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On Certain New Classes of Functions with Fixed Point Defined by Generalized Differential Operator

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Abstract

The primary objective for this artical, the authors present a new certain differential operator $S_w^k f(z)$ with a new subclass $S_w^* M(\xi,\gamma)$, the main motivation for this article is to explore the several essential results and attributes. Moreover, we infer numerous results in the hadamard products.

Keywords: Analytic Functions, Differential Operator, Fixed Point, Hadamard Products, Positive Coefficients,

1. Introduction and Preliminaries

Now, we will go to present preface for univalent functions, with important contributions to get some important properties in this is article.

Let A defined the class of a univalent functions $f(z)$ of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \quad \left(z \in U,\ p = \{1,2,3,...\}\right), (1.1) \tag{1.1}$$

which is functions in the $U = \{z \in C : |z| < 1\}$.

A univalent functions in A is defined to be starlike class of order ϑ if and only if

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > \vartheta \qquad (z \in U), \tag{1.2}$$

$\vartheta(0 \le \vartheta < p)$.

So also, a univalent functions in A is defined to be convex class of order ϑ if and only if

$$\operatorname{Re}\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \vartheta \qquad (z \in U), \tag{1.3}$$

$\vartheta(0 \le \vartheta < p)$.

2. Methodology

let w be define as

$A(w) = \{f \in H(U) : f(w) = f'(w) - 1 = 0\}$. studied see[1] presented the accompanying classes

$$S_w = \{f \in A(w) : f \text{ is univalent in } U\}, \tag{1.4}$$

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$$SA_w = \left\{ f \in A(w) : Re\left(\frac{(z-w)f'(z)}{f(z)} \right) > 0 \right\} \quad (z \in U), \tag{1.5}$$

$$VR_w = \left\{ f \in A(w) : Re\left\{ 1 + \frac{(z-w)f''(z)}{f'(z)} \right\} > 0 \right\} \quad (z \in U) \tag{1.6}$$

Later is authors Acu and Owa[2] were studied the classes broadly.

Let analytic functions $f(z)$ defined of $A(w)$ comprising functions:

$$f(z) = (z-w) + \sum_{n=2}^{\infty} a_n (z-w)^n$$
$$((z\text{-w}) \in U^* = \{z : z \in C \text{ and } 0 < z < 1\}). \tag{1.7}$$

The function $f(z)$ in S_w is defined to be starlike class:

$$Re\left\{ -\frac{(z-w)f'(z)}{f(z)} \right\} > \vartheta \; ((z\text{-w}) \in U^*) \tag{1.8}$$

$\vartheta(0 \le \vartheta < 1)$. We define by $S_w^*(\vartheta)$ the class of all starlike of order ϑ.

So also, a analytic function f of S_w is defined to be convex class:

$$Re\left\{ -1 - \frac{(z-w)f''(z)}{f'(z)} \right\} > \vartheta \; (z\text{-w} \in U^*) \tag{1.9}$$

$\vartheta(0 \le \vartheta < 1)$. We define by $VR_w(\vartheta)$ the class of all convex of order ϑ.

We take see of that the class $S_0^*(\vartheta)$ and different other subclasses of $S_w^*(\vartheta)$ have been concentrated rather broadly by many authors see[2,5-12].

We denote the contribution is generalized operator:

$$S_w^0 f(z) = f(z),$$

$$S_w^1 f(z) = [1 - (\lambda - \delta)(\beta - \alpha)] f(z) + [(\lambda - \delta)(\beta - \alpha)](z-w) f'(z),$$

and for $k = 1, 2, 3, \ldots$

$$S_w^k f(z) = S(S_w^{k-1} f(z)),$$
$$= (z-w) + \sum_{n=2}^{\infty} ((\lambda - \delta)(\beta - \alpha)(n-1) + 1)^k a_n (z-w)^n, \tag{1.10}$$

for
$f \in A, \alpha, \beta, \delta, \lambda \ge 0, \lambda > \delta, \beta > \alpha,$

and $n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

***Remark* 1:** (i) When $w = 1$, they presented by two authors Ramadan and Darus[13] the new contribution.

With the new contribution is the differential operator, we denote $S_w M(\xi, \gamma)$ class:

***Definition* 1:** let $f(z) \in S_w$ be given by (1.7). The class $S_w M(\xi, \gamma)$ is denoted by:

$$S_wM(\xi,\gamma)=$$

$$\left\{ f(z)\in S_w : \left| \frac{\dfrac{(z-w)(S_w^k f(z))''}{(S_w^k f(z))'}+2}{\dfrac{(z-w)(S_w^k f(z))''}{(S_w^k f(z))'}+2\gamma} \right| < \xi \right\},$$

(1.11)

For some $0<\xi\leq 1, 0\leq\gamma\leq 1,$ and $n\in\mathbb{N}_0$ for all $z-w\in U^*$.

