

Image Fusion using Variational Mode Decomposition

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Abstract

Background/Objectives: This paper introduced an image fusion algorithm based on Variational Mode Decomposition (VMD). **Methods/Statistical Analysis:** Image fusion is one of the image enhancement methods which results the image with better quality derived from a set of degraded images. Fused image contains more information than input images and it is efficient for visual perception and computer vision applications. This paper proposed an image fusion technique based on VMD for multi focus images. VMD has been a recently introduced non-recursive decomposition method, which decomposes the image into separate spectral bands called Intrinsic Mode Function (IMF) or modes. The modes are generated with respect to the associated central frequencies and they are band limited. **Findings:** A fusion rule based on weighing scheme is performed at the decomposition level for increasing the features by decreasing the mutual information. The reconstruction of the IMFs results the final fused image. The performance analysis of the proposed fusion method is experimented using standard objective quality metrics. The efficiency of the proposed method is determined by comparing the method with some state of the art methods. **Application/Improvements:** The image fusion using VMD is applicable to multi-resolution, multi model multi-sensor images.

Keywords: Fusion Rule, Image Fusion, Image Quality Metrics, 2D-Variational Mode Decomposition, Variational Mode Decomposition

1. Introduction

The main objective of digital image processing is to extract useful information from the image. Image fusion will improve the quality of the images by combining the information either from different images of the same scene or the images which are captured by different sensors¹. The fused image is single, composite and has better information compared to the multiple input images². Images captured by different methods have diverse characteristics in texture, color, spatial and spectral properties. In certain cases the image acquisition technique fails and produce

degraded images, which interrupt further processing³. The goal of fusion is to enhance such kind of images by bringing all relevant information from the set degraded images. The application of fusion spreads over the area of remote sensing, robot vision, vehicle or robotic guidance, medical imaging, industrial defect detection, military surveillance etc⁴. The input for fusion is a set of multi sensor, multimodal, multi temporal and multi resolution images. Fusion is categorized into two named as spatial and transform domain. The fusion method for spatial domain is performed directly onto the source image, which directly deals with pixels. The algorithms such as the simple average, select minimum,

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select maximum, Brovey method, Principal Component Analysis etc. comes under spatial fusion technique, which limits the fusion by the spatial distortion⁵. This type of limitation can be efficiently solved by transform domain methods. But in transform domain, for further processing⁶ the images are transformed into the other relevant domain. Commonly used transform methods are Discrete Wavelet Transform, Morphological Pyramid, Laplacian Pyramid, Stationary Wavelet Transform, Shift-Invariant Discrete Wavelet Transform etc⁷. This paper introducing an image fusion technique based on a fully intrinsic and adaptive decomposition algorithm called VMD. In VMD, each image is decomposed into its elementary patterns called Intrinsic Mode Function (IMF) which are extracted concurrently⁸. VMD is formulated as an optimization problem for minimizing the sum of bandwidths of the IMF, such that the sum of the IMF must be the original signal. Fusion is done at the decomposition stage by combining the appropriate portion of the corresponding mode of each image. The result should be a composite image with enhanced information⁹. In order to integrate the appropriate pixels from the source images, a fusion rule based on a weighing scheme are used which enhanced the pixel intensity by reducing the mutual information from the set of images. The image quality metrics are used for determining the performance of the proposed algorithm.

2. Related Works

The inevitable application of image processing such as image restoration, image enhancement, remote sensing applications, machine vision, robotic vision, medical image processing, transmission and encoding etc. promote research in the field of image fusion². The most primitive techniques for image fusion are select maximum, select minimum and simple averaging which results considerably less enhanced features than advanced technique such as pyramid and transforms methods. The resultant image is blurred due to the spatial distortions⁴. Fusion based on Principal Component Analysis (PCA) uses an orthogonal transformation in which the principal components are projected on to corresponding data and its summation results the fused image¹⁰. Pyramid transform based image fusion explored good results than primitive fusion methods. The pyramid transform of the input image is generated the pyramid transforms of the fused images¹¹. The inverse pyramid transform forms the final fused image. Morphological pyramid, Gradient

