,0}, P_{1,1}, \cdots\cdots, P_{1,q}\right)$$

$$\prod\nolimits_3 = \left(P_{2,0}, P_{2,1}, \cdots\cdots, P_{2,q}\right)$$

$$\prod\nolimits_4 = \left(P_{3,0}, P_{3,1}, \cdots\cdots, P_{3,q}\right)$$

$$\prod\nolimits_5 = \left(P_{4,0}, P_{4,1}, \cdots\cdots, P_{4,q}\right)$$

$$\prod\nolimits_{i-1} = \left(P_{i-1,0}, P_{i-1,1}, \cdots\cdots, P_{i-1,q}\right)$$

$$\prod\nolimits_{i} = \left(P_{i,0}, P_{i,1}, \cdots\cdots, P_{i,q}\right)$$

$$\prod\nolimits_{i+1} = \left(P_{i+1,0}, P_{i+1,1}, \cdots\cdots, P_{i+1,q}\right)$$

$$\prod\nolimits_{n-2} = \left(P_{n-2,0}, P_{n-2,1}, \cdots\cdots, P_{n-2,q}\right)$$

$$\prod_{n-1} = \left(P_{n-1,0}, P_{n-1,1}, \cdots\cdots, P_{n-1,q}\right)$$

$$\prod_{n} = \left(P_{n,1}, P_{n,2}, \cdots\cdots, P_{n,q}\right)$$

Therefore, we have the following set of equations:

$$A\prod_{1} + G\prod_{2} = 0 \quad (4.2)$$

$$B\prod_{i-1} + C_{i-1}\prod_{i} + D_{i-1}\prod_{i+1} = 0,$$

$$(2 \le i \le n-3) \quad (4.3)$$

$$F\prod_{n-2} + E_{n-2}\prod_{n-1} + D_{n-2}\prod_{n} = 0 \quad (4.4)$$

$$F\prod_{n-1} + E_{n-1}\prod_{n} + D_{n-1}\prod_{n+1} = 0 \quad (4.5)$$

$$F\prod_{n} + L\prod_{n+1} = 0 \quad (4.6)$$

On solving the equations (4.2) -(4.6), we get the following steady state solutions:

$$\prod_{2} = \frac{-A\prod_{1}}{G} \quad (4.7)$$

$$\prod_{3} = \frac{(-BG + C_1 A)\prod_{1}}{D_1 G} \quad (4.8)$$

$$\prod_{4} = \frac{(D_1 BA - C_2 C_1 A + C_2 BG)\prod_{1}}{D_1 D_2 G} \quad (4.9)$$

$$\prod_5 = \frac{\left(D_2B^2G - D_2BC_1A - C_3D_1BA + C_3C_2C_1A - C_2C_3BG\right)\prod_1}{D_1D_2D_3G} \quad 4.10)$$

$$\prod_{i+1} = \frac{\left(-C_{i-1}\prod_i - B\prod_{i-1}\right)}{D_{i-1}} \quad (4.11)$$

$$\prod_{i+2} = \frac{\left((C_iC_{i-1} - D_{i-1}B)\prod_i + C_iB\prod_{i-1}\right)}{D_{i-1}D_i} \quad (4.12)$$

$$\prod_{n+1} = \frac{-F\prod_n}{L} \quad (4.13)$$

$$\prod_n = \frac{-E\prod_{n-1}}{E_{n-1} - D_{n-1}F} \quad (4.14)$$

$$\prod_{n-1} = \frac{\left(-E^2E_{n-1} + F^2D_{n-1}\right)\prod_{n-2}}{E_{n-2}E_{n-1} - D_{n-1}E_{n-2}F - D_{n-2}E} \quad (4.15)$$

$$\prod_{n-2} = \frac{\left(-BLE_{n-2}E_{n-1} + BFD_{n-1}E_{n-2} + BLFD_{n-2}\right)\prod_{n-3}}{C_{n-3}E_{n-2}E_{n-1} - D_{n-1}E_{n-2}C_{n-3} - LFD_{n-2}C_{n-3} - E^2D_{n-3}E_{n-1} + E^2D_{n-3}D_{n-1}} \quad (4.16)$$

From the normalizing condition, we have

$$\left[\left(P_{0,0}, P_{0,1}, \cdots, P_{0,q}\right)e_1 + \left(P_{1,0}, P_{1,1}, \cdots, P_{1,q}\right)e_2 + \cdots\cdots + \left(P_{i,0}, P_{i,1}, \cdots, P_{i,q}\right)e_i + \cdots\cdots + \left(P_{n,0,}, P_{n,1}, \cdots, P_{n,q}\right)e_n\right] = 1 \quad (4.17)$$

On simplifying eq. (4.17), we get

$$\left[\prod_1 e_1 + \prod_2 e_2 + \ldots + \prod_i e_i + \cdots + \prod_n e_n\right] = 1 \quad (4.18)$$

From eq. (4.7) -(4.16) and eq. (4.18), we can recursively find the value of $\prod_1$.

