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Flexible Flowshop Scheduling Model with Four Stages

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Abstract

Objectives: In this paper, we consider the Flexible Flowshop Scheduling (FFS) problem with parallel machines. The main objective of this paper is to obtain a good schedule of jobs to minimize the makespan of FFS problem. **Methods/Statistical analysis**: In this study, two heuristic algorithms have been developed of FFS to reduce the makespan. First, we constructed the new heuristic algorithm based on Minimum Processing Time Selective Approach (MPTSA) and Longest Processing Times (LPT) approach to find the optimal or near optimal sequence for minimization of makespan of FFS problem with parallel machines. Next, we developed the heuristic algorithm using PALMER approach. In the PALMER approach we sequence the jobs based on Longest Slope Value (LSV) and obtained the value of objective function. **Findings**: We compared both the heuristic algorithms with the help of numerical illustrations. We solved the same numerical by both the heuristic algorithm and result show that our constructed heuristic algorithm has resulted in a better industrial production makespan. The percentage improvement of our constructive heuristic algorithm is also calculated. Gantt chart is also generated to verify the effectiveness of constructed heuristic algorithm. **Application/Improvements:** Our constructed heuristic algorithm is more effective to reduced the makespan of FFS problems as compare to classic heuristic algorithm as Palmer approach and provide an important tool for decision makers in production management.

Keywords: Flexible Flowshop Scheduling, Gantt Chart, LPT Approach, Makespan, MPTSA Approach, Parallel Machines, Percentage Improvement

1. Introduction

In the last three and half decades, many researchers worked in flexible flowshop scheduling problems and numerous papers have been published on the problem of flexible flowshop scheduling with two stages and three stages. The main reason to study the FFS problem is to get the efficient and reasonable scheduled to get the immense economic benefits in production and manufacturing industries because we do not need to deplete the inordinate physical resources. The Flexible Flowshop Scheduling (FFS) is alternatively named as Blended flowshop scheduling problem, Hybrid flowshop scheduling and multistage scheduling[1]. The general flow shop scheduling problem is a production problem where a set of *n* jobs have to be processed with identical flow pattern on *m* machines and Flexible flowshop scheduling is the generalization of the simple and classical flowshop scheduling with parallel processor environments[2]. The first research was appeared in 70's In the simple flowshop scheduling environment, each machine center or stage consist only one machine and flexible flowshop environment consist the several parallel or unparallel machines on some production stages or all production stages or center. It is the condition for FFS that at least one machine production stage or center has to be consist more than one machines. The flexible flowshop scheduling with parallel machines has been studied by numerous researches. In the con-

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sidered that[3] and[4] are the first who studied the flexible flowshop scheduling problem with two machine centers. In[5] used the branch and bound algorithm for FFS to optimize the makespan as well as other criterion and in[6] modified the method of[5]. FFS is considered as an NP-hard and combinatorial problem[5,7] which are difficult to solve these problems in polynomial times. Hence, heuristics algorithms are developed to solve NP hard problems to obtain the near optimal solutions in reasonable time. Consequently, many heuristic algorithms are developed. In[8] developed a method to obtain a near optimal solution through a multi-stage process in the minimum total time. In[9] studied two stage flexible flowshop scheduling with setup times. In[10-12] also concerned FFS with identical processor to obtain the good solution in polynomial times. In[13] studied the flexible flowshop scheduling model with learning and forgetting effect. In[14] surveyed the hybrid flowshop scheduling.

Here we deal with parallel machines FFS and we represent the FFS environment with parallel machines and it is shown in the Figure 1. In this figure we shown q multiple center or stages and each stage consist t parallel machines and i jobs are to be processed in the series of q multiple center to optimize the given objective function.

Where, $\{(q = 1.2.3 \ldots . l)$ and

$$t = (1, 2, 3, \ldots \ldots, m) \, and \; i = (1, 2, 3, \ldots \ldots, n)\}$$

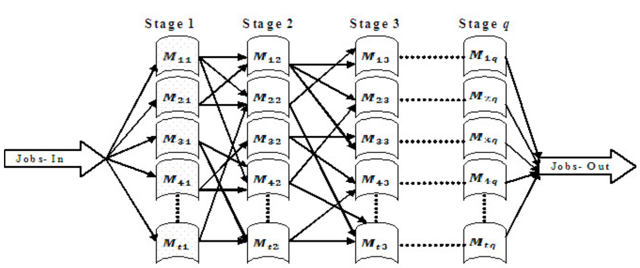


Figure 1. Flexible flowshop scheduling environment of multistage processor with parallel machines

In[7] devolved a heuristic algorithm for two stages. In this paper we extend the work of Sriskandarajah and Sethi from two stage machine center to more than three stages machine center in flexible flowshop scheduling environment using Minimum Processing Time Selective Approach (MPTSA) and Longest Processing Times (LPT) approach to minimize the makespan.