pyramid, Laplacian Pyramid etc. are the different types of pyramid transform techniques proposed for image fusion¹². The Discrete Wavelet Transform (DWT) is an efficient algorithm used for fusion in which the image is decomposed into four sub-bands at each level³. Either a low pass or high pass sub-band provides sufficient features for image fusion. In the decomposition level of the DWT, different fusion rule is applied and reconstruction results the fused image¹³. The Discrete Cosine Transform (DCT) is also a transform based approach proposed for fusion. The averages of the DCT coefficients of the input images results a composite fused image¹⁴. The Empirical Mode Decomposition (EMD) has been a recent developed decomposition algorithm, which is introduced for image fusion¹⁵. EMD decomposes the signals into sub-signals called Intrinsic Mode Function (IMF) and a fusion rule is performed in the decomposition stage to obtain the fused image⁹. The extension of EMD called complex EMD is also put forward for image fusion which is more effective than EMD. But the input data for fusion should be complex¹⁶. All these transforms methods have better results when compared with the primitive methods. The image quality is evaluated by using two different quality metrics such as quantitative and qualitative analysis¹⁷. In qualitative analysis, performance of fused image is determined by comparing the fused image and raw input images¹⁸ and quantitative analysis determine the quality based on the full reference method.

3. VMD Algorithm

VMD is a decomposition method, which decomposes a signal into a discrete number of sub-signals or constituent modes, which are called Intrinsic Mode Function (IMF). The modes are extracted concurrently using an iterative optimization algorithm called Alternate Direction Method of Multipliers (ADMM). Each IMF has a compact frequency support around a central frequency and it is fixed using a limited bandwidth around its characteristic center frequencies. The obtained modes have specific directional and oscillatory characteristics and have limited spatial support. The local frequency and amplitude of the modes varied smoothly⁸. VMD can perfectly reconstruct the original signal together the ensemble of modes. The IMF is the product form of two narrow band functions such as Amplitude Modulated (AM) slow varying function and Frequency Modulated (FM) fast varying function. The mathematical formulation of IMF is written as:

$$\min_{\omega, \omega_k} \left\| \sum_k \left[\left(\delta(t + \frac{j}{\omega}) * u_k(t) \right) e^{-j\omega_k t} \right] \right\| \quad (1)$$

s.t. $\sum u_k = f$

Here f is the original signal, u is the mode, k is the number of modes, ω is the center frequency of the signal. The unknown functions are k central frequencies and the k functions centered at those frequencies. VMD can decompose both 1D and 2D signals. The 2D VMD aims to decompose of the images into consistent modes with respect to its bandwidth. To extract the modes of images a unique extension of the 1D analytic signal for 2D is used¹⁹. Shift the image frequencies to baseband through heterodyne demodulation for single sided Fourier spectra. The 2D analytical signal is obtained by fixing the half plane of frequency domain into zero. The 2D analytic signal is defined in the frequency domain as:

$$s.t. \sum_k \hat{u}_{AS,k}(\bar{\omega}) = \begin{cases} 2\bar{u}_k(\omega), & \text{if } \bar{\omega}, \bar{\omega}_k > 0 \\ \hat{u}_k(\omega), & \text{if } \bar{\omega}, \bar{\omega}_k = 0 \\ 0, & \text{if } \bar{\omega}, \bar{\omega}_k < 0 \end{cases} \quad (2)$$

$= (1 + \text{sgn}(\bar{\omega}, \bar{\omega}_k)) \hat{u}(\bar{\omega})$

Equation (3) can also be defined by means of some Fourier property called frequency mixing and heterodyne demodulation which is given as:

$$u_{AS,k}(\bar{x}) = u_k(\bar{x}) * \left(\delta(\langle \bar{x}, \bar{\omega}_k \rangle) + \frac{j}{\Pi \langle \bar{x}, \bar{\omega}_k \rangle} \right) \delta(\langle \bar{x}, \bar{\omega}_{k,\perp} \rangle) \quad (3)$$

The constrained variational problem for 2D VMD is formulated as:

$$\min_{u_k, \bar{\omega}_k} \left\{ \sum_k \left\| \nabla \left[u_{AS,k}(\bar{x}) e^{-j \langle \bar{\omega}_k, \bar{x} \rangle} \right] \right\|_2^2 \right\}$$

s.t. $\sum_k u_k = f$ (4)

This problem is optimized by ADMM.