5. Performance Measures

Some of the performance measures are as follows:

- Probability of a fresh call being successful is framed as:

$$P_{FS} = b\lambda \sum_{i=0}^{n-3} \sum_{j=0}^{q} (m-i-j)P(i,j) \tag{5.1}$$

- Probability of a fresh call being unsuccessful is framed as:

$$P_{FU} = b\theta\lambda \sum_{j=0}^{q} (m-n+3-j)P(n-3,j) \tag{5.2}$$

- Probability of a fresh call being blocked or abandoned is framed as:

$$P_{FB} = b(1-\theta)\lambda \sum_{j=0}^{q} (m-n-j)P(n,j) \tag{5.3}$$

- Probability of a repeated call being successful is framed as:

$$P_{RS} = \alpha_0 \sum_{i=0}^{n-1} \sum_{j=0}^{q} jP(i,j) \tag{5.4}$$

- Probability of a repeated call being unsuccessful is framed as:

$$P_{RU} = \theta\alpha_0 \sum_{j=0}^{q} jP(n,j) \tag{5.5}$$

- Probability of a repeated call being blocked or abandoned is framed as:

$$P_{RB} = (1-\theta)\alpha_0 \sum_{j=0}^{q} jP(n,j) \tag{5.6}$$

- Probability of a handover call being successful is framed as:

$$P_{HS} = \lambda_H \sum_{i=0}^{n-1} \sum_{j=0}^{q} (m-i-j)P(i,j) \tag{5.7}$$

- Probability of a handover call being unsuccessful is framed as:

$$P_{HU} = \theta\lambda_H \sum_{j=0}^{q} (m-n-j)P(n,j) \tag{5.8}$$

- Probability of a handover call being blocked or abandoned is framed as:

$$P_{HB} = (1-\theta)\lambda_H \sum_{j=0}^{q} (m-n-j)P(n,j) \tag{5.9}$$

6. Numerical Analysis

In this section, we provide numerical results of the steady state equations framed in (3.1) -(3.15). Efficiency of a particular model can be best described by its numerical analysis. MATLAB software has been used to find the results. The effect on performance measures by various parameters have been shown by means of tables and graphs. The set of default parameters is taken as λ_F =1, λ_H =1, θ =0.2, α_0 =0.5, μ =0.7, and b =1.

6.1 Effect of Retrial Rate on Various Performance Measures

The effect of retrial behavior over different performance measures have been evaluated. Figure 2 - Figure 4 are plotted to examine the sensitiveness of the retrial behavior towards fresh calls being successful (P_{FS}) and fresh calls being blocked or abandoned.

In Figure 2 by taking λ_H =0.2, μ =0.2, α_0 =0.3, m=50, a relationship between retrial rate (θ) and fresh calls has been developed. As θ increases, the probability of a fresh call being successful decreases. The effect on the abandonment and blocking of handover calls is shown in Figure 3. As the retrial rate increases, the probability of a call being abandoned decreases. The Figure 4 shows the impact of retrials on repeated calls. As the retrial rate increases, the probability of a repeated call being successful also increases.

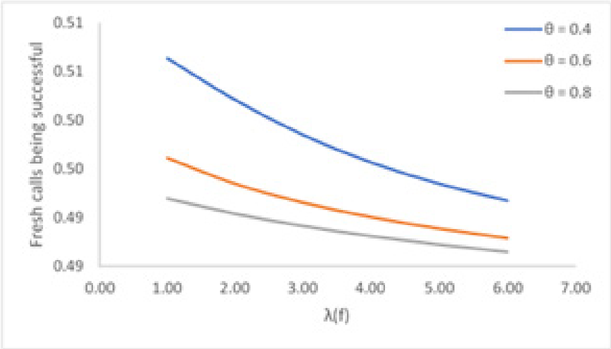

Figure 2. Impact of θ on P_{FS}.

