

Regularized Least Square Approach for Remote Sensing Image Denoising using Wavelet Filters

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Abstract

Noise in remote sensing images (aerial and satellite) is caused due to various reasons such as atmospheric interference or lack of quality in sensors used to capture them. Removal of noise in an efficient way is a big challenge for researchers. In this paper, one dimensional signal denoising based on weighted regularized least square method is mapped to two dimensional image denoising. **Objectives:** This paper introduces a novel image denoising technique based on least square weighted regularization. **Methods/Statistical Analysis:** The proposed technique for image denoising based on Least Square (LS) approach is experimented on five different satellite and aerial images corrupted by gaussian noise with varying noise levels and regularization parameter lambda (λ) for different wavelet filter coefficients such as 'haar', 'symlet', 'daubechies' and coiflet. The effectiveness of the proposed method of image denoising is compared against the existing second order filter [based on LS] and conventional wavelet based image denoising technique based on the standard metric called Peak Signal to Noise Ratio (PSNR). **Findings:** From the experimental result analysis obtained it is inferred that the wavelet filters outperforms the second order filter and the conventional wavelet based image denoising. The complexity of the mathematics is low in our proposed method for image denoising. **Applications/Improvements:** The proposed denoising technique can be adopted as a faster pre-processing step in most of the image processing applications.

Keywords: Gaussian Noise, Image Denoising, Least Square, Peak Signal-to-Noise Ratio, Wavelet Filter Coefficients

1. Introduction

Noise, normally generated during the image acquisition process alters the actual intensity values of the image thus degrading the visual perception as well as the information carried in it¹. The number of intensity values corrupted decides the quantity of noise. The quality of the satellite and aerial image is reduced due to the interference of noise at the time of their transmission and reception¹. This prevents us from gathering most of the necessary information from the satellite and aerial images². Hence, an efficient image denoising method is essential to improve the quality of these images. The different categories of image noise are the Gaussian noise, salt and pepper noise, speckle noise but, in real time the satellite and aerial images are most commonly affected by the additive white Gaussian noise alone³.

There are various traditional image denoising methods such as the wavelet and Total Variation (TV) based

denoising methods, which retrieves the true intensity values of the image, but the major hurdle in these methods is its high computational time and high mathematical complexity. The least square based denoising method (proposed method) is used as an alternative to provide the best possible denoised outcome of the noisy image at the cost of lesser computational time⁴.

The invertible property of wavelet filters helps to recover the denoised image by wavelet transformation⁵. The haar filter is the fundamental wavelet filter used for image denoising due to its low computing requirements and good edge detecting property⁶. The other forms of wavelet filters that can be applied for image denoising are the Daubechies, Symlet, Coiflets, Discrete Meyer wavelet, Biorthogonal and Reverse biorthogonal^{7,8}.

In this paper, the one-dimensional approach proposed by Selesnick has been extended to two-dimensional image denoising using the concept of least square weighted regularization with different wavelet coefficients. The

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proposed method is compared with the methodology involving second order sparse matrix and the existing wavelet based denoising method. The accuracy in denoising is measured through the image quality metric, PSNR. Also, the proposed method has the advantage of low mathematical complexity.

This paper deals with denoising of aerial and satellite colour images using least square weighted regularization technique. The quality of the denoised image is determined on the basis of visual perception and PSNR. A comparative study of denoising using different filter coefficients at different noise levels and regularization parameter lambda (λ) is performed depending on the PSNR values obtained.

2. Wavelets Used

In the proposed method, two dimensional image denoising is carried out using least square weighted regularization where, the second order difference matrix is replaced by coefficients of different wavelet filters such as 'haar', 'daubechies', 'symlet', and 'coiflet'.

The efficiency of the second order differential coefficients in denoising is compared with these wavelet coefficients based on the peak signal to noise ratio.

The haar wavelet is the simplest and symmetric⁶. Daubechies wavelets are the functions that are orthogonal with finite vanishing moments conditions⁶. The modified form of daubechies that is efficient in denoising application is symlet which is symmetric, orthogonal and biorthogonal⁹. Coiflet has 2N moments equal to zero and its scaling function has 2N-1 moments equal to zero, where N represents the length of the wavelet¹⁰. The different wavelet filter coefficients used are:

- Haar Filter : [-0.7071 0.7071]
- Daubechies Filter : [-0.4830 0.8365 -0.2241 -0.1294]
- Symlet Filter : [-0.3327 0.8069 -0.4599 -0.1350 0.0854 0.0352]
- Coiflet Filter : [0.0727 0.3379 -0.8526 0.3849 0.0727 -0.0157]