4. Proposed Approach

In the proposed approach, fusion of multi-focus images is performed by 2D VMD. The VMD decomposes the input images into corresponding IMF. To enhance the features of the fused image, a fusion rule is applied on each IMF. The rule used in the proposed method is based on a weighing scheme. Each IMF is multiplied with a set of weighted coefficients which improve the quality of the image by decreasing the mutual information from both the modalities. Fused image is obtained by combining all these modes⁹. In the weighing scheme, the weighted coefficient is fixed by satisfying certain criteria which are explained as follows.

The fused image represented as F can be mathematically defined as:

$$F(x, y) = \sum_{i=1}^M [\alpha_i(x, y)A_i(x, y) + \beta_i(x, y)B_i(x, y)] \quad (5)$$

Where (x, y) indicate the spatial location of the image. A and B are the input images for fusion. According to the fusion rule the criteria for fusion is explained in Equation 7. The weighted coefficients are $\alpha_i(x, y)$ and $\beta_i(x, y)$ which should satisfy the condition as $\alpha_i(x, y) + \beta_i(x, y) = 1$ here $\alpha_i(x, y) \geq 0$ and $\beta_i(x, y) \geq 0$. The weighted coefficients are determined based on the variance of input image.

$$\alpha_i(x, y) = 0, \quad \text{if } \text{var}\{A_i(x, y)\} - \text{var}\{B_i(x, y)\} < -\xi$$

$$\alpha_i(x, y) = 0.5, \quad \text{if } |\text{var}\{A_i(x, y)\} - \text{var}\{B_i(x, y)\}| < \xi \quad (6)$$

$$\alpha_i(x, y) = 1, \quad \text{if } \text{var}\{A_i(x, y)\} - \text{var}\{B_i(x, y)\} > \xi$$

Where $\xi > 0$ and $\text{var}\{A_i(x, y)\}$ is the local variance of A_i at (x, y) and $\text{var}\{B_i(x, y)\}$ is the local variance of B_i at (x, y) .

The framework of the proposed approach is given in Figure 1. In that A and B are the two multi-focus images subjected for fusion¹¹. The proposed method for fusion called VMD is applied to both input images. VMD decomposed the image A into A_1, A_2, \dots, A_M as the IMF similarly the image B is decomposed into B_1, B_2, \dots, B_M . The IMF for fusion is selected based on the intensity of pixels in the modes. The final fused image is obtained as a result of a fusion rule. The performance of the fused image is evaluated by standard image quality metrics.

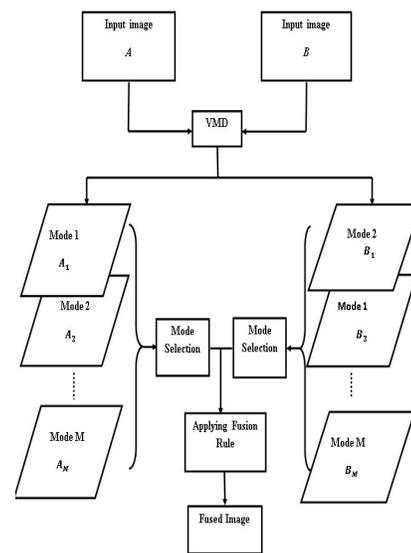


Figure 1. Proposed framework for image fusion.

5. Results and Analysis

The experiment is performed using two different datasets of multi-focus images which is represented as dataset 1 and dataset 2. Figure 2 represents the input images of dataset 1 and Figure 3 represents the input images of dataset 2. VMD is decomposed the images into 5 different modes as mode 1 to mode 5 respectively. The modes selected for fusion based on the values of pixel intensity and the central frequency. The modes with poor intensity are neglected since it does not contribute information for fusion. A fusion rule is applied to the selected modes based on a weighing scheme. The weighted coefficient is individually multiplied with selected modes and the summation of the modes results fused image. The weighing scheme reduced the mutual information from the both modalities and improved the features of the images.

To evaluate the performance of the proposed method, an image quality metric with full reference method is adopted². The quality measures used as Root Mean Square Error (RMSE), Percentage Fit Error (PFE), Correlation (CORR), Maximum Difference (MD), Signal to Noise Ratio (SNR), Peak Signal to Noise Ratio (PSNR), Quality Index (QI), Normalize Absolute Error (NAE), Mean Absolute Error (MAE), Average Difference (AD), Mutual Information (MI), Structural Content (SC) and Structural Similarity Index Metric (SSIM)¹⁸.