2. Practical Significance of this Model

This paper primarily concerned with production and industrial scheduling problems because now days multi-stage production facilities are established in most of the industries like food industries, rubber[15], paint companies, steel[16], textile industries, glass industries[17], photographic film industries[18], ice cream production[19], Element Analysis of a Four Wheeler Automobile Car Chassis etc. In the classical flowshop scheduling problems, we have only one machine in a one stage so we can produce only one product at a time in the flow line but in the flexible flowshop scheduling or multistage flowshop scheduling we have parallel machines in every stages so we can produced multiple products at a time in the flow line and increase the productivity of our product. So we can say that, FFS is the induction of the classical flowshop scheduling problems.

The practical situation of FFS occurs in our day to day life where four stages production flowshop used with parallel machines processor. We provide an example of denting car repairing in automobile car repairing shop. If someone's car got dented or scratched, then it proceed through four stages for repairing the car in the automobile repair shop and each stages consist the parallel machines. The four stages of removing the scratched or dent of the car are as follows in Figure 2.

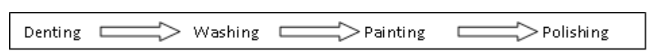


Figure 2. The four stages of removing the scratched or dent of the car

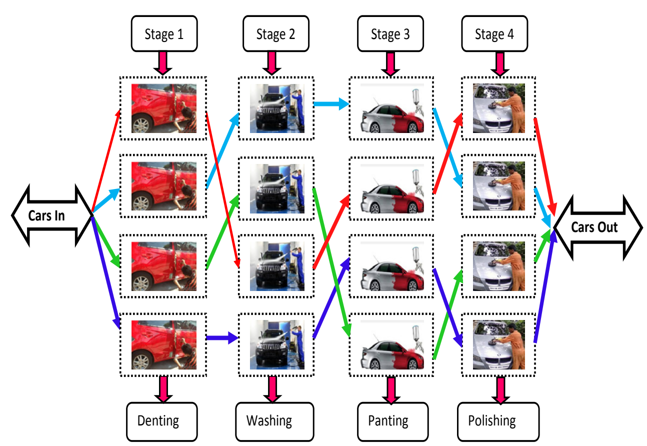


Figure 3. FFS model of automobile dent car repairing

If automobile service center has more than one machines of denting, washing, painting and polishing to increase the capacity or productivity of shop floor then it become the flexible flowshop scheduling. The first stage of repairing the scratched car is denting; the automobile worker first, set the dent to its original position with help of hammer or other techniques. In the second stage, they wash the car and then paint the dent area of the car in third stage. Finally, in the fourth stage they polished or wax the car and make it spotless car. The procedure of automobile dent car repairing is shown in the Figure 3. Where we show that four same machines are parallel arrange in every stages.

3. Assumptions Used

- All the jobs are processed through stage 1 > stage 2 > stage 3 > stage 4 with same production flow line.
- To make job on the second stage, it must be completed on the first stage.
- Any machine cannot be processed more than one operation at a time
- The machine is constantly in use until all the jobs are completed that means breakdown is not occurring or allowed.
- Processing of job must be completed and cannot be interrupted.(preemption is not allowed)
- Setup times are included with processing times.
- Machines and jobs are available at any time on the purview or horizon.
- The jobs related information (Processing time of jobs) is known beforehand that means we work on deterministic scheduling problem not on stochastic scheduling.
- Each machine center has the same number of identical machines and all the machines are paralleled process in that particular stage. Furthermore, at least two production stages should be in the FFS model.
- Each job processed more than two tasks.