2. Mathematical Background

In this section, the least square weighted regularization based signal denoising algorithm is being discussed. The one dimensional signal denoising approach using least

square was proposed by Selesnick⁴. In this paper, the one dimensional least square approach is extended to the two dimensional image denoising. The problem formulation for one dimensional signal denoising is given by.

$$\min \|y - x\|_2^2 + \lambda \|Dx\|_2^2 \tag{1}$$

where, D is the second order difference matrix given by,

$$D = \begin{bmatrix} 1 & -2 & 1 & & & \\ & 1 & -2 & 1 & & \\ & & \ddots & \ddots & \ddots & \\ & & & 1 & -2 & 1 \end{bmatrix}$$

In the formulation represented in Equation (1) 'y1' is the noisy image and 'x' is the denoised image. The basic idea behind this formulation is to obtain the denoised image 'x' on passing the noisy image 'y1' as the input. The process of denoising happens in the first part of Equation (1) and the second term corresponds to the regularization process which depends on the given value of λ which must be greater than zero.

In the first iteration the output will be equal to the input hence, in the absence of the regularization term the process of denoising will remain incomplete.

The minimization the first term of the objective function in Equation (1) forces the output signal to be similar to the input signal and the minimization of the second term leads to the smoothing of the noisy input signal thus, producing a denoised output signal.

The role of the regularization parameter λ is to increase the extent of denoising as the noise level in the signal increases which is depicted in the least square formulation for signal denoising given by:

$$x = (I + \lambda D^T D)^{-1} y1 \tag{2}$$

In Equation (2) if λ is zero, the denoising fails. Hence, the output signal will be equal to the input noisy signal. But, the weightage given to the second term of Equation (1) increases with increase in λ value which in-turn results in a denoised output. Therefore, higher the value of lambda (λ), smoother is the denoised output image. If λ is very small, less smoothing is achieved hence forcing the denoised image to be same as input noisy image. Hence, λ should be chosen such that, denoised output image will be similar to the input image along with the removal of noise.

4. Proposed Method

In the proposed method, the least square weighted regularization with different filter coefficients is applied for satellite and aerial image denoising. The methodology adopted for image denoising of individual colour planes is as shown in Figure 1.

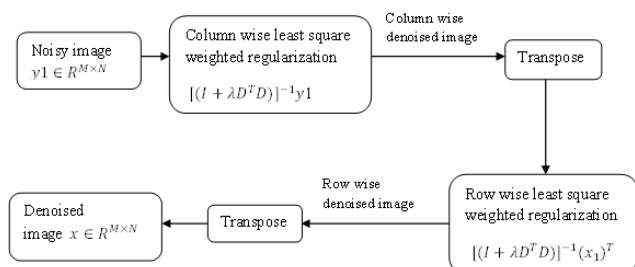


Figure 1. Block diagram of the proposed method.

4.1 Algorithm

The algorithm for denoising of satellite and aerial images is as follows:

- Noise is added to the red, green and blue planes of the colour image.
- The three noisy planes are concatenated to form a noisy colour image which is stored in variable 'y'.
- Let 'y1' represent one of the colour planes of the input noisy image of size $m \times n$.

- This noisy image is passed as an input to the equation in block 2 where weighted least square regularization is applied column wise on the image.
- The column wise denoised image 'x1' obtained from step 2 is of size $m \times n$.
- The column wise denoised image 'x1' is transposed to obtain the $x1^T$ of size $n \times m$, for row wise denoising of the image.
- The row wise denoised image is transposed to obtain the denoised output image 'x' of size $m \times n$. All the colour planes of the input noisy image are processed individually as explained in the step (3) to (7). The resultant denoised planes are concatenated to obtain the output denoised colour image.

5. Experimental Results and Analysis

This section sets a brief description of accuracy assessment measures and analysis of the least square weighted regularization denoising technique.

In our proposed method, satellite and aerial image denoising is performed using different wavelet filter coefficients based on the Least Square (LS) method. The experiment is carried out on five different aerial and satellite images shown in Figures 2, 3 respectively, using the Gaussian noise type with different noise levels over a



Figure 2. Input aerial images used for analysing the proposed method (a) aerial 1 (b) aerial 4 (c) aerial 2 (d) aerial 3 (e) aerial 5.



Figure 3. Input satellite images used for analysing the proposed method (a) sat 1 (b) sat 2 (c) sat 3 (d) sat 4.

fixed range of regularization parameter λ . The Gaussian noise is incorporated into the image with mean at zero and different variance values such as 0.01, 0.05, 0.1, and 0.25.