- RMSE: RMSE is used to compute the variation of the pixel intensity in the image. The output image is closer to the reference image when the RMSE value is zero. Hence, best fusion has less RMSE value⁵.
- PFE: PFE computes the ratio of the norm of the difference of corresponding pixel values of the reference and fused image to the norm of the reference image.
- MAE: Mean Absolute Error is computed as the ratio of the mean absolute error of respective pixels in the reference image and fused images. It should be a less value¹⁰.
- CORR: The Correlation computes the similarity features of the reference image and fused image. The benchmark correlation value is one when the fused and reference are exactly alike.
- SNR: The SNR is the ratio between the information and noise between the fused images. The higher the SNR ensures the fused image has high quality.

- PSNR: PSNR is the ratio of the gray level of corresponding pixels in the reference and fused images. The value is high for good fusion²⁰.
- MI: Mutual Index is the measure of similarity of the pixel intensity of fused and reference image. The highest value gives better fusion.
- QI: Quality Index measures the amount of pixel information present in the reference image is transformed into the fused image⁵.
- SSIM: SSIM is an efficient, quality metric for fusion, which is defined as the comparison of local patterns of the pixel intensities between the reference and fused image. The best fusion has QI and SSIM value nearer to 1²¹.
- AD: Average Difference used to find the average of the difference of the pixel intensity of fused and reference image, which is less value for better fusion.
- SC: Structural Content calculated as the ratio of sum of square of pixel intensity of the reference image to fused image. The SC value becomes one for the fused image is more identical to the reference.
- MD: The Maximum Difference gives the corresponding pixel error¹⁸. It is minimum for good fusion.
- NAE: Normalized Absolute Error is the sum ratio of the error value and perfect value. The value should be close to zero for perfect fusion.

The image is decomposed into five modes such as IMF 1 to IMF 5 as shown in Figure 4. It is observed that low frequency information is captured in IMF 1 and 2 respectively, and IMF 3 contains the high frequency information, namely edge information and IMF 4 and 5 does not contain much information of the original image. Hence IMF 1, 2 and 3 are used for fusion. The Figures 5 and 6 are the input images and corresponding IMF of the dataset 1 and Figure 7 is its result. Similarly the Figures 8 and 9 are the input image and corresponding IMF of the dataset 2 and Figure 10 is its corresponding results.

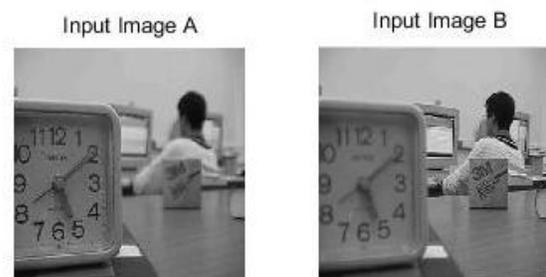


Figure 2. Input images of dataset 1.



Figure 3. Input images of dataset 2.

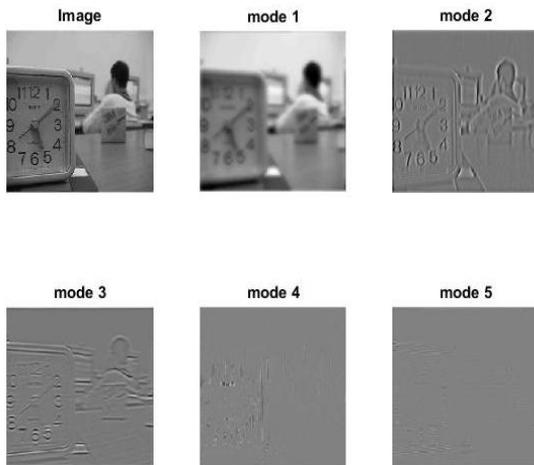


Figure 4. VMD decomposition of image.

