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PSO and DE Based Model Order Reduction and Discrete Time Structure Specified Controller Design

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Abstract

Background/Objectives: The objective is to reduce the order of the plant and design discrete time PID controller using Particle Swarm Optimization (PSO) and Differential Evolution. **Methods/Statistical analysis**: Two popular optimization techniques – PSO and Differential Evolution (DE) are used to obtain the reduced order model of the physical plant and its discrete time structure specified PID controller. The objective function used for model order reduction is an integral square error between outputs of original and reduced order model. Whereas, the controller designs, integral square error between output and reference signal is chosen for tracking purpose. **Findings:** Nominal models for physical systems are highly complex and their order is usually very high. Designing controllers for such models in industries require huge amount of cost and large number of hardware components. The optimization techniques will prevent this issue, as shown in this paper. The optimal reduced model and optimal controllers are extremely useful for the industrial applications. **Application/Improvements:** In this paper, this methodology is proposed to design discrete time controller for a process of eight orders. But, this approach can be effectively applied to the complex plants for implementation of controller.

Keywords: DE Optimization, Discrete Time Systems, Order Reduction, PSO Optimization, Transfer Function

1. Introduction

Basically, control systems are broadly classified into two types: Continuous time and Discrete time Systems. Discrete time control systems contain both continuous and discrete signals; whereas, continuous time systems deal with only continuous time signals. But, with the advent of digital computers, the digital processors are replacing analog processors. Wide flexible and complex control algorithms can easily be implemented in digital controllers. The analog processors face with the problem of rewiring of hardware components in case of changes in system parameters; which is not the case of digital processor, as it requires only reprogramming. Researchers have discovered various types of digital controller techniques. Some of the techniques are presented here. Robust controllers, since 1950's are used to make the system robustly stable, inspite of parametric variations, disturbances and noise signals. RST controllers are the polynomial two degree of freedom controllers to achieve tracking and regulation independently. Adaptive controllers have also become popular for the online control of the process using complex adaptation algorithms. The most widely used controllers in industries are PID controllers. The PID controllers (Proportional-Integral-Derivative) are extensively employed in AGC (Automatic Generation Control) of power plants for limiting the deviation in frequency of generators. Since, the tuning of PID parameters is a tedious process. But, optimal control design has become extremely useful in finding the parameters of the PID controller, as shown in this paper. The optimal PID controller designed in this paper, is structure specified as the order of the controller is limited to two. This helps in reducing the cost and complexity of the hardware implementation. The two proposed optimization methods PSO and DE are used to obtain the PID parameters by

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minimizing the fitness function i.e. integral square error between actual output and step reference. Hence, with this optimal control design, the tracking properties can be achieved successfully. The time domain responses are given in Section VI.

Model order Reduction is also presented in this paper. This technique is one of the very important concepts of control system, that analyzes the characteristics of dynamic systems, for reducing their structural complexity, while preserving their input-output properties[1,2]. The order of nominal model for physical plant is very high. Therefore, the controller design for the control engineers, is a difficult task. Moreover, the robust controller techniques like Hinf controllers result in higher order. This again increases cost and hardware complexity. One of the most essential steps in the design of the reduced order model is to obtain the optimal values of the coefficients of reduced transfer function. This is not an easy procedure and often needs many iterations[11]. The PSO and DE optimizations are used to get these values that give the model of reduced order to two, by minimizing the fitness function i.e. integral square error of the output. Then, using the same optimization technique, the discrete time PID controller has been designed for the same reduced order model and original model. The PSO and DE methods are used due to its simplicity as well as ease of implementation.

The organization of this paper is as follows: Section II describes the PSO algorithm. Then, Section III explains the DE algorithm. In Section IV, algorithm for the model order reduction of discrete time control system is presented. In Section V, Design example of the order 8 is shown. In Section VI, the simulation and results for the reduced order and original plants are presented. Section VII gives conclusion.

2. Particle Swarm Optimization (PSO)

PSO is based on intelligence of the swarm. It is a stochastic algorithm. The particle here represents: a bird in a flock or bee in a swarm. The particle uses its own intelligence as well as the past memory to find the best path to food location. This information is transmitted to all the particles and all the particles will converge to the best and optimal solution.