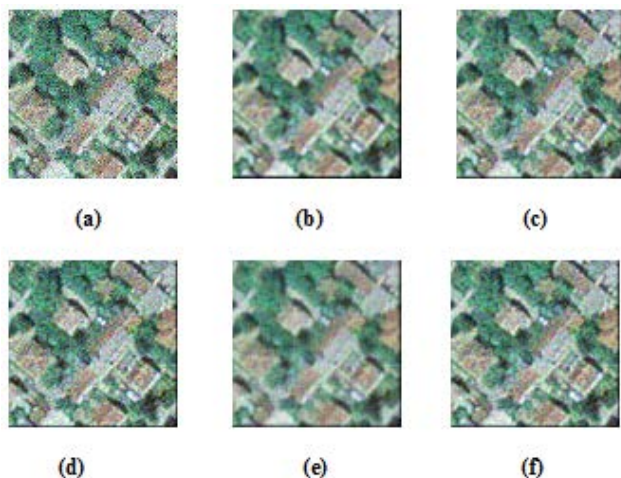


Figure 4. (a) Noisy aerial image at noise level 0.1 (b-f) Denoised images using 2d, db, haar, symlet and coiflet filters respectively.

The experiment is conducted for different wavelet filters like daubechies, haar, symlet and coiflet (proposed method) whose performance is compared with the existing technique of denoising based on second order differential coefficients using the LS method shown in Figures 4, 5. A comparison on the performance of wavelet based denoising and least square based denoising is

also made to prove the efficiency of the latter. Initially, the denoising of the ten different noisy images at four different noise levels is carried out using the second order differential coefficients and the above mentioned wavelet filter coefficients.

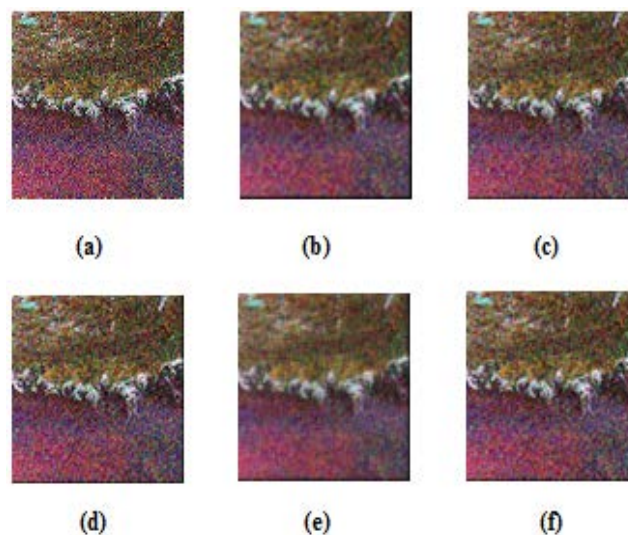


Figure 5. Noisy satellite image at noise level 0.1 (b-f) denoised images using 2d, db, haar, symlet and coiflet filters respectively.

The extent of denoising measured based on the standard metric called PSNR for noise levels 0.1 and 0.25 is tabulated in Table 1. The results in Table1 projects that the proposed method provides a higher PSNR for the

Table 1. Peak signal to noise ratio (PSNR) of denoised images and noisy images for noise levels 0.1 and 0.25 using different wavelet filters

Noise Level	0.1						0.25					
	PSNR of Noisy Image	Existing Method		Proposed Method (using wavelets)			PSNR of Noisy Image	Existing Method		Proposed Method (using wavelets)		
Filter		2d	dB2	sym3	haar	coif1		2d	dB2	sym3	haar	coif1
AERIAL IMAGE												
aerial 1	11.5398	21.1383	21.3396	20.9800	21.2171	21.3081	9.1031	18.4717	18.5082	18.2608	18.4919	18.4277
aerial 2	11.1612	24.465	24.7169	23.6116	26.1916	24.5671	8.9921	22.1526	22.3206	21.0772	23.3367	22.1438
aerial 3	11.3451	21.9087	21.7018	21.7276	22.0493	17.7735	9.036	19.4514	19.4561	19.1943	19.4012	22.0952
aerial 4	11.9301	20.3440	19.9721	20.2779	15.0318	20.4161	9.2417	17.0221	17.0744	16.9668	17.1042	17.0883
aerial 5	11.3175	22.7318	23.0531	22.2470	23.8904	23.8904	9.0616	20.2486	20.5183	19.7879	21,2117	21.2117
SATELLITE IMAGE												
sat 1	11.6719	19.6146	19.7314	19.5149	19.8277	19.6799	9.1584	17.5229	17.562	17.3929	17.6719	17.4842
sat 2	11.3397	20.1925	20.3343	20.097	20.4210	20.2713	9.0515	18.5151	18.5386	18.3098	18.6646	18.4834
sat 3	11.8591	19.4651	19.5932	19.4358	19.5900	19.4596	9.1925	16.9577	16.9968	16.8690	16.9951	17.0021
sat 4	11.8445	20.5791	20.6479	20.4814	20.8156	20.6406	9.2104	17.7666	17.7637	17.4941	17.9287	17.7404
sat 5	11.692	20.3108	20.4022	20.1879	20.4570	20.3745	9.1679	17.6902	17.7706	17.5618	17.8236	17.7503

