

# Design and Implementation of Second order Microwave Differentiator

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## Abstract

**Objective:** To design and implementation of second order digital differentiator at microwave frequencies. **Method:** We proposed simple accurate formulations for Discrete-time Processing (DSP) of second order digital differentiators in the Z-domain. The transmission lines are constructed by using chain scattering parameters to map with second order digital differentiator in order work at microwave frequencies. So the developed equations were instrumented by using micro strip lines to form a second order microwave differentiator and simulated by using Advanced Design System (ADS) software. **Findings:** A simplified mathematical expressions are derived to design the digital differentiator at microwave frequencies. The designed second order microwave differentiator magnitude response is similar to second order digital differentiator. **Applications:** It can be used in target detection of marine radar.

**Keywords:** Chain Scattering Parameters, Digital Differentiator, Micro Strip, Microwave Differentiator, Stub etc.

## 1. Introduction

In order to obtain the time derivative of the incoming signal Digital Differentiators<sup>1</sup> are used. Majority of Engineering fields like Digital Signal Processing<sup>2</sup>, Digital Image Processing (DIP), Control Systems, Bio-medical engineering, Instrumentation etc., make user of Digital Differentiators. In order to estimate the acceleration and velocity of objects from its position in radars and sonars, differentiators<sup>3,4</sup> are used. The expression for digital differentiator can be given as,  $H(j\omega) = j\omega$  for  $0 \leq \omega \leq \pi$ . In order to obtain ideal characteristics, FIR type differentiators should have higher order so they have less real-time applications. Al-Alaoui<sup>5,7</sup> designed a second order IIR type digital differentiator by interpolation process is given by

$$H_a(z) = \frac{1.25(z^2 - 1)}{T(z^2 + z + 0.25)}$$
The considerable drawback of this digital differentiator is that they can work efficiently in low frequency applications. In this paper an attempt is

made to implement the second order digital differentiator at microwave frequencies.

The organization of this paper is as follows: Section 1 depicts preface to chain scattering parameters and digital differentiators. Design and implementation of second order digital differentiator at microwave frequencies by using microstrips is explained in Section 2. Section 3 accord with Results and conclusions. References are specified in Section 4.

The relationship between incident waves and emergent waves of a Scattering Matrix which is represented in two port network as,

$$\begin{pmatrix} b(1) \\ b(2) \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a(1) \\ a(2) \end{pmatrix} \quad (1)$$

The drawback with scattering matrix is that they are not suitable in the analysis of representing the transmission lines which are connected in cascade. So this issue can be overwhelmed by using chain scattering matrix. The two port network of a Chain Scattering Matrix<sup>8</sup> defined as:

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$$\begin{pmatrix} a(1) \\ b(1) \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} b(2) \\ a(2) \end{pmatrix} \quad (2)$$

The affinity between chain scattering and scattering parameters will be:

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} = \begin{pmatrix} \frac{1}{S_{21}} & -\frac{S_{22}}{S_{21}} \\ \frac{S_{11}}{S_{21}} & S_{12} - \frac{S_{11}S_{22}}{S_{21}} \end{pmatrix} \quad (3)$$

The Chain Scattering Matrix will be calculated for different Transmission Line configurations<sup>9-10</sup> in this Section.

### 1.1 Transmission Line

The chain scattering matrix for a Transmission line with impedance  $Z_2$  is given by:

$$\begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix}_{T.L} = \frac{1}{(1 - \Gamma^2)D^{-1}} \begin{pmatrix} 1 - \Gamma^2 D^{-2} & -\Gamma + \Gamma D^{-2} \\ \Gamma - \Gamma D^{-2} & -\Gamma^2 + D^{-2} \end{pmatrix} \quad (4)$$

where  $\Gamma = \frac{Z_2 - Z_0}{Z_2 + Z_0}$ . The overall chain scattering matrix and its Transfer function can be calculated by connecting open circuited stubs, short circuited stubs, transmission Lines in a cascaded form and substituting  $D^{-2} = z^{-1}$ , If the network contains, K open circuited stubs, L Short circuited stubs and M Transmission line sections, then the Transfer function is shown to be:

$$S_{21}(z) = \frac{1}{T_{11}(z)} = z^{-\frac{M}{2}} \frac{\prod_{k=1}^K (1 + z^{-1}) \prod_{l=1}^L (1 + z^{-1})}{\sum_i A_i z^{-i}} \quad (5)$$

where,

$$A_i = \frac{a_i}{\prod_{m=1}^M (1 - \tau_m^2)} \quad (6)$$

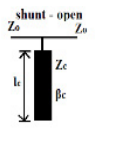
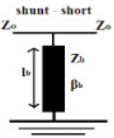
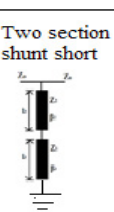
are functions of characteristic impedances of stubs and Transmission lines. From the Transfer function it is evident that,

- zeros at  $z = -1$  are contributed by open-circuited stubs.
- zeros at  $z = 0$  are contributed by short-circuited stubs.