In this PSO mechanism, the particle is represented by its velocity and position. The velocities are updated first and then the position of the particles. It is started with a random swarm size N. A swarm is consisting of N particles that are randomly moving around in a multi dimensional search space. Each particle checks its position in space of the problem, which computes the fitness function. The best position obtained by other particles so far, is Gbest, whereas, the best position that has been received by the current particle is Pbest. The global best particle connects all particles of the population together. The PSO technique involves cohesion, separation and alignment. The velocity and position of a particle is modified as:

$$v_{id}^{k+1} = v_{id}^{k} + c_1 * r_1 * (p_{id}^{k} - x_{id}^{k}) + c_2 * r_2 * (p_{gd}^{k} - x_{id}^{k}) \quad (1)$$

$$x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k} \quad (2)$$

Here v is the velocity of particle- i, p_{id} and g_{id} are the Pbest and Gbest for it i^{th} particle ,x is the position, k is iteration and c_1,c_2 are the cognition and social learning rates[2]. These parameters are variable or constant as '2' . The variables r_1,r_2 are the random numbers in the range (0,1). The solution will converge if the best fitness function value is achieved or maximum number of population is reached.

3. Differential Evolution (DE)

DE is a metaheuristic approach and population based search strategy similar to standard evolutionary algorithms. DE uses mating, mutation and natural selection to search in parameter space in a parallel fashion for the best solution to the problem to be optimized[14]. One main distinguishing feature between DE and other EAs is that mating and mutation takes place in a single step. DE is self-organizing and it uses very simple mathematical formulae of vector differential of two individuals. This method is applied to every member of a population. Another difference is the selection process which is deterministic, in that a direct comparison between parent and child exists such that the better is allowed in the population. DE is a relatively new evolutionary algorithm, it offers fast convergence, robustness and simplicity.

$$x_{i,0}^{j} = l^{j} + rand \times \left(u^{j} - l^{j}\right), \quad j = 1,2,\ldots\ldots,n, \quad \forall\ i, \quad (3)$$

where rand $\in$ [0,1] is a uniformly distributed random value generated for each j and u^j and l^j are the respective upper and lower limits for the j^{th} variable or component. DE is used for multidimensional real-valued functions but does not require for the optimization problem to be differentiable. DE can also be used on non continuous problems.

4. Model Order Reduction of the Discrete Time System

Assume the given transfer function of discrete time system of order n:

$$G(z) = \frac{b_1 z^{n-1} + b_2 z^{n-2} + \ldots\ldots + b_{n-1} z + b_n}{z^n + a_1 z^{n-1} + \ldots\ldots + a_{n-1} z + a_n} \quad (4)$$

Now, the aim is to obtain a reduced $\mathbf{r^{th}}$ order transfer function with r < n:

$$G_r(z) = \frac{d_1 z^{n-1} + d_2 z^{n-2} + \ldots + d_{r-1} z + d_r}{z^r + c_1 z^{r-1} + \ldots\ldots c_{r-1} z + c_r} \quad (5)$$

This reduced model has similar essential characteristics of the original model $\mathbf{G(z)}$. The transient responses of this reduced plant are as close as possible to that of original plant for same input[1]. To get the optimal coefficients of reduced order plant, the original higher order model is reduced with the Evolutionary techniques such as PSO and DE. The fitness function is chosen as Integral Squared Error (ISE) of difference between the original and reduced order responses given by the expression:

$$J = \sum [y_0(k) - y_r(k)]^2 \quad (6)$$

Where $y_0(k)$ is the step response of the plant at instant k. $y_r(k)$ is the step response of the reduced plant at instant k.

This objective is chosen because it denotes the power consumption and squaring the error will result in weightage to larger errors as compared to other smaller errors. Also squaring makes computation easier to handle.

In this paper, the direct method of model order reduction is used which does not require the transformation of model to other form.