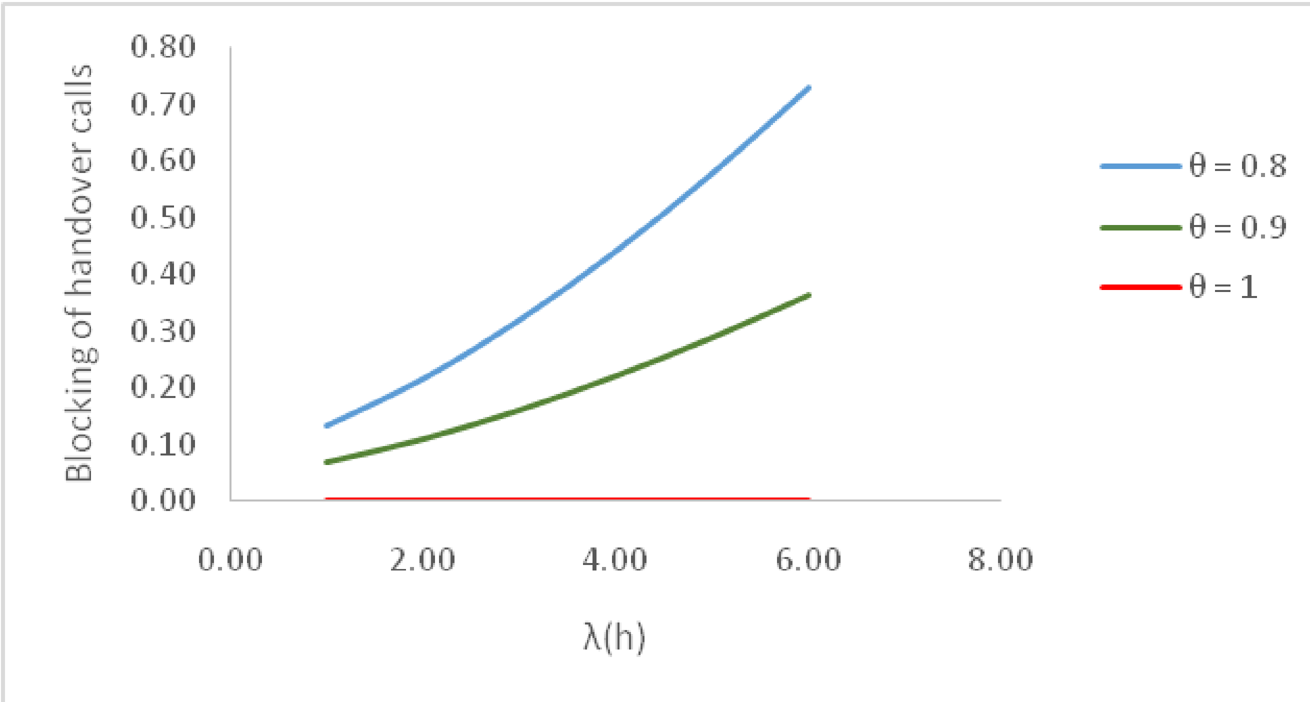

Figure 3. Impact of θ on P_{HB}.