***Remark* 1:** (i) When $w=0,$ we now have this is class introduced and studied by two authors Atshan and Sulman[14].

Let $A^*(w)$ define the subclass of $A(w)$ comprising of the form:

$$f(z)=(z-w)+\sum_{n=2}^{\infty}a_n(z-w)^n, \quad (a_n\geq 0).$$

(1.12)

additionally, we denote $S_w^*M(\xi,\gamma)$ by

$$S_w^*M(\xi,\gamma)=S_wM(\xi,\gamma)\cap A^*(w).$$

(1.13)

We use techniques similar to this used see[16-21].

3. Main Results

In this mathematics work, we have inside this is article properties with many results for the subclass $S_wM(\xi,\gamma)$ of the form:

$$G(z)=\frac{c+p}{z-w^c}\int_0^{z-w}u^{c-1}f(u)du \qquad \text{for all } c>-1,$$

for the class $S_w^*M(\xi,\gamma)$ is considered.

4. Coefficient Inequalities

In this part, the result gives an adequate condition a function, regular in U^*, to be in $S_w^*M(\xi,\gamma)$.

Theorem 2.1.

Let $f(z)\in S_w$ *.Then* $f(z)\in S_w^*M(\xi,\gamma) \iff$

$$\sum_{n=2}^{\infty}((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k.n\left[n-\xi(n+2\gamma-1)+1\right]|a_n|$$
$$\leq 2(\xi\gamma-1),(2.1)$$

(2.1)

where $0<\xi\leq 1, 0\leq\gamma\leq 1,$ and $n\in\mathbb{N}_0$

Proof. Let us suppose that inequality (2.1) is true. Further assume that

$$M(f,f')=\left|(z-w)(S_w^k f(z))''+2(S_w^k f(z))'\right|$$
$$-\xi\left|(z-w)(S_w^k f(z))''+2\gamma(S_w^k f(z))'\right|$$

Then for |z-w|=r<1, we get

$$|(z-w)(\sum_{n=2}^{\infty}n(n-1)((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k.a_n(z-w)^{n-2})|$$

$$+2\left|\left(1+\sum_{n=2}^{\infty}n((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k.a_n(z-w)^{n-1}\right)\right|$$

$$-\xi\left|(z-w)\left(\begin{array}{l}\sum_{n=2}^{\infty}n(n-1)((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k\\ .a_n(z-w)^{n-2}\end{array}\right)\right|$$

$$+2\gamma\xi\left|\left(1+\sum_{n=2}^{\infty}n((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k.a_n(z-w)^{n-1}\right)\right|$$

$$\leq \sum_{n=2}^{\infty}(z-w)^{n-1}n.((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k$$
$$.a_n\left[n+1-\xi n+\xi(1-2\gamma)\right]+(2-\xi 2\gamma)\leq 0.(2.2)$$

(2.2)

Hence by the principle of maximum modulus,

$f(z)\in S_w^* M(\xi,\gamma).$

On the other hand, suppose that analytic functions $f(z)$ denoted by (1.12) and (1.10), we get

$$\left|\frac{\dfrac{(z-w)(S_w^k f(z))''}{(S_w^k f(z))'}+2}{\dfrac{(z-w)(S_w^k f(z))''}{(S_w^k f(z))'}+2\gamma}\right|$$

$$(z-w)\left(\sum_{n=2}^{\infty}n(n-1)((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k \; .a_n(z-w)^{n-2}\right)$$

$$+2\left(1+\sum_{n=2}^{\infty}n((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .a_n(z-w)^{n-1}\right)$$

$$(z-w)\left(\sum_{n=2}^{\infty}n(n-1)((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k \; .a_n(z-w)^{n-2}\right)$$

$$+2\gamma\left(1+\sum_{n=2}^{\infty}n((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .a_n(z-w)^{n-1}\right)$$