Since last decade, Evolutionary Algorithms (EAs) have given its usefulness for solving all types of optimization problems. This is the most powerful research field, an area based on analogies with nature or social systems. The original higher order discrete-time system is reduced to model of lower order (2nd order) using optimization techniques such as PSO and DE[1,12] and then a discrete time structure specified PID controller is designed for the reduced order. The same PID controller, as designed for reduced model with the same parameters is then used for the higher order model to obtain the desired specifications.

5. Design Example

An example is taken for model order reduction of the discrete-time system[1]. Then a PID controller is designed for the reduced order model by minimizing ISE between original and reduced model output. The same PID controller is also used to control the original higher order model with the same structure and same controller parameters. The transfer functions of the process plant with numerator and denominator coefficient.:

$$\begin{aligned} \text{num} &= [0.1625\ 0.125\ -0.0025\ 0.005250\ -.02263\ -0.00088\ 0.003\ -0.000413]; \\ \text{den} &= [1\ -0.6307\ -0.4185\ 0.078\ -0.057\ 0.1935\ 0.09825\ -0.0165\ 0.00225]; \end{aligned} \quad (7)$$

The objective is to find out the 2nd order reduced model by using PSO and DE of the form[1]:

$$G_r = \frac{A_r z + B_r}{C_r z^2 + D_r z + E_r} \quad (8)$$

Firstly, discrete-time model of 8th order is taken[2] and is reduced to 2nd order by using optimization techniques PSO and DE. The objective is chosen as:

$$J = \sum [y_0(k) - y_r(k)]^2 \quad (9)$$

The transfer functions for reduced model obtained from the technique PSO and DE are given in Table 1:

The results and simulations are explained in later sections. The value of J is less in case of PSO as compared to DE.

5.1 Discrete PID Controller Design using PSO and DE

The PID controllers are extremely popular in industrial application and power grids. The integral term is used to improve the steady state accuracy whereas the derivative term improves the transient response. The k_p ,k_i and k_d are optimized by using PSO and DE algorithm. The ISE between output and step input is minimized for tracking ,as explained in Figure 1. The structure of PID controller is:

Table 1. Reduced models using PSO and DE

Method	Reduced Model	ISE
PSO algorithm	$\frac{0.0050898z - 0.0024351}{0.032614z^2 - 0.0573272z + 0.027169}$	0.000741
DE algorithm	$\frac{0.004832z - 0.002472}{0.029458z^2 - 0.05172z + 0.02477}$	0.000839

$$\text{numc} = [Kp+Ki+Kd \ \ -Kp-2*Kd \ \ Kd]\ ; \qquad \text{denc} = [1\ \ -1 \quad 0]\ ; \tag{9}$$

The resulting PID parameters for reduced order model obtained are shown in Table 2:

6. Simulations and Results

MATLAB software has been used to implement the models and their controllers. The results for the step response of the original and reduced order models are shown. The Figure 2 and Figure 3 shows that the discrete time step responses between original and reduced order models are overlapping. The Figure 4 and Figure 5 shows that the discrete time PID controller designed using reduced model have improved steady state and transient responses. The Figure 6 and Figure 7 shows the same PID controller used with the original model also gave satisfactory responses.

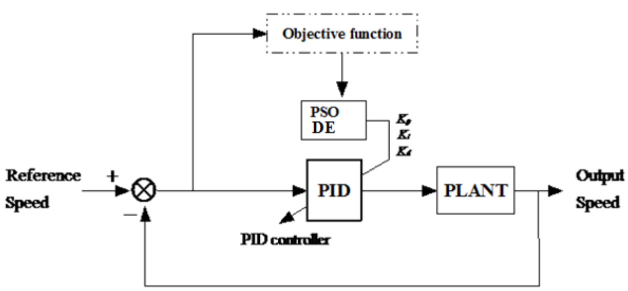

Figure 1. Block diagram of plant with discrete time PID controller tuned by PSO and DE algorithm.