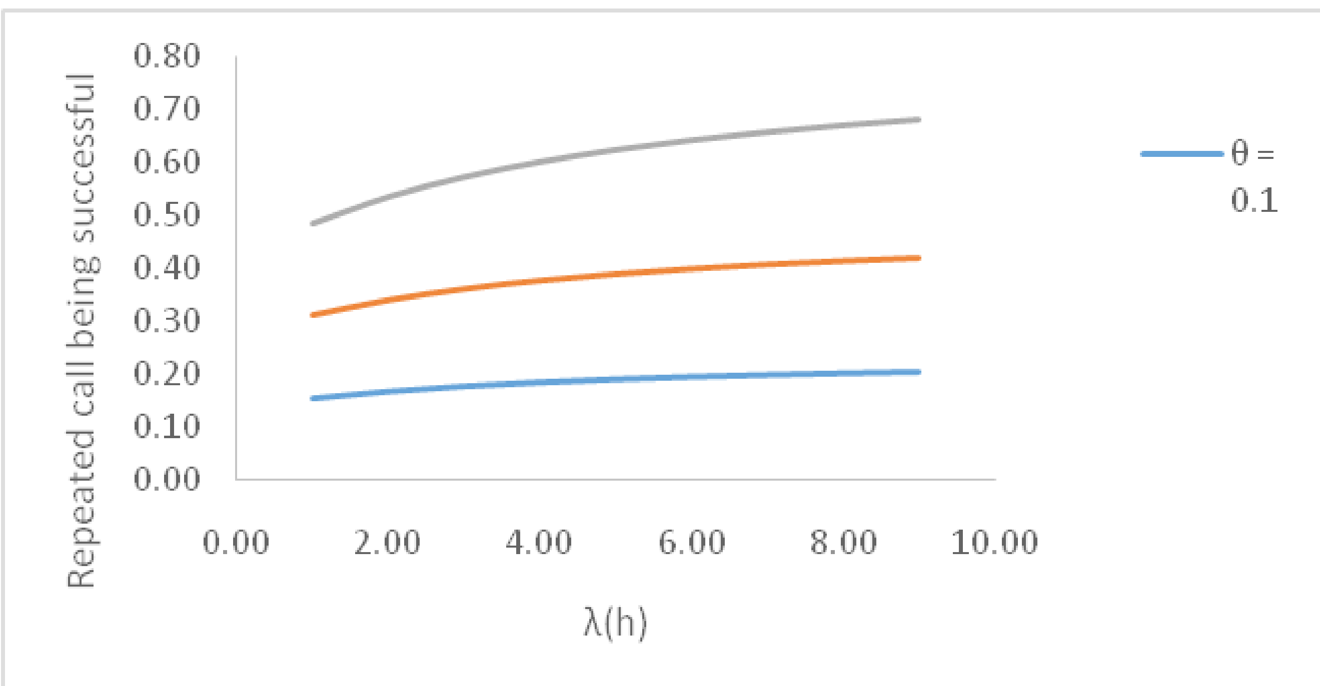

Figure 4. Impact of θ on P_{RS}.