$\leq \xi.$

Since $|Re(z-w)|\leq|z-w|$ for all $(z-w)$, we have

$$Re\left\{\frac{\begin{array}{l}(z-w)\\ .\left(\sum_{n=2}^{\infty}n(n-1)((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .a_n(z-w)^{n-2}\right)\\ +2\left(1+\sum_{n=2}^{\infty}n((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .a_n(z-w)^{n-1}\right)\end{array}}{\begin{array}{l}(z-w)\\ .\left(\sum_{n=2}^{\infty}n(n-1)((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .a_n(z-w)^{n-2}\right)\\ +2\gamma\left(1+\sum_{n=2}^{\infty}n((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .a_n(z-w)^{n-1}\right)\end{array}}\right\}$$

$\leq \xi.$

we pick the value of z on the real axis so that

$\dfrac{(z-w)(S_w^k f(z))''}{(S_w^k f(z))'}$ is real.

The inequality (2.2) and letting $r\to 1^-$, we get the coefficient inequality (2.1).

This finishes the proof for (2.1)

Corollary 2.2 Let $f(z)$ the function be defined by (1.12). If $f\in S_w^* M(\xi,\gamma)$, then

$$a_n\leq \frac{2(\xi\gamma-1)}{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .n\left[n-\xi(n+2\gamma-1)+1\right]},$$
$$(n\in\square). \quad (2.2)$$

(2.2)

This is attained for $f(z)$ given by

$$f(z)=(z\quad w)+\frac{2(\xi\gamma\quad 1)}{((\lambda-\delta)(\beta-\alpha)(n-1)+1)\;.n\left[n-\xi(n+2\gamma-1)+1\right]}(z\quad w),$$
$$(\quad\in\square). \quad (2.3)$$

(2.3)

5. Distortion Theorem

In the this part in article presented a second important result for $S_w^* M(\xi,\gamma)$.

Theorem 3.1. Let the functions $f(z)$ given by (1.12) be in the class $S_w^* M(\xi,\gamma)$,

where $0<\xi\le 1, 0\le\gamma\le 1,$, and $n\in \square_0$. Then for $|z - w| = r<1$, we have

$$r-\frac{2(\xi\gamma-1)}{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .n\left[n-\xi(n+2\gamma-1)+1\right]}r$$

$$\le |f(z)|$$

$$\le r+\frac{2(\xi\gamma-1)}{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .n\left[n-\xi(n+2\gamma-1)+1\right]}r$$

and

$$1-\frac{(\xi\gamma-1)}{(\xi\gamma+1)}\le |f'(z)|\le 1+\frac{(\xi\gamma-1)}{(\xi\gamma+1)}.$$

Proof. Since $f\in S_w^* M(\xi,\gamma)$, , from Theorem 2.1 gets the inequality

$$\sum_{n=2}^{\infty} a_n \le \frac{2(\xi\gamma-1)}{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k n\left[n-\xi(n+2\gamma-1)+1\right]}. \tag{3.1}$$

Thus, for $|z - w| = r<1$, also by (3.1) we get

$$|f(z)|\le |z-w|+\sum_{n=2}^{\infty} a_n |z-w|^n \le r+r\sum_{n=2}^{\infty} a_n$$

$$\le r+\frac{2(\xi\gamma-1)}{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .n\left[n-\xi(n+2\gamma-1)+1\right]}r$$

and

$$|f(z)|\ge |z-w|-\sum_{n=2}^{\infty} a_n |z-w|^n \le r-r\sum_{n=2}^{\infty} a_n$$

$$\ge r-\frac{2(\xi\gamma-1)}{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .n\left[n-\xi(n+2\gamma-1)+1\right]}r.$$

from the coefficient inequality (2.1)., it follows that

$$2(\xi\gamma+1)\sum_{n=2}^{\infty} na_n$$

$$\le \sum_{n=2}^{\infty}((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .n\left[n-\xi(n+2\gamma-1)+1\right]a_n$$

$$\le 2(\xi\gamma-1).(3.2)$$

(3.2)

Hence

$$|f'(z)|\le 1+\sum_{n=2}^{\infty} na_n |z-w|^{n-1}$$

$$\le 1+\sum_{n=2}^{\infty} na_n .$$

$$\le 1+\frac{(\xi\gamma-1)}{(\xi\gamma+1)}.$$

$$|f'(z)|\ge 1-\sum_{n=2}^{\infty} na_n |z-w|^{n-1}$$

$$\ge 1-\sum_{n=2}^{\infty} na_n$$

$$\ge 1-\frac{(\xi\gamma-1)}{(\xi\gamma+1)}.$$

This finishes the proof for 3.1.