4. Notations and Parameters Used

- $i =$ the number of jobs $(i = 1 \ to \ n)$
- $q =$ the number of machine centers or stages $(q = 1 \ to \ 4)$
- $t =$ the number of parallel machines in the stage $(t = 1 \ to \ 4)$
- $j_i = i$ th independent jobs
- $m_{ti} =$ The processing time of i th job. Where, $i = 1, 2, \ldots\ldots, n$.
- $S_q =$ The q th machine center or stage.
- $M_{tq} =$ The t th parallel machine in q th stage.
- $F_{S_q} =$ The categorized flexible flowshop of q^{th} stage.
- $C_{\max(FLEX)} =$ The total completion time of i th job on all the stages also called makespan of flexible flowshop scheduling.
- $C_{\max(F_{S_q})} =$ The makespan of q th stage categorized FFS.
- $\max\left(C_{\max(F_{S_q})}\right) =$ maximum makespan among the all the makespan of categorized FFS
- $\max\left(C_{\max(F_{S_q})const}\right) =$ maximum makespan among the all the makespan of categorized FFS using constructive heuristic algorithm.
- $\max\left(C_{\max(F_{S_q})palmer}\right) =$ maximum makespan among the all the makespan of categorized FFS using PALMER heuristic algorithm
- $Y_i =$ The slope of i th independent jobs.
- $m_{tq} =$ Processing time of the job on t machine of q stage where $q = 1 \ to \ 4$.
- $F_{Flow \ S_q} =$ processing flow pattern of F_{S_q}, where $q = 1 \ to \ 4$

5. Performance Measure of Flexible Flowshop Scheduling with Parallel Machines

5.1 Completion Time Based Measures

We measure the Total Completion Time or Total Flow Time or Makespan for FFS problem and also compute the maximum makespan of every categorized FFS.

- The makespan of FFS problem is the sum of the makespan's of all the categorized flowshops.

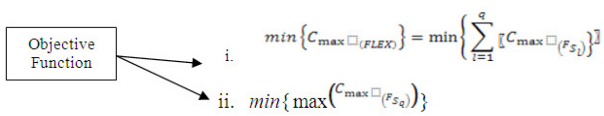
$$C_{\max(FLEX)} = C_{\max(F_{S_1})} + C_{\max(F_{S_2})} + C_{\max(F_{S_3})} + \ldots\ldots\ldots\ldots\ldots + C_{\max(F_{S_q})}$$

Or
$$C_{\max(FLEX)} = \sum_{i=1}^{q} C_{\max(F_{S_i})}$$

$$\max\left(C_{\max(F_{S_q})}\right) = \max\{C_{\max(F_{S_1})}, C_{\max(F_{S_2})}, C_{\max(F_{S_3})}, \; C_{ma}\}$$

6. Objective Function

The objective of this chapter is to obtained the sequence of jobs to minimize the makespan of the flexible flowshop scheduling problem and also minimize the maximum makespan of categorized FFS.



7. Problem Description

In this chapter we studied the four stages flexible flowshop scheduling problem with a parallel machines in each center. We consider that a set of n independent jobs $j_i(j_1, j_2, j_3, j_4, \ldots\ldots j_n)$ with processing time $p_i(p_1, p_2, p_3 \ldots\ldots p_n)$ respectively are executed in q stages or center $(S_1, S_2, \cdots \cdots .. S_q)$ and each stage having $M_{tq}(M_{1q}, \quad M_{2q}, \ldots, \quad M_{mq})$ homogeneous parallel machines where $i = (1 \; to \; n), \quad q = (1 \; to \; l) \; and \; t = (1 \; to \; m)$.

All the jobs have to processed through four operations as S_1 to S_4 stages. Furthermore, we also assumed that each stage of flexible flowshop scheduling problem is categorized as single stage flowshop scheduling with four machines. Hence, q stages having the F_{S_q} simple flowshop scheduling problems like,

$(S_1 \rightarrow F_{S_1})$, $(S_2 \rightarrow F_{S_2})$, $(S_3 \rightarrow F_{S_3})$, $(S_4 \rightarrow F_{S_4})$,……….. $(S_q \rightarrow F_{S_q})$. Hence, four stages flexible flowshop scheduling problem categorized to four single stage flexible flowshop scheduling problems.

8. Mathematical Model of the Four Stages Flexible Flowshop Scheduling in the Matrix Form

The mathematical model of four stages FFS is shown in Table 1 and jobs processing times into the four stages shown in Table 2.