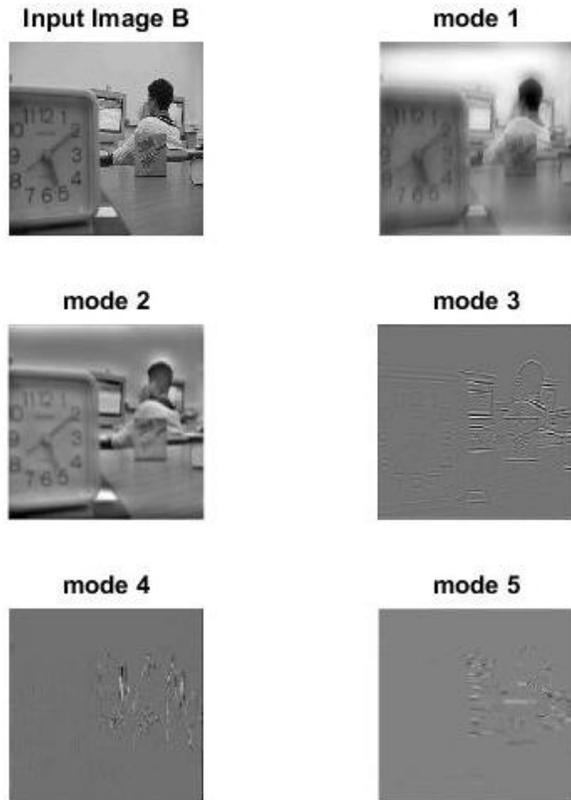


Figure 6. Input image B of dataset 1 and its IMFs.

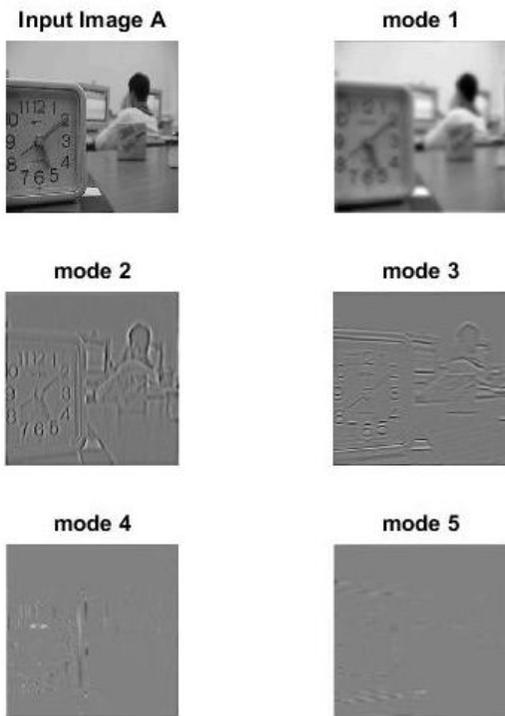


Figure 5. Input image A of dataset 1 and its IMFs.

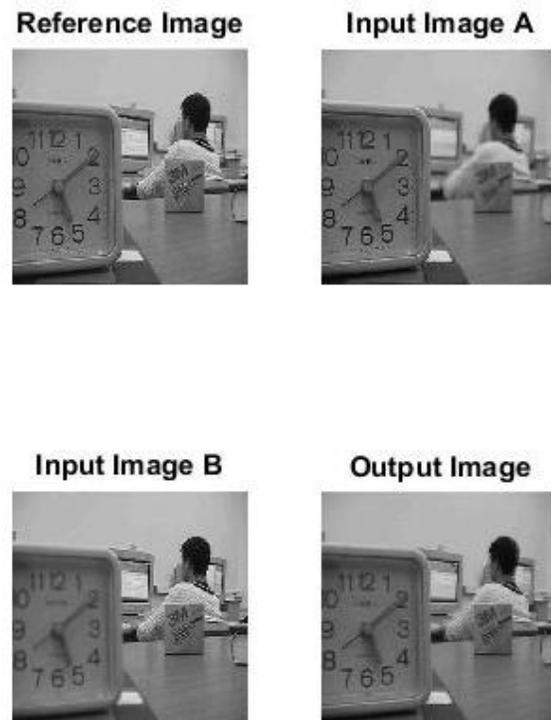


Figure 7. Result of dataset 1.