6. Radii of Starlikeness and Convexity

In the this part in article presented a third important result for $S_w^* M(\xi,\gamma)$.

Theorem 4.1. Let $f(z)$ define in (1.12) for

$S_w^* M(\xi,\gamma)$, where $0<\xi\le 1, 0\le\gamma\le 1$,

and $n\in \mathbb{N}_0$, then function $f(z)$ is starlike, $\eta(0\le\eta<1)$ in $|z-w|<r_1$, where

$$r_1=\inf_{n\ge 2}\left\{\begin{bmatrix}(1-\eta)((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k\\ \times n\left[n-\xi(n+2\gamma-1)+1\right]\end{bmatrix} \div\left[2(n+\eta)(\xi\gamma-1)\right]\right\}^{\frac{1}{n}} \tag{4.1}$$

This is sharp for $f(z)$ in (2.3).

Proof. Now,

$$\left|\frac{(z-w)f'(z)}{f(z)}-1\right|\le 1-\eta, \tag{4.2}$$

for $|z-w|<r_1$. We get

$$\left|\frac{(z-w)f'(z)}{f(z)}-1\right|=\left|\frac{-\sum_{n=2}^{\infty}(1-n)a_n(z-w)^n}{(z-w)+\sum_{n=2}^{\infty}a_n(z-w)^n}\right|$$

$$\le\frac{\sum_{n=2}^{\infty}(1-n)a_n|z-w|^n}{1+\sum_{n=2}^{\infty}a_n|z-w|^n}. \tag{4.3}$$

from (4.3).

If

$$\sum_{n=2}^{\infty}(1-n)a_n|z-w|\le(1-\eta)\left(1+\sum_{n=2}^{\infty}a_n|z-w|\right)$$

or

$$\sum_{n=2}^{\infty}\frac{(n+\eta)}{(1-\eta)}a_n|z-w|^n\le 1. \tag{4.4}$$

From (2.1), (4.4)

If

$$\frac{(n+\eta)}{(1-\eta)}|z-w|^n \le\frac{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k.n\left[n-\xi(n+2\gamma-1)+1\right]}{2(\xi\gamma-1)}. \tag{4.5}$$

Solving (4.5) for $|z-w|^n$, we acquire

$$|z-w| \le\left\{\frac{(1-\eta)((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k.n\left[n-\xi(n+2\gamma-1)+1\right]}{2(n+\eta)(\xi\gamma-1)}\right\}^{\frac{1}{n}}$$

$(n\ge 2)$.

This finishes the proof 4.1.

Theorem 4.2. Let $f(z)$ define in (1.12) in $S_w^* M(\xi,\gamma)$, where, $0<\xi\le 1, 0\le\gamma\le 1$,

and $n\in\mathbb{N}_0$, then functions is convex, $\eta(0\le\eta<1)$ in $|z-w|<r_1$,, where

$$r_2 = \inf_{n\geq 2} \left\{ \begin{bmatrix} (1-\eta)((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k \\ \times(n[n-\xi(n+2\gamma-1)+1]) \end{bmatrix} \right. \\ \left. \div[2n(n+\eta)(\xi\gamma-1)] \right\}^{\frac{1}{n}}$$

(n ≥ 2).

(4.6)

This is sharp for $f(z)$ in (2.3).

Proof. By do the same system utilized in theorem 4.1,

$$\left|\frac{(z-w)f''(z)}{f'(z)}+2\right| \leq 1-\eta, \tag{4.7}$$

for $|z - w| < r_2$, in theorem 2.1.

Consequently the proof of theorem 4.2

7. Closure Theorems

This part in article presented the closure theorems for the class $S_w^* M(\xi,\gamma)$.