denoised images when compared to the existing second order differential coefficients. For sat 1 of satellite images, the PSNR improvement obtained on using the existing method is 8.3645 dB and the improvement on using the daubechies, symlet, haar and coiflet filter coefficients are 8.4036 dB, 8.2345 dB, 8.5135 dB and 8.3258 dB respectively. Similar improvement in PSNR values is noticed for the other nine images using the different wavelet filter coefficients. This is also supported by the graph in Figure 6 which is a plot of the average peak signal to noise ratio of all images for each filter over different

noise levels.

Table 1 and Figure 6 depicts that, different wavelet filter coefficients outperforms the existing method at different noise levels.

From Figure 6 it can be inferred that the daubechies filter provides the best overall PSNR for noise levels 0.25 and 0.1 whereas, the haar filter provides the best PSNR for noise level 0.05.

Figure 7 represents the comparison of least square based and wavelet based denoising methods at maximum noise level of 0.25 for the daubechies filter coefficients. On

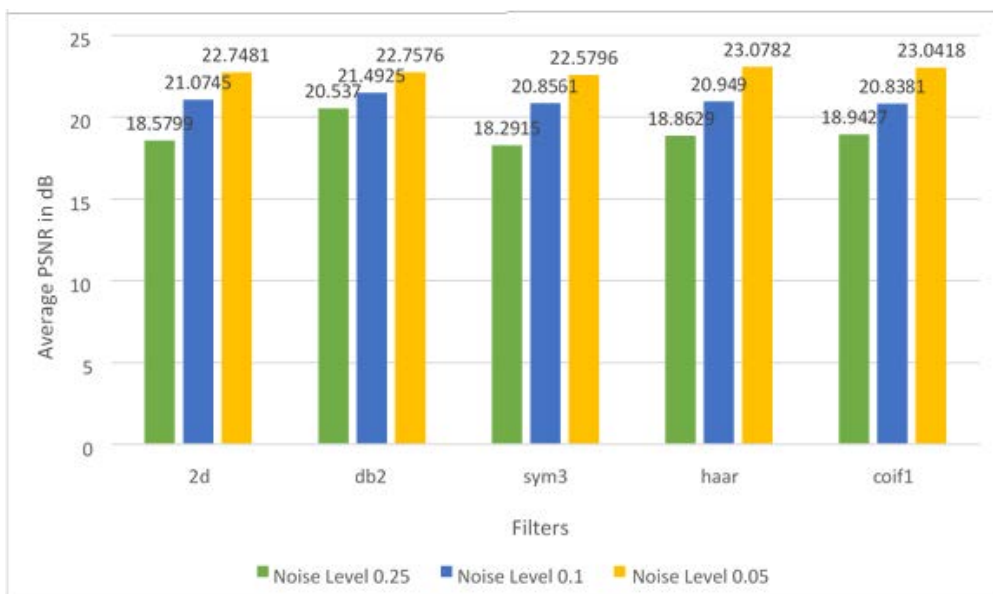


Figure 6. Performance analysis of different filters in LS based denoising.