Table 1. Four stages flexible flowshop scheduling model in a matrix form

Jobs (i)	Stage S_1 Machine M_{t1}	Stage S_2 Machine M_{t2}	Stage S_3 Machine M_{t3}	Stage S_4 Machine M_{t4}	Flow pattern of jobs
j_1	M_{11}	M_{12}	M_{13}	M_{14}	⇨ $F_{Flow\,S_1}$
j_2	M_{21}	M_{22}	M_{23}	M_{24}	⇨ $F_{Flow\,S_2}$
j_3	M_{31}	M_{32}	M_{33}	M_{34}	⇨ $F_{Flow\,S_3}$
j_4	M_{41}	M_{42}	M_{43}	M_{44}	⇨ $F_{Flow\,S_4}$
j_n	⇩ F_{S_1}	⇩ F_{S_2}	⇩ F_{S_3}	⇩ F_{S_4}	⇦ Categories of jobs

Table 2. Jobs processing times into four stages

Jobs (i)	Stage (S_1)	Stage (S_2)	Stage (S_3)	Stage (S_4)
	Processing time m_{i1}	Processing time m_{i2}	Processing time m_{i3}	Processing time m_{i4}
j_1	m_{11}	m_{12}	m_{13}	m_{14}
j_2	m_{21}	m_{22}	m_{23}	m_{24}
j_3	m_{31}	m_{32}	m_{33}	m_{34}
j_4	m_{41}	m_{42}	m_{43}	m_{44}
j_n	m_{n1}	m_{n2}	m_{n3}	m_{n4}

9. Constructive Heuristic Algorithm for Flexible Flowshop Problem with Parallel Machines

Here, we construct the new heuristic algorithm to solve the flexible flow-shop problems of more than three machine centers as makespan criterion. This algorithm is based on Minimum Processing Time Selective Approach (MPTSA) and Longest Processing Times (LPT) approach.

Step 1: Categorization of Flexible Flowshop Scheduling
First, we categorize the four stages flexible flow shop problem into the four single stage flexible flowshop scheduling problem. If there are four machines centers (stages) and each of them machine center has four homogeneous parallel machines. Hence, there will be F_{S_q} single stage flexible flowshop scheduling problems (where $= 1 \ to \ 4$) Which we categorized as follows in Figure 4.

$$F_{S_1} = M_{11} + M_{21} + M_{31} + M_{41}$$
$$F_{S_2} = M_{12} + M_{22} + M_{32} + M_{42}$$
$$F_{S_3} = M_{13} + M_{23} + M_{33} + M_{43}$$
$$F_{S_4} = M_{14} + M_{24} + M_{34} + M_{44}$$

Figure 4. Categorization of four stages FFS into single stage FFS

We need to categorize the four stage flexible flowshop scheduling model into the four different single stage flexible flowshop scheduling problem to assign all the jobs.

Step 2: Job Assigning Process using MPTSA
In this step we used the Minimum Processing Time Selective Approach (MPTSA). According to this approach, first we will select to that job which has minimum processing time among the entire machines centers and then, we will assign to this job on that particular flowshop scheduling categories where, it belongs to. In the similar way, we will assign to all the jobs in the F_{S_q} categories which it belongs and jobs processing flow pattern of each category of flowshop (F_{S_q}). For example: If job j_3 has the minimum processing times $[\![{}^{"}m]\!]_{\downarrow 34}{}^{"}$ (means third job on four stages) then, j_3 have to assign into the $[\![{}^{"}F]\!]_{\downarrow}(S_{\downarrow}4)^{"}$ flow shop scheduling categories.

Step 3: Applying LPT rule
After assigning to all the jobs, we applied the Longest Processing Times (LPT) approach(decreasing order of processing times of jobs) between the assigned jobs on each flow shop scheduling categories (F_{S_q}) of every machines center stage.

Step 4: Job processing flow pattern and construct the In – Out table
The flow pattern of the jobs of every (F_{S_q}) category will remain the same throughout the job process in all the stags as shown in Figure 5.