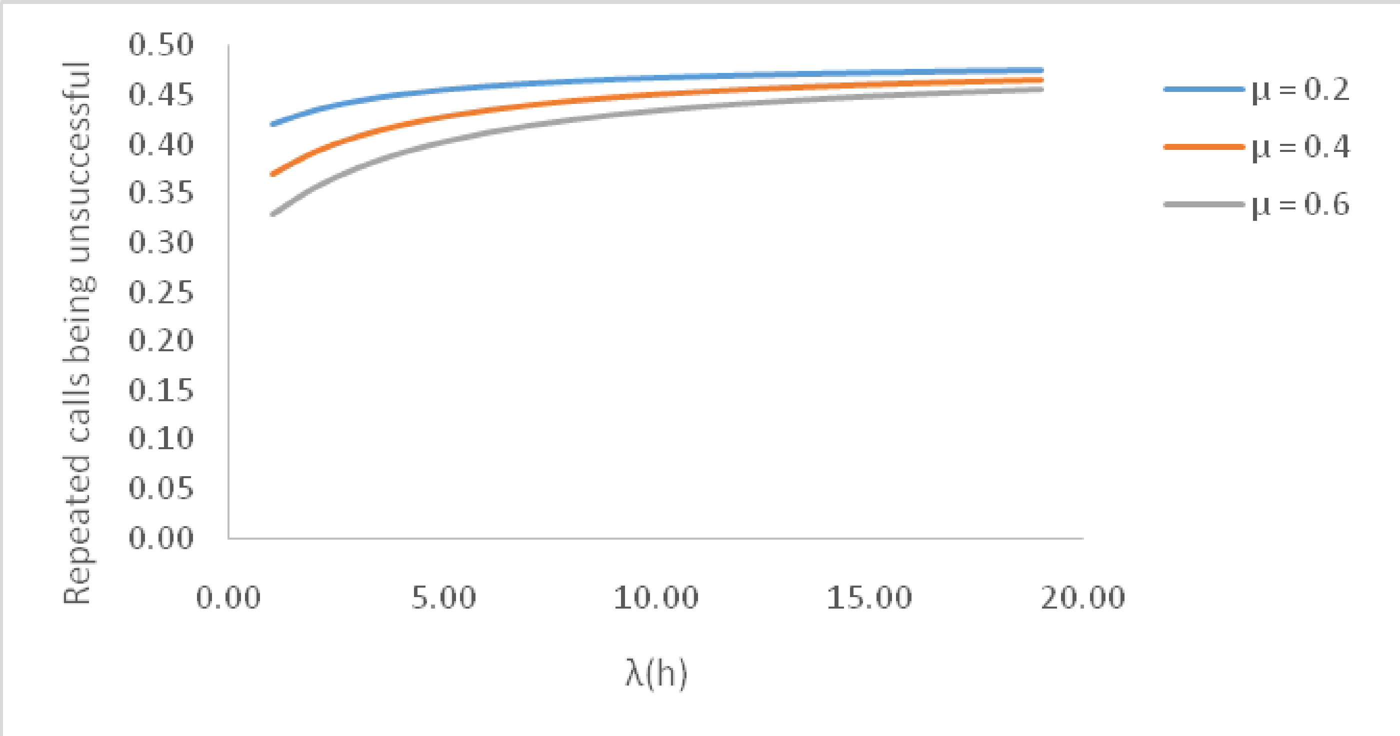

Figure 5. Impact of μ on P_{RU}.

6.2 Effect of Service Rate on Various Performance Measures

The effectiveness and performance of a cellular mobile model can be judged by its service rate i.e. the rate at which the customers are served. Higher the service rate, more calls can be answered hence improving the QoS. In Figure 5, the effect of service rate on repeated calls is examined. As the service rate increases, the probability of the repeated calls being unsuccessful decreases, which is quite obvious.

Table 1 shows the effect of service rate on fresh calls and handover calls. The service rate has been taken as μ = 0.1, 0.2, 0.3 and other parameters are taken as θ=0.5, α_0=0.9, m=50. It is clear from Table1 that as the service rate increases, the probability of a call to be successful also increases. On the other hand, we see that as the arrival rates of fresh calls as well as hand over calls increases, the probability of a handover call being successful is more as compared to the fresh calls indicating that higher priority is given to handover calls.

Table 2 shows the effect of service rate on abandonment of fresh calls and handover calls. The service rate has been taken as μ = 0.7, 0.8, 0.9 and other parameters are taken to be θ=0.9, α_0=0.1, m=50. We can see from Table 2 that as the service rate increases, the probability of call to be abandoned decreases. Moreover, as the arrival rate of fresh calls and handover calls increases, the prob-

Table 1. Impact of μ on P_{FS} and P_{HS}

P_{FS}				P_{HS}			
λ_F	$\mu = 0.7$	$\mu = 0.8$	$\mu = 0.9$	λ_H	$\mu = 0.7$	$\mu = 0.8$	$\mu = 0.9$
1	0.0066	0.0508	0.1644	1	0.2286	0.4747	0.7287
2	0.0067	0.0516	0.1655	2	0.2472	0.5130	0.7886
3	0.0068	0.0522	0.1678	3	0.2624	0.5440	0.8369
4	0.0069	0.0526	0.1687	4	0.2751	0.5696	0.8765
5	0.0070	0.0529	0.1693	5	0.2858	0.5910	0.9094
6	0.0070	0.0533	0.1698	6	0.2949	0.6091	0.9371

Table 2. Impact of μ on P_{FB} and P_{HB}

P_{FB}				P_{HB}			
λ_F	$\mu = 0.7$	$\mu = 0.8$	$\mu = 0.9$	λ_H	$\mu = 0.7$	$\mu = 0.8$	$\mu = 0.9$
1	0.4051	0.3574	0.3097	1	0.3096	0.2829	0.2586
2	0.5108	0.4646	0.4191	2	0.4054	0.3735	0.3441
3	0.6136	0.5679	0.5240	3	0.5073	0.4709	0.4370
4	0.7146	0.6687	0.6256	4	0.6136	0.5736	0.5359
5	0.8143	0.7677	0.7248	5	0.7231	0.6801	0.6396
6	0.9133	0.8655	0.8224	6	0.8351	0.7896	0.7461

ability of a handover calls to be abandoned is less as compared to that of a fresh call.

6.3 Effect of 'b' Factor on Various Performance Measures

The effect of factor 'b' in smart phones is displayed by means of Figure 6. We have taken μ =0.2 and m=50. It shows the impact of b on the fresh calls being served. We take the values of b to be 0 and 1. When b = 0, we see that the probability of a fresh call being answered is 0, which means that a fresh call will not reach the server and hence will be blocked. On the other hand, when b = 1, we see that the probability is comparatively high resulting in the fresh calls being answered.

6.4 Influence on the Population Size

As we consider our population size to be finite, we analyse the effect of population size on the performance measures. By taking θ =0.2, μ =0.2, α_0 =0.5, in Figure 7, we have examined the impact of population size on the handover calls. As the population size increases, the probability of a handover call being unsuccessful also increases, indicating that with infinite population size, the QoS of the model is most likely to worsen.