Table 2. PID parameters obtained with PSO and DE

Controller	K_p	K_i	K_d
PID (PSO)	2.7674	0.636	2.7133
PID (DE)	2.901	0.6289	2.6196

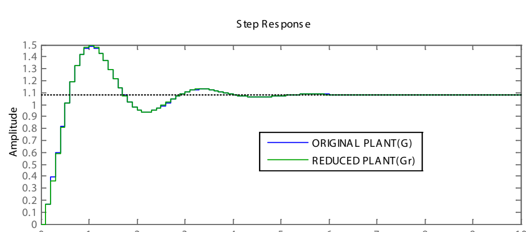

Figure 2. Step response of the original and reduced plant using PSO.

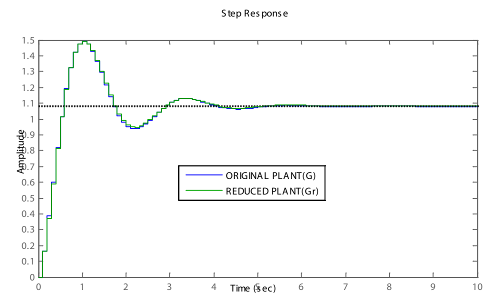

Figure 3. Step response of the original and reduced plant using DE.

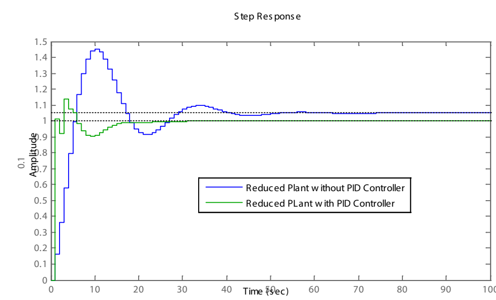

Figure 4. Step response of the reduced plant with and without PID using PSO.

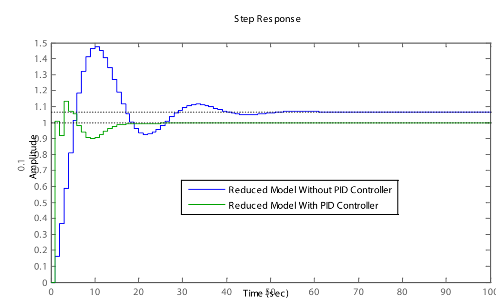

Figure 5. Step response of the reduced plant with and without PID using DE.

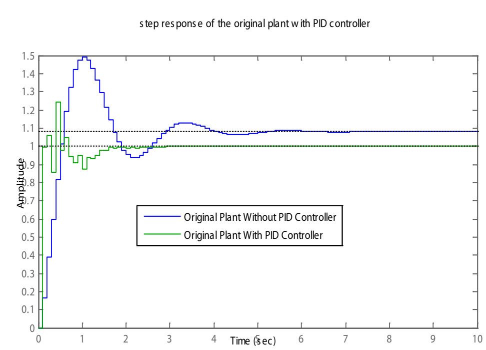

Figure 6. Step response of the original plant with and without PID controller using PSO.

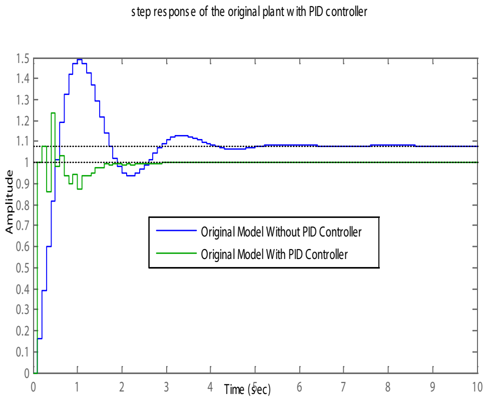

Figure 7. Step response of the original plant with and without PID controller using DE.

7. Conclusion

This paper shows the flexibility of two optimization techniques in obtaining the optimal solutions of coefficient of reduced plant and PID controller. PSO is better than the DE for this particular problem as an error/ objective function is less in case of PSO. But, both the results are satisfactory. The steady state and transient responses are improved with the use of structure specified discrete time PID controller designs. Even, with the reduction of the order of model and controller, hardware cost and components used, are reduced. Moreover, model order reduction can be utilized for the various complicated industrial processes using optimization techniques .

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