Figure 5. Flow pattern of jobs in FFS

The processing or flowing pattern of the jobs (which we assigned in the flowshop scheduling categories F_{S_q}) on the different machines on different stages are represented in the Table 3. In the table arrow show that F_{S_1} follow the processing pattern of $F_{Flow\ S_1}$ and similarly,

$F_{S_2} \Rightarrow F_{Flow\ S_2}$, $F_{S_3} \Rightarrow F_{Flow\ S_3}$ and $F_{S_4} \Rightarrow F_{Flow\ S_4}$

Table 3. Job processing flow pattern of each category of flow shop (F_{S_q})

	$F_{Flow\ S_1}$ ⇩	$F_{Flow\ S_2}$ ⇩	$F_{Flow\ S_3}$ ⇩	$F_{Flow\ S_4}$ ⇩
F_{S_1} ⇨	M_{11}	M_{21}	M_{31}	M_{41}
F_{S_2} ⇨	M_{12}	M_{22}	M_{32}	M_{42}
F_{S_3} ⇨	M_{13}	M_{23}	M_{33}	M_{43}
F_{S_4} ⇨	M_{14}	M_{24}	M_{34}	M_{44}

The Job processing flow pattern of each category of flow shop (F_{S_q}) is shown in Table 3. Now, Construct the In – Out table for each category of flow shop (F_{S_q}) scheduling according to Table 3 of job processing flow pattern.

Step 5: Compute the Makespan $C_{\max (FLEX)}$ and max $(C_{\max (F_{S_q})})$
Now calculate the total completion times or makespan of the flexible flowshop scheduling problem and maximum makespan among all the flowshop categories F_{S_q}

$$C_{\max(FLEX)} = C_{\max(F_{S_1})} + C_{\max(F_{S_2})} + C_{\max(F_{S_3})} + C_{\max(F_{S_4})}$$

$$\max\left(C_{\max(F_{S_q})}\right) = \max\{C_{\max(F_{S_1})}, C_{\max(F_{S_2})}, C_{\max(F_{S_3})}, C_{\max(F_{S_4})}\}$$

10. Proposed Heuristic Algorithm for Flexible Flowshop Scheduling Problem with Parallel Machines Using Palmer Approach

Step 1: In this algorithm Step 1 and Step 2 are the same as our above constructive heuristic algorithm.

Step 3: Now compute the slope Y_i for n^{th} jobs ($i = 1\ to\ n$) for every category of flow shop scheduling (F_{S_q}) on each machine center stage (S_q) as follows;

$$Y_i = -\sum_{k=1}^{q}\{q-(2k-1)\}m_{i[(F]S_q)}$$

Step 4: Now sequenced the jobs in every flow shop scheduling categories (F_{S_q}) according to Longest Slope Value (LSV) that means non-increasing order or descending order of Y_i such that, $Y_1 \geq Y_2 \geq Y_3 \geq Y_4 \ldots\ldots\ldots\ldots \geq Y_n$

Step 5: After scheduled the jobs using LSV rule, now we construct the In – Out table same as step 4 of above constructed heuristic algorithm.

Step 6: Calculate the Makespan $C_{\max(FLEX)}$ and max $\left(C_{\max(F_{S_q})const}\right)$ according to step 5 of above constructed heuristic algorithm.

11. Numerical Illustrations

Consider the 10 jobs $[(j]_1\ to\ j_{10})$ with processing times m_{1i}, m_{2i}, m_{3i} and m_{4i} each of them having four tasks (4-stages of machine center) Each machine center has four machines. Ten jobs four stages Flexible flowshop scheduling problem is described in Table 4.

11.1 Numerical Solved by First Constructed Heuristic Algorithm

As per step 1: First we categories the flexible flow shop scheduling problem as per step 1,

Table 4. Ten jobs four stages flexible flowshop scheduling

Jobs	Stage (S_1)	Stage (S_2)	Stage (S_3)	Stage (S_4)
	m_{1i}	m_{2i}	m_{3i}	m_{4i}
j_1	6	2	9	5
j_2	8	3	4	2
j_3	5	4	7	8
j_4	6	5	2	4
j_5	5	2	4	1
j_6	3	4	1	2
j_7	1	3	5	2
j_8	2	7	4	5
j_9	8	4	3	6
j_{10}	2	1	6	3

$$F_{S_1} = M_{11} + M_{21} + M_{31} + M_{41}$$
$$F_{S_2} = M_{12} + M_{22} + M_{32} + M_{42}$$
$$F_{S_3} = M_{13} + M_{23} + M_{33} + M_{43}$$
$$F_{S_4} = M_{14} + M_{24} + M_{34} + M_{44}$$

As per step 2: In this step we used the MPTSA. According to MPTSA rule we assign all the jobs in our defined flowshop scheduling categories which we obtained in above step and it is shown in Table 5.