6.5 Effect of α_0 on Various Performance Measure

In Figures 7 - Figure 9 the impact on repeated calls and handover calls is illustrated. In Figure 8, the abandon-

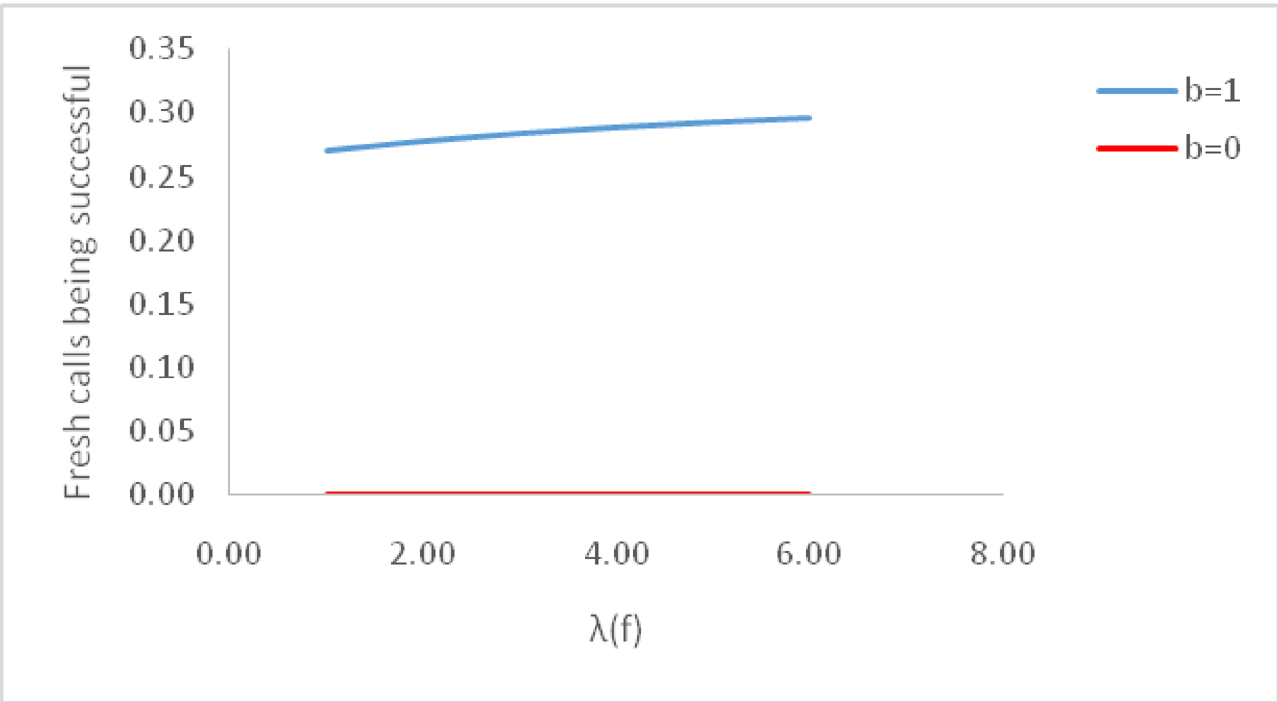

Figure 6. Impact of b on P_{FS}.