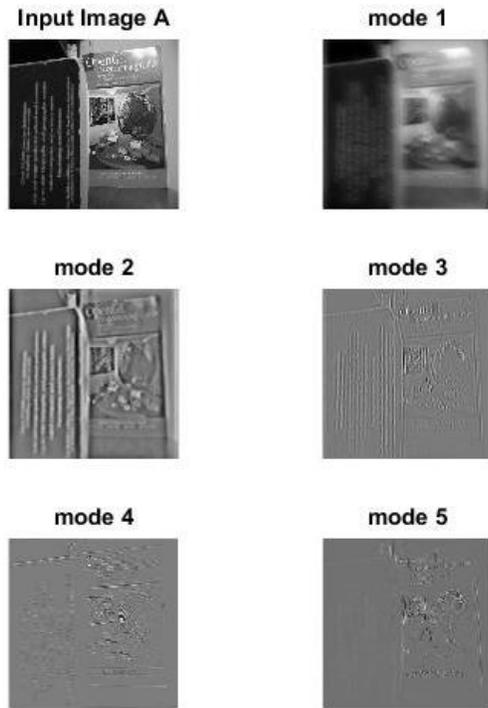


Figure 8. Input image A of dataset 2 and its IMFs.

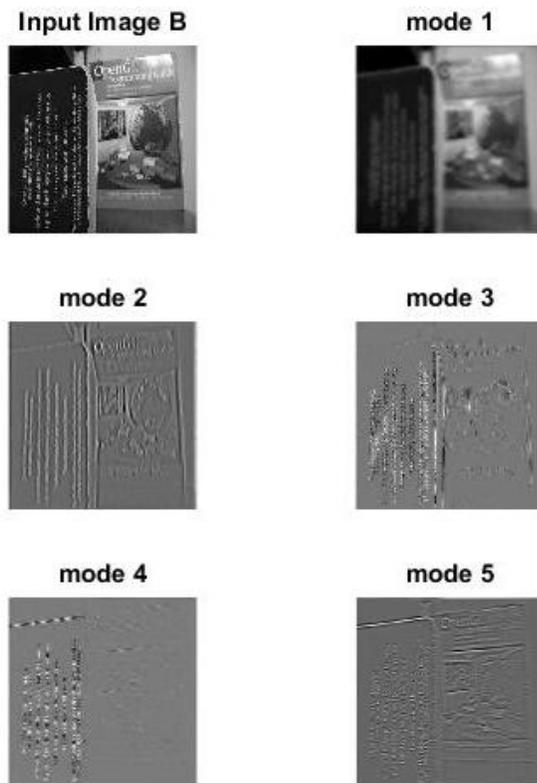


Figure 9. Input image B of dataset 2 and its IMFs.

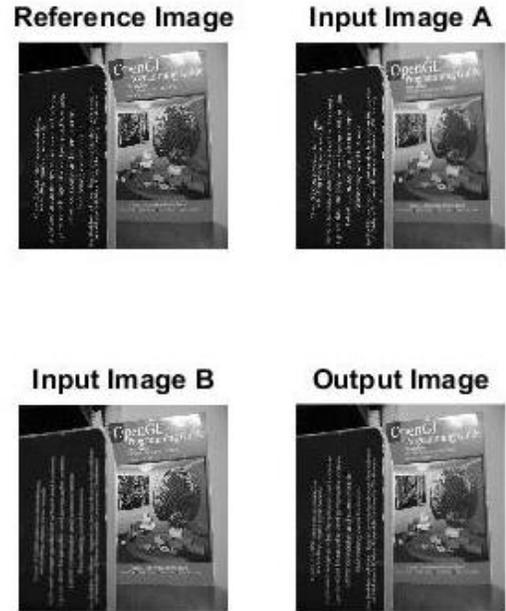


Figure 10. Result of dataset.

The performance analysis of the proposed method is experimented in terms of above quality metrics and comparing with state of the art methods such as Simple Average (SA), Principal Component Analysis (PCA), Select Minimum (SM), Gradient Pyramid (GP), FSD Pyramid (FSDP), Laplacian Pyramid (LP), Morphological Different Pyramid (MDP), Ratio Pyramid (RP), Shift-Invariant Discrete Wavelet Transform (SIDWT) and Contrast Pyramid (CP)¹².