Table 5. Assigning the Jobs in categorized FFS

Machines Center Stage $(q = 1\ to\ 4)$	Flowshop Categories (F_{S_k})	Assigned Jobs
1	F_{S_1}	$j_7,\ j_8$
2	F_{S_2}	$j_1,\ j_3,\ j_{10}$
3	F_{S_3}	$j_4,\ j_6,\ j_9$
4	F_{S_k}	$j_2,\ j_5$

As per Step 3: Apply the LPT rule between the jobs which we assign to different categories in the above step is as follows in Table 6.

Table 6. Apply LPT rule in assigned jobs

Machines Center Stage K	Flowshop Categories (F_{S_k})	Assigned Jobs		Sequenced Jobs
1	F_{S_1}	j_7, j_8	Apply LPT RULE	$j_7 > j_8$
2	F_{S_2}	j_1, j_3, j_{10}		$j_3 > j_1 > j_{10}$
3	F_{S_3}	j_4, j_6, j_9		$j_9 > j_4 > j_6$
4	F_{S_4}	j_2, j_5		$j_2 > j_5$

As per step 4: Construct the In–Out table for the sequences of all the flowshop categories which we obtained in step 3 and it is shown in Table 7.

As per Step 5: To calculate the makespan of four stages flexible flowshop scheduling problem and max

$\left(C_{max(F_{S_q})const}\right)$ is as follows;

$$C_{max(FLEX)c} = C_{max(F_{S_1})} + C_{max(F_{S_2})} + C_{max(F_{S_3})} + C_{max(F_{S_4})} = 19 + 34 + 27 + 20 = 100\ units$$

$$\max\left(C_{max(F_{S_q})}\right) = \max\left\{C_{max(F_{S_1})}, C_{max(F_{S_2})}, C_{max(F_{S_3})}, C_{max(F_{S_4})}\right\}$$

$$\max\left(C_{max(F_{S_q})const}\right) = \max\{19, 34, 27, 20\} = 34\ units$$

12. Numerical Solved by Proposed Heuristic Algorithm Using Palmer Approach

As per Step 1 & 2 assign to all the jobs in our defined flow-shop scheduling categories (F_{S_q})

As Per Step 3: Calculate the slope Y_i for every job in all the flowshop scheduling categories (F_{S_q}) as shown in Table 8.

As per Step 4: Sequenced the jobs according to Longest Slope Value(LSV), descending order of Y_i such that, $Y_1 \geq Y_2 \geq Y_3 \geq Y_4 \ldots\ldots\ldots\ldots\ldots\ldots \geq Y_n$. It is shown in Table 9.

As per Step 5: Construct the In –Out Table according to Table 9. In – Out table is shown in Table 10.

As per Step 6: To calculate the makespan of four stages flexible flowshop scheduling problem and max max

$\left(C_{max(F_{S_q})palmer}\right)$ is as follows;

$$C_{max(FLEX)c} = C_{max(F_{S_1})} + C_{max(F_{S_2})} + C_{max(F_{S_3})} + C_{max(F_{S_4})} = 19 + 36 + 28 + 22 = 105\ units$$

$$\max\left(C_{max(F_{S_q})}\right) = \max\left\{C_{max(F_{S_1})}, C_{max(F_{S_2})}, C_{max(F_{S_3})}, C_{max(F_{S_4})}\right\}$$

Table 7. In – Out Table for FFS using constructive heuristic algorithm

	Jobs $(j)_i$	S_1	S_2	S_3	S_4	
		In --- Out	In --- Out	**In --- Out**	**In --- Out**	
F_{S_1}	j_8	0 --- 2	2 --- 6	6 --- 12	12 --- 15	
	j_7	2 --- 3	6 --- 12	12 --- 17	17 --- 19	$C_{max(F_{S_1})}$
F_{S_2}	j_3	0 --- 5	5 --- 9	9 --- 16	16 ---24	
	j_1	5 --- 11	11 ---13	16 --- 25	25 ---30	
	j_{10}	11 --- 13	13 --- 14	25 --- 31	31 --- 34	$C_{max(F_{S_2})}$
F_{S_3}	j_9	0 --- 8	8 --- 12	12 --- 15	15 --- 21	
	j_4	8 --- 14	14 --- 19	19 --- 21	21 --- 25	
	j_6	14 --- 17	19 --- 23	23 --- 24	25 --- 27	$C_{max(F_{S_3})}$
F_{S_4}	j_2	0 --- 8	8 --- 11	11 --- 15	15 --- 17	