Neglecting delay factor of M serial Transmission Lines  $z^{-M/2}$ , Equation(5) can be rewritten as:

$$S_{21}(z) = \frac{(1 - z^{-1})^K}{\sum_{i=0}^{M+K} A_i z^{-i}} \quad (7)$$

**Table 1.** Chain -scattering parameter illustration of transmission lines.

Transmission Line when open circuited	$\frac{1 + z^{-1}}{(1 + c) + (1 - c)z^{-1}}$	
Transmission Line when Short circuited	$\frac{1 - z^{-1}}{(1 + c) - (1 - c)z^{-1}}$	
A two-section shunt short Transmission Line	$H(z) = S_{21}(z) = \frac{2c_1(1 - z^{-2})}{(1 + 2c_1) - 2\Gamma_1 z^{-1} + (1 - 2c_1)z^{-2}}$	

### 1.2 A Two-Section Shunt Short Transmission Line

Table 1 parades the chain-scattering-parameter illustration of transmission line in different forms, in Z-domain. The presumed allusion characteristic impedance is 50Ω. For a Transmission line shunted with 2-section shorted line with  $Z_0, Z_1, Z_2$  as characteristic impedances. If the Transmission lines are assumed to be of finite length, then the Transfer function is shown to be:

$$S_{21}(z) = \frac{2c_1(1 - z^{-2})}{(1 + 2c_1) - 2\Gamma_1 z^{-1} + (1 - 2c_1)z^{-2}} \quad (8)$$

Where  $c_1 = \frac{Z_1}{Z_0}$  and  $\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$  the above Transfer function will have a dual pole with negative value given by:

$$z_p = \frac{\Gamma_1}{1 + 2c_1} = \frac{-\sqrt{(1 - 4c_1^2)}}{1 + 2c_1} \quad (9)$$

## 2. Implementation of Second order Microwave Differentiator

The expression for Al-Alaoui Second order digital differentiator is given by

$$H_d(z) = \frac{1.25(z^2 - 1)}{T(z^2 + z + 0.25)} \quad (10)$$

The above equation is having a double pole at  $z = -0.5$ . Comparing Equations (8) and (10), the values of  $Z_1 = 0.625 \Omega$  and  $Z_2 = 25.378 \Omega$ .

The implementation of second order differentiator by using micro-strip line is shown below. The corresponding values of the impedances are stored as width and length of micro strips. These values can be determined by synthesis method.

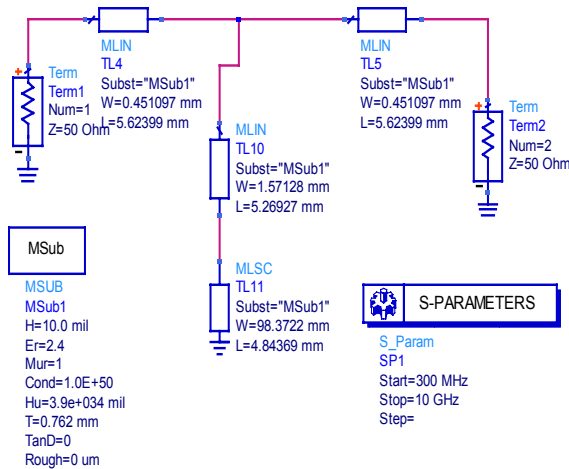


Figure 1. Implementation using ADS software.

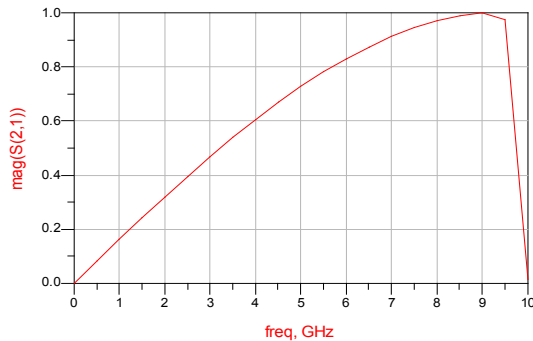


Figure 2. Magnitude response of  $S_{2,1}$ .