The functions are defined by:

$$f_j(z) = (z-w) + \sum_{n=2}^{\infty} a_{n,j}(z-w)^n \qquad (a_{n,j} \geq 0), (5.1)$$

for $z - w \in U^*$. (5.1)

Theorem 5.1. Let $f_j(z)$ defined in

for $S_w^* M(\xi,\gamma)$, where, $0<\xi\leq 1, 0\leq\gamma\leq 1$, ,and

$n \in \mathbb{N}_0$ $j = 1,2,\ldots,l$. Then the function $G(z)$ is:

$$G(z) = (z-w) + \sum_{n=2}^{\infty} b_n(z-w)^n \qquad (b_n \geq 0) \tag{5.2}$$

is a member of the class $S_w^* M(\xi,\gamma)$, where

$$b_n = \frac{1}{l}\sum_{j=1}^{l} a_{n,j} \quad (n \geq 1).$$

Proof. Since $f_j(z) \in S_w^* M(\xi,\gamma)$, Theorem 2.1 that

$$\sum_{n=2}^{\infty}((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k . n\left[n-\xi(n+2\gamma-1)+1\right]a_n \\ \leq 2(\xi\gamma-1).$$

for every $j = 1,2,\ldots,l$. Hence,

$$\sum_{n=2}^{\infty}((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k . n\left[n-\xi(n+2\gamma-1)+1\right]b_n$$

$$= \sum_{n=2}^{\infty}((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k . n\left[n-\xi(n+2\gamma-1)+1\right] \\ .\left\{\frac{1}{l}\sum_{j=1}^{l} a_{n,j}\right\}$$

$$= \frac{1}{l}\sum_{j=1}^{l}\left(\begin{array}{l}\sum_{n=2}^{\infty}((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k \\ .n\left[n-\xi(n+2\gamma-1)+1\right]a_{n,j}\end{array}\right)$$

$$\leq \frac{1}{l}\sum_{j=1}^{l}\left[2(\xi\gamma-1)\right] = 2(\xi\gamma\text{-}1),$$

Which (in see of Theorem 2.1) means that $G(z) \in S_w^* M(\xi,\gamma)$.

Theorem 5.2. The $S_w^* M(\xi,\gamma)$ is closed under convex

linear combination,

where $0<\xi\leq 1, 0\leq\gamma\leq 1$, , ,and $n \in \mathbb{N}_0$.

Proof. assume that

$f_j(z)(j=1,2)$ by (5.1) in $S_w^*M(\xi,\gamma)$

$$H(z)=\tau f_1(z)+(1-\tau)f_2(z) \qquad (0\le\tau\le 1),(5.3)$$

is also in the class $S_w^*M(\xi,\gamma)$. Since for $(0\le\tau\le 1)$,

$$H(z)=(z-w)+\sum_{n=2}^{\infty}\{\tau a_{n,1}+(1-\tau)a_{n,2}\}(z-w)^n, \tag{5.4}$$

we note that

$$\sum_{n=2}^{\infty}((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .n\left[n-\xi(n+2\gamma-1)+1\right]\{\tau a_{n,1}+(1-\tau)a_{n,2}\}$$

$$=\tau\sum_{k=1}^{\infty}((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .n\left[n-\xi(n+2\gamma-1)+1\right]a_{n,1}+$$

$$(1-\tau)\sum_{k=1}^{\infty}((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .n\left[n-\xi(n+2\gamma-1)+1\right]a_{n,2}$$

$$\le\tau\left[2(\xi\gamma-1)\right]+(1-\tau)\left[2(\xi\gamma-1)\right]$$

$$=\left[2(\xi\gamma-1)\right].$$

Hence $H(z)\in S_w^*M(\xi,\gamma)$.. we have the proof of theorem 5.2.

8. Extreme Points

Now, in the this part in article presented this is result for $S_w^*M(\xi,\gamma)$.

Theorem 6.1. *Let* $f_0(z)=(z-w)$

and

$$f_n(z)=(z-w)+\frac{2(\xi\gamma-1)}{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .n\left[n-\xi(n+2\gamma-1)+1\right]}(z-w)^n,$$

$n=1,2,\ldots,(z-w)\in U^*$.