The quality metrics of dataset 1 and 2 are tabulated in Tables 1 and 2 respectively. The Tables 1 and 2 inferred that the PSNR value of the proposed method is high, which means that the proposed method outperforms the comparing algorithm in terms of PSNR. As well as the SNR got high value when it compared with the other algorithms. The NAE and MAE are closer to zero, which means the fused image is closer to the reference image. Also the RMSE and PFE got less value for the proposed method. From the Tables 1 and 2, it is clear that the proposed image fusion approach outperforms the comparing algorithm.

6. Conclusion

This work proposed a novel and efficient image fusion technique as VMD. VMD decomposed the image into constituent modes. The modes have poor pixel intensity is discarded

Table 1. Quantitative measurements of dataset 1

	RMSE	PFE	MAE	CORR	SNR	PSNR	MI	QI	SSIM	AD	SC	MD	NAE
VMD	10.49	4.55	4.03	0.99	26.63	37.89	1.35	0.46	0.96	0	1	85.56	0.02
PCA	10.95	4.79	4.24	0.99	26.39	37.77	1.34	0.45	0.96	0	1	85.15	0.02
SA	10.95	4.79	4.24	0.99	26.39	37.77	1.34	0.45	0.96	0	1	85.16	0.02
SM	13.42	5.87	4.22	0.99	24.63	36.88	1.33	0.44	0.95	0.01	1	88.05	0.02
GP	17.5	7.67	7.29	0.99	22.23	35.70	1.32	0.41	0.92	0.02	1	92.71	0.03
FSDP	20.53	8.98	7.12	0.99	20.93	35.04	1.32	0.41	0.92	0.03	0.99	93.06	0.03
LP	18.47	8.08	5.85	0.99	21.85	35.45	1.33	0.41	0.94	0.02	0.97	92.54	0.03
MDP	32.34	14.15	11.15	0.99	16.99	33.07	1.30	0.40	0.89	0.04	0.95	95.64	0.05
RP	19.09	8.34	6.21	0.99	21.57	35.36	1.32	0.41	0.94	0.02	0.97	92.06	0.02
SDWT	12.68	5.55	4.63	0.99	25.12	37.13	1.34	0.45	0.95	0	1	84.37	0.02
CP	19.07	8.34	6.20	0.99	21.57	35.35	1.32	0.41	0.94	0.02	0.97	92.79	0.03

Table 2 Quantitative measurements of dataset 2

	RMSE	PFE	MAE	CORR	SNR	PSNR	MI	QI	SSIM	AD	SC	MD	NAE
VMD	0.03	5.01	0.01	0.99	25.99	63.98	1.76	0.78	0.99	0	1	0.3	0.02
PCA	0.03	5.16	0.01	0.99	25.75	63.86	1.75	0.79	0.99	0	1	0.33	0.03
SA	0.03	5.18	0.01	0.99	25.71	63.83	1.75	0.79	0.99	0	1	0.33	0.03
SM	0.03	6.29	0.01	0.99	24.01	62.99	1.73	0.75	0.99	0.01	1.04	0.65	0.02
GP	0.04	8.79	0.02	0.99	21.12	61.54	1.61	0.64	0.99	0.01	1.06	0.61	0.05
FSDP	0.05	10.26	0.02	0.99	19.78	60.87	1.61	0.63	0.99	0.02	0.96	0.33	0.05
LP	0.05	9.44	0.02	0.99	20.5	61.23	1.65	0.71	0.99	0.02	0.94	0.13	0.05
MDP	0.07	14.31	0.04	0.99	16.89	59.42	1.54	0.67	0.99	0.04	0.89	0.09	0.08
RP	0.05	9.76	0.02	0.99	20.21	61.08	1.64	0.71	0.99	0.02	0.94	0.23	0.05
SDWT	0.04	7.72	0.02	0.99	22.25	62.10	1.65	0.73	0.99	0	1.01	0.52	0.04
CP	0.05	9.76	0.02	0.99	20.21	61.08	1.64	0.71	0.99	0.02	0.94	0.23	0.05

from the fusion. A fusion rule based on weighing scheme is used for improving the performance of fusion by reducing the mutual information. The performance analysis is based on different quality metrics and compared with state of the art methods. The experimental results show that the proposed can perform image fusion on standard datasets efficiently than the comparing algorithm concluded that the proposed approach gives better performance in terms of quality metrics.

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