Table 2. Variation of magnitude of S (2,1) with frequency.

Frequency	S(2,1)
500.0 MHz	0.081
1.000 GHz	0.162
1.500 GHz	0.241

2.000 GHz	0.319
2.500 GHz	0.395
3.000 GHz	0.469
3.500 GHz	0.539
4.000 GHz	0.606
4.500 GHz	0.669
5.000 GHz	0.728
5.500 GHz	0.782
6.000 GHz	0.831
6.500 GHz	0.875
7.000 GHz	0.913
7.500 GHz	0.946
8.000 GHz	0.972
8.500 GHz	0.991
9.000 GHz	1
9.500 GHz	0.975

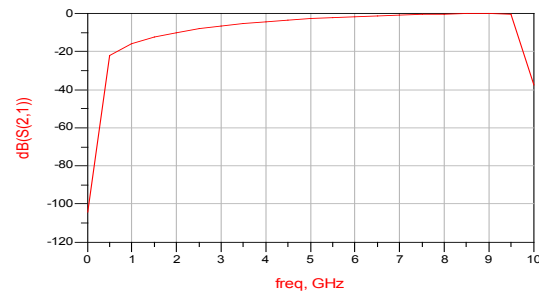


Figure 3. Frequency response of  $S_{2,1}$ .

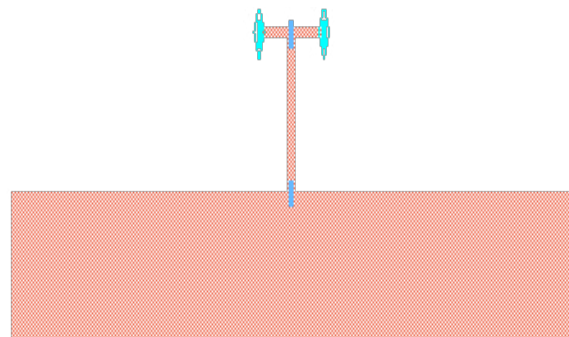


Figure 4. Layout of second order differentiator.

### 3. Results and Conclusions

To mimic transmission lines, the designed second order microwave differentiator was formed by using micro strip as shown in Figure 1. The transmission line shunted with two section shorted signal line having impedance is given as  $0.625\Omega$  &  $25.378\Omega$  and  $50\Omega$  respectively where  $50\Omega$  is the characteristics impedance. These impedances are transformed into equivalent micro strips. Then these micro strips are placed both sides symmetrically shown in figure 1. ADS environment is used to perform simulation. Table 2 describes the variation of magnitude of  $S_{21}$  with frequency and the graph is plotted for the same shown in Figure 2. Variation of gain of  $S_{21}(f)$  in dB of a 2nd order microwave differentiator with frequency is shown in Figure 3. It is noticed that the magnitude response of mentioned 2nd order microwave differentiator is analogous to the magnitude response of 2nd order digital differentiator. By using ADS software, the physical layout is generated for the same as shown in Figure 4.

### 4. References

1. Proakis JG, Manolakis DG. Digital Signal Processing, 3rd Edition, PHI Publications; Newdelhi, 1999.
2. Begum JT, Naidu SH, Vaishnavi N, Sakana G, Prabhakaran N. Design and Implementation of Reconfigurable ALU for Signal Processing Applications, Indian Journal of Science and Technology. 2016; 9(2):1–6.
3. Skolnik MI. Introduction to Radar Systems, 3rd Edition. McGraw-Hill: New York; 2002.
4. Hong JS, Lancaster MJ. Micro-strip Filters for RF and Microwave Applications, John Wiley and Sons, 2001.
5. Al-Alaoui MA. Novel Digital Integrator and Differentiator, IEEE Electronic Letters. 1993; 29:376–78.
6. Kumar B, Roy SCD. Design of Digital Differentiators for Low Frequencies, Proceedings IEEE. 1988; 76:287–89.
7. Krishna BT, Rao SS. On Design and Applications of Digital Differentiators, Proceedings of 4th ICoAC, 2012.
8. Chang DC, Hsue CW. Design and Implementation of Filters using Transfer Functions in the Z Domain, IEEE Transactions on Microwave Theory Techniques. 2001; 49:979–85.
9. Hsue CW et.al. Implementation of 1st Order and 2nd Order Microwave Differentiators, IEEE Microwave Theory and Techniques. 2004; 52:1443–47.
10. Hsue CW, Cheng TR, Chen HM. A Second Order Microwave Differentiator, IEEE MW and Wireless Components Letters. 2003; 13:137–39.