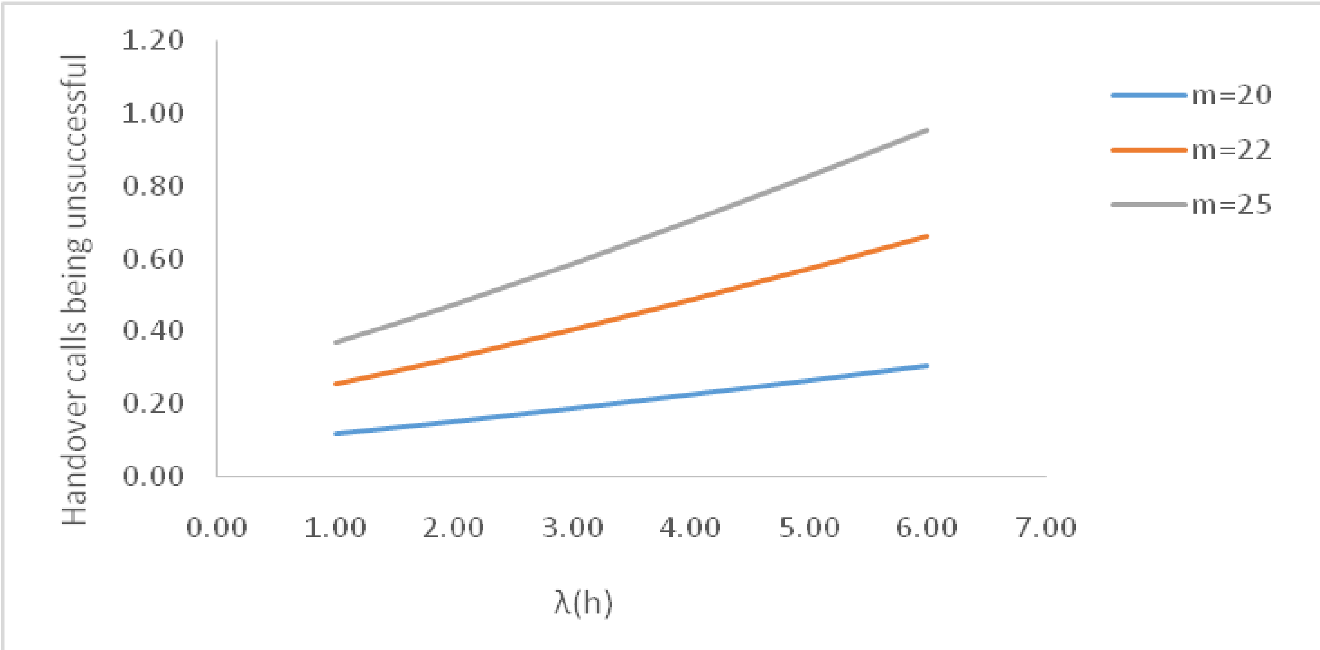

Figure 7. Impact of m on P_{HS}.

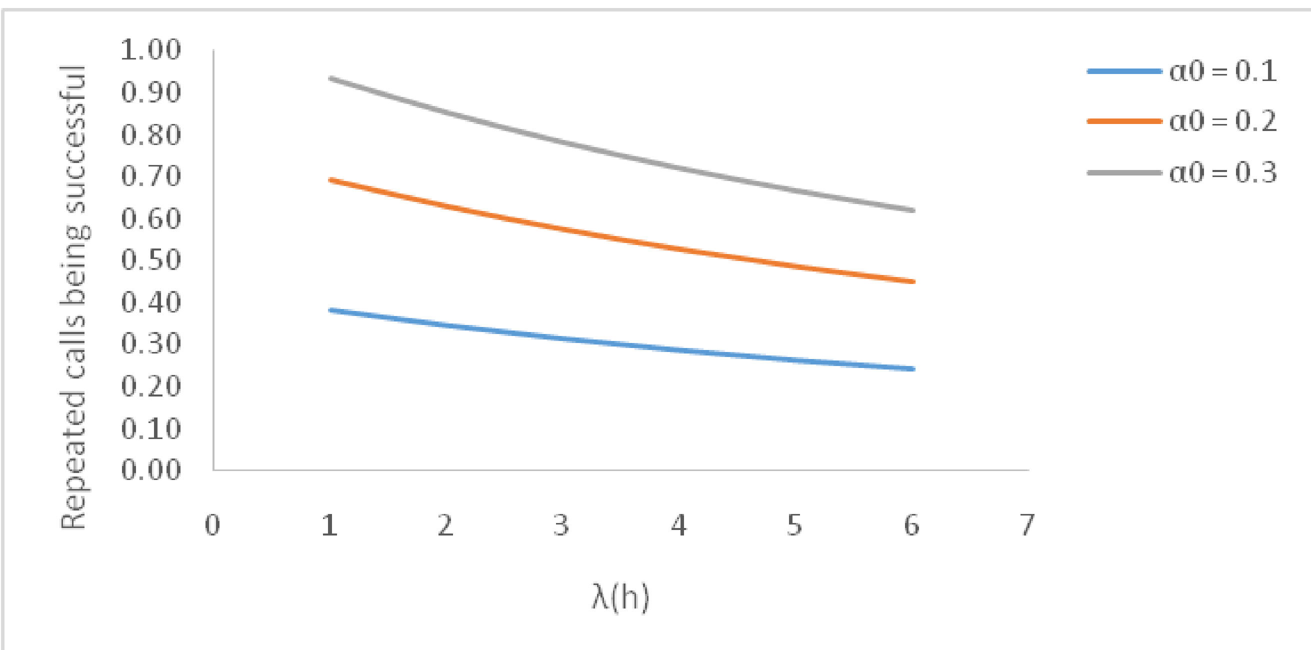

Figure 8. Impact of on P_{RS}.