Table 8. Calculate the slope value using palmer approach

Jobs $(j)_i$ $k = 1$		m_{1i} $k = 2$	m_{2i} $k = 3$	m_{3i} $k = 4$	m_{4i}	Y_i
$\{4-[(2\times1)-1]\}$ $=3$			$\{4-[(2\times2)-1]\}$ $=1$	$\{4-[(2\times3)-1]\}$ $=-1$	$\{-[(2\times4)-1]\}$ $=-3$	
F_{S_1}	j_7	3	3	− 5	− 6	5
	j_8	6	7	− 4	− 9	6
F_{S_2}	j_1	18	2	− 9	− 15	4
	j_3	15	4	− 7	− 24	12
	j_{10}	6	1	− 6	− 9	8
F_{S_3}	j_4	18	5	− 2	− 12	− 9
	j_6	9	4	− 1	− 6	− 6
	j_9	24	4	− 3	− 18	− 7
F_{S_4}	j_2	24	3	− 4	− 6	− 17
	j_5	15	2	− 4	− 3	− 10

Table 9. Sequenced the jobs according to Longest Slope Value (LSV)

Machines Center Stage K	Flowshop Categories (F_{S_q})	Assigned Jobs	Sequenced the Jobs With respect to PALMER Slope Value $(Y)_i$	Sequenced Jobs
1	F_{S_1}	j_7, j_8		$j_7 > j_8$
2	F_{S_2}	j_1, j_3, j_{10}		$j_3 > j_{10} > j_1$
3	F_{S_3}	j_4, j_6, j_9		$j_6 > j_9 > j_4$
4	F_{S_4}	j_2, j_5		$j_5 > j_2$

Table 10. In – out table for FFS using palmer approach

	Jobs $(j)_i$	S_1 In --- Out	S_2 In --- Out	S_3 In --- Out	S_4 In --- Out	
F_{S_1}	j_8	0 --- 2	2 --- 6	6 --- 12	12 --- 15	
	j_7	2 --- 3	6 --- 12	12 --- 17	17 --- 19	$C_{max}(F_{S_1})$
F_{S_2}	j_3	0 --- 5	5 --- 9	9 --- 16	16 ---24	
	j_{10}	5 --- 7	9 ---10	16 --- 22	24 ---27	
	j_1	7 --- 13	13 --- 15	22 --- 31	31 --- **36**	$C_{max}(F_{S_2})$

F_{S_3}	j_6	0 --- 3	3 --- 7	7 --- 8	8 ---10	
	j_9	3 --- 11	11 --- 15	15 --- 18	18 --- 24	
	j_4	11 --- 17	17 --- 22	22 --- 24	24 --- **28**	$C_{max}(F_{S_3})$
F_{S_4}	j_5	0 --- 5	5 --- 7	7 --- 11	11 --- 12	
	j_2	5 --- 13	13 --- 16	16 --- 20	20 --- **22**	$C_{max}(F_{S_4})$

$$\max\left(C_{\max}(F_{S_q})_{palmer}\right) = \max\{19,36,28,22\} = 36\ units$$

13. Percentage Improvement

The percentage improvement of the first constructed heuristic algorithm over second proposed heuristic algorithm using Palmer approach is calculated for the makespan. The percentage improvement is calculated as follows:

i. Percentage improvement of the total makespan

$$= \frac{\left[\left(C_{\max(FLEX)_p}\right) - C_{\max}(FLEX)_c\right]}{C_{\max(FLEX)_p}} \times 100$$

$$\%\ \mathrm{IMP} = \frac{(105 - 100)}{105} \times 100 = 4.8\ \%$$

ii. Percentage improvement of the maximum makespan

$$= \frac{\left[\max\left(C_{\max}(F_{S_q})_{palmer}\right) - \max\left(C_{\max}(F_{S_q})_{const}\right)\right]}{\max\left(C_{\max}(F_{S_q})_{palmer}\right)} \times 100$$

$$\%\ \mathrm{IMP} = \frac{(36 - 34)}{36} \times 100 = 5.6\ \%$$

It is observed that our constructive heuristic algorithm provides 4.8 % improvement in the makespan. In other word we can say our constructive heuristic algorithm gives 4.8% better result as compare to PALMER based heuristic algorithm

14. Comparatative Study Between Constructive and Palmer Based Heuristics Algorithm in the Matrix Form

The Comparatative study between constructive and palmer based heuristics algorithm is shown in Table 11.