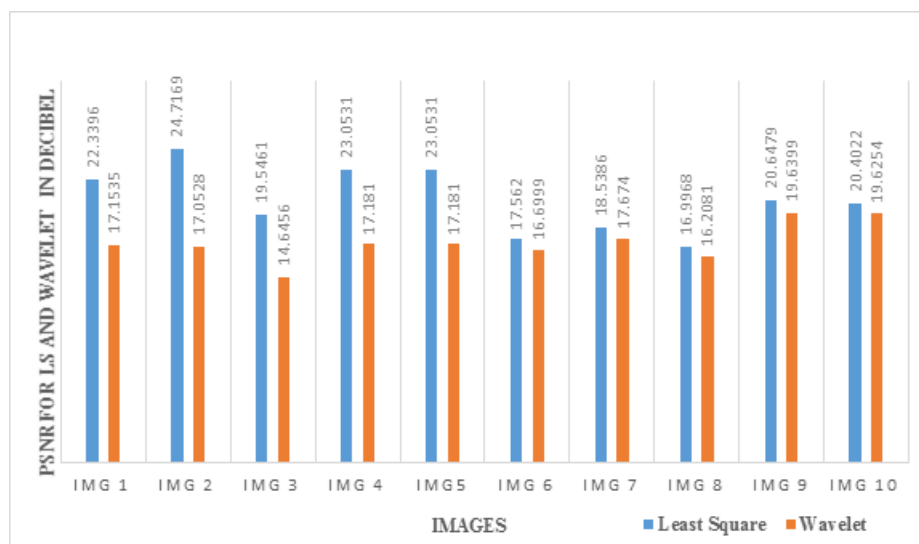


Figure 7. Comparison of the performance of LS and Wavelet based denoising for daubechies filter at 0.25 noise level.

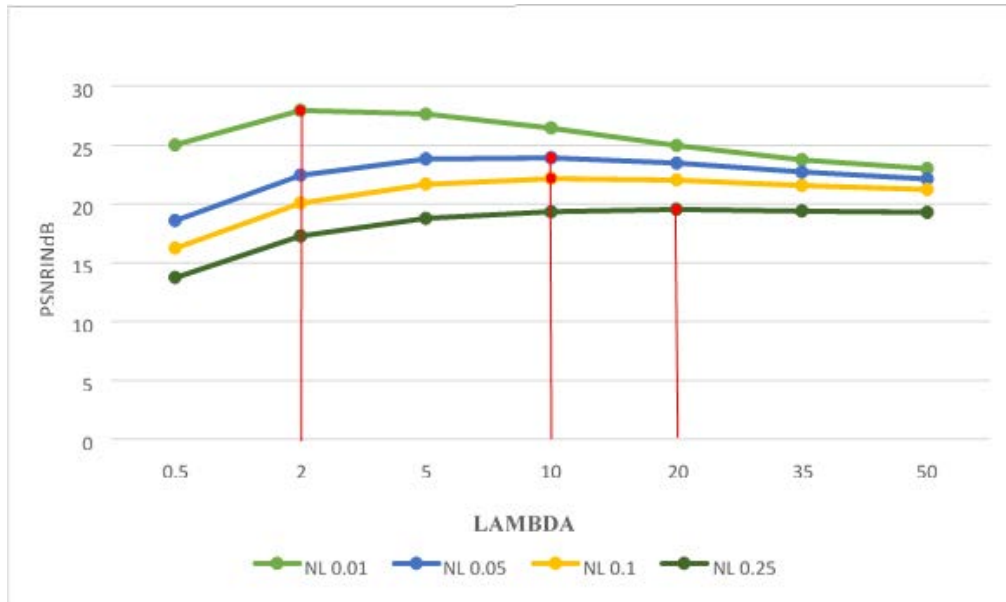


Figure 8. Variation of PSNR with lambda for aerial image 4.

analysing the PSNR values in Figure 7, it can be deduced that the proposed method (least square based denoising) is much more efficient than the existing method of wavelet based denoising. Figure 8 represents the variation of PSNR for aerial 4 at different noise levels over a range of regularization parameter λ (0.05,2,5,10,20,35,50). For noise level 0.01, lambda 2 gives the highest PSNR and for noise level 0.05 and 0.1, lambda of 10 gives the highest PSNR whereas, lambda 20 provides the highest PSNR for noise level 0.25. Figure 8 depicts the variation of PSNR as λ varies for aerial 4 using the daubechies filter. It supports the fact that as lambda value increases PSNR decreases which is predicted by the least square formulation in Equation (1) where, as lambda value increases the weightage given to the diffusion part of Equation (1) increases which in-turn reduces the quality of the image hence the PSNR.

6. Conclusion

This paper has proposed a method for image denoising using the weighted regularized least square method. The one dimensional approach proposed by Selesnick is extended for two dimensional image denoising where, first the least square method is applied row-wise and then column-wise for a range of noise levels and λ values. Here, apart from using the second order

differential coefficients, denoising is done by replacing the 'D' matrix with coefficients of the daubechies, haar, symlet and coiflet filters and the efficiency of these filters in denoising is compared with the second order coefficients. The performance of the proposed method is also compared with the wavelet based denoising method and its effectiveness is brought out by improvement in the standard metric called the peak signal to noise ratio, which is also supported by the visual analysis.

7. References

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