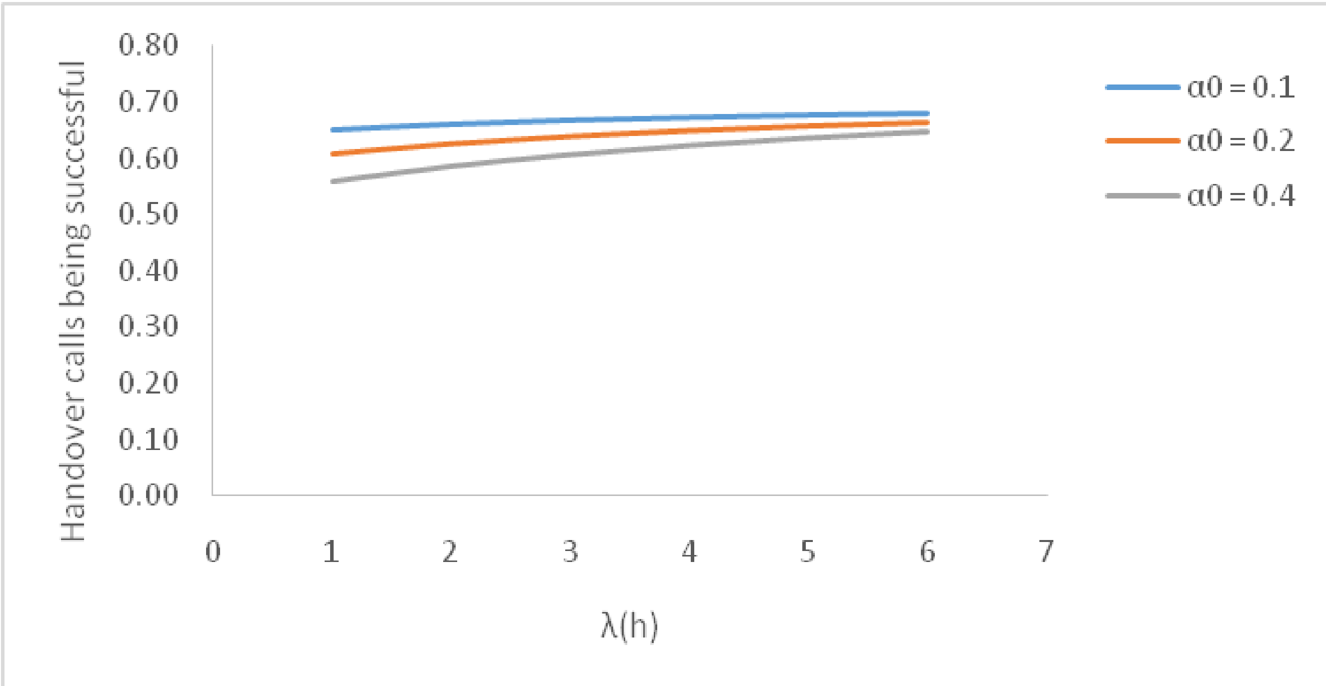

Figure 9. Impact of α_0 on P_{HS}.

ment rates are taken to be α_0 =0.1, 0.2, 0.3. It is clear from the graph that as α_0 increases, the probability of a repeated call being successful also increases. In Figure 9, the α_0 is taken to be 0.1, 0.2, 0.4 and other parameters as μ = 0.1, m=25. It is clear from the figure that as the α_0 increases, the probability of a handover call to be successful reduces.

7. Conclusions

A cellular mobile model has been investigated wherein two guard channels have been taken. A 'b' factor known as the spam factor is introduced which decides the entering or barring of calls to the handset. Moreover, some performance measures are formed so as to analyze the effect of different parameters on them. We saw that with a rise in the retrial rate, the probability of fresh calls being successful was very less. On the other hand, an increase in the retrial behavior led to a decrease in the blocking of handover calls. Also, an effect of service rate on handover calls and fresh calls was determined. As the service rate increased, the probability of a fresh call and a handover call to be successful also increased. An effect of the number of customers generating a call on fresh calls was determined.

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