Table 11. Comparatative study between constructive and palmer based heuristics algorithm

Makespan	Constructive Heuristic Algorithm	Palmer Based Heuristic Algorithm
$C_{\max}(F_{S_1})$	19	19
$C_{\max}(F_{S_2})$	34	36
$C_{\max}(F_{S_3})$	27	28
$C_{\max}(F_{S_4})$	20	22
$C_{\max}(FLEX)$	100 units	105 units
% Improvement		
% IMP of the total makespan		4.8 %
% IMP of the maximum makespan		5.6 %

15. Gantt Chart

Gantt Chart of Flexible Flowshop Scheduling using Constructive Heuristic Algorithm is shown in Figure 6. In the Gantt chart we show the result of our constructive heuristic algorithm using Table 11. With the help of Gantt chart we also calculate the Ideal time of machines and it is 18 units.

16. Conclusion

In this paper, we developed the two heuristic algorithms for the flexible flowshop scheduling with four stages to optimize the makespan and maximum makespan of categorized flexible flowshop. First constructive heuristic

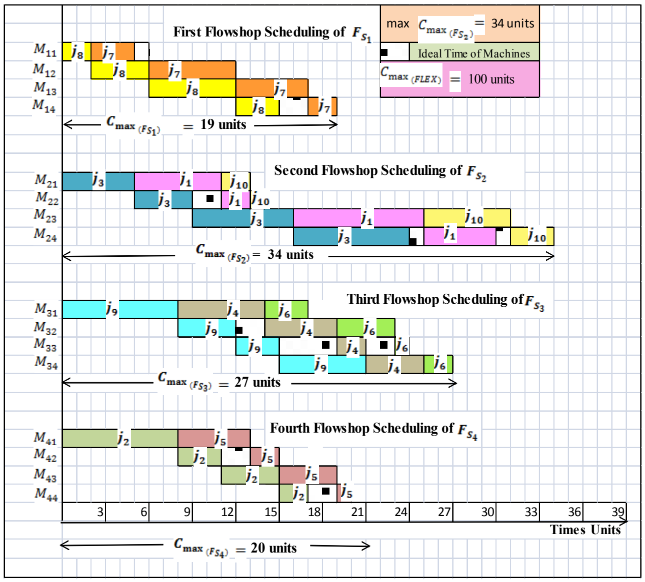


Figure 6. Gantt chart of flexible flowshop scheduling using constructive heuristic algorithm

algorithm is based on MPTSA and second proposed heuristic algorithm is based n PAOLMER approach. In this four stages flexible flowshop scheduling model we used four parallel machines in each single stage. To construct these heuristic algorithms first, we divide the four stage FFS problem into single stage FFS problem then calculate the makespan of each single FFS. Consequently, obtained the makespan of four stages FFS problem. The Comparatative study between both the heuristic algorithm shows that our constructive heuristic algorithm is outperformed and give the better result for the makespan as compared to PALMER based heuristic algorithm which is shown in Table 11. The performance improvement of our constructive heuristic is 4.8 % in the makespan of FFS and 5.6 % in the maximum makespan of categorized FFS. Hence, our constructive heuristic algorithm is good for minimization of makespan for four stages FFS. The Gantt chart is also generated to see the effectiveness of our constructive heuristic algorithm.

17. Future Research

For the future research we can extend to this work for more than four stages. In this model, we used parallel machines processor in each stage. Hence, we can also use unparallel, machines processor for future study. Here we used flexible flowshop environment so we can also used some other environments instead of flowshop like job shop, open shop, mixed shop and fuzzy environment. Furthermore, exact algorithms (like branch and bound algorithm, tabu search etc.), heuristic algorithms (like NEH algorithm, CDS algorithm etc), metaheuristic algorithm (like Genetic algorithm, Ant Colony optimization technique, Simulated Annealing, swarm Optimization teaching etc.) and hyper metaheurictics algorithms are also apply with diffident parameters like break down of machines, set up times, transportation times, job block etc.

18. References

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