$f\in S_w^*M(\xi,\gamma) \Leftrightarrow$ *it can be say in the form*

$$f(z)=\phi_0 f_0(z)+\sum_{n=1}^{\infty}\phi_n f_n(z)\,(6.1)$$

where $\phi_n\ge 0$ *and* $\phi_0+\sum_{n=2}^{\infty}\phi_n=1$.

Proof. Assume that

$$f(z)=\sum_{n=1}^{\infty}\phi_n f_n(z)$$

where $\phi_n\ge 0(n\ge 0)$ and $\sum_{n=2}^{\infty}\phi_n=1$. Then

$$f(z)=\sum_{n=1}^{\infty}\phi_n f_n(z)=\phi_1 f_1(z)+\sum_{n=2}^{\infty}\phi_n f_n(z)$$

$$=(z-w)+\sum_{n=2}^{\infty}\frac{2(\xi\gamma-1)}{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .n\left[n-\xi(n+2\gamma-1)+1\right]}.\phi_n(z-w)^n.$$

Then we have

$$\sum_{n=2}^{\infty}\frac{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^{k}.n\left[n-\xi(n+2\gamma-1)+1\right]}{2(\xi\gamma-1)}\phi_n$$

$$.\frac{2(\xi\gamma-1)}{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^{k}.n\left[n-\xi(n+2\gamma-1)+1\right]}$$

$$=\sum_{n=2}^{\infty}\phi_n=1-\phi_0\le 1.$$

Hence $f\in S_w^*M(\xi,\gamma)$.

On the other hand, we suppose that f in (1.12) in $S_w^*M(\xi,\gamma)$, Using Corollary 2.2, we get

$$a_n\le\frac{2(\xi\gamma-1)}{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^{k}.n\left[n-\xi(n+2\gamma-1)+1\right]}.$$

Setting

$$\phi_n=\frac{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^{k}.n\left[n-\xi(n+2\gamma-1)+1\right]}{2(\xi\gamma-1)}a_n,(n\ge 2)$$

and

$$\phi_0=1-\sum_{n=2}^{\infty}\phi_n$$

We have

$$f(z)=\phi_0 f_0(z)+\sum_{n=2}^{\infty}\phi_n f_n(z).$$

We have the proof .

9. Convolution Properties

In the this part in article presented the convolution properties.

Theorem 7.1.

$$f_j(z)(j=1,2)\ (7.1) \tag{7.1}$$

we denote by $(f_1\otimes f_2)(z)$ the convolution for the functions $f_1(z)$ and $f_2(z)$;that is,

$$(f_1\otimes f_2)(z)=(z-w)-\sum_{n=p+1}^{\infty}a_{n,1}a_{n,2}z^n.(7.2) \tag{7.2}$$

Theorem 7.2. *Let* $f_j(z)(\mathrm{j}=1,2)$ *in* (7.1) *be in the class* $S_w^*M(\xi,\gamma)$.

Then $(f_1\otimes f_2)(z)\in S_w^*M(\xi,\gamma),$, where

$$ñ=\frac{1}{\xi}+\frac{\left(2(\xi\gamma-1)\right)^2}{(2\xi)((\lambda-\delta)(\beta-\alpha)+1)^{k}.2\left[2-\xi(1+2\gamma)+1\right]}. \tag{7.3}$$

The result is sharp for $f_j(z)(\mathrm{j}=1,2)$ by

$$f_j(z)=(z-w)+\sum_{n=2}^{\infty}\frac{\left(2(\xi\gamma-1)\right)^2}{(2\xi)((\lambda-\delta)(\beta-\alpha)+1)^{k}.2\left[2-\xi(1+2\gamma)+1\right]}z^n,(j=1,2). \tag{7.4}$$

Proof. So as to demonstrate theorem 7.2. , utilizing the method utilized before see[15], the largest ñ

$$\sum_{n=2}^{\infty}\left[\begin{matrix}\left[\begin{matrix}((\lambda-\delta)(\beta-\alpha)(n-1)+1)^{k}\\ \times n\left[n-\xi(n+2\gamma-1)+1\right]\end{matrix}\right]\\ \div 2(\xi ñ-1)\end{matrix}\right]a_{n,1}a_{n,2}\le 1, \tag{7.5}$$

For $f_j(z) \in S_w^* M(\xi, \tilde{n})(j=1,2)$.

since $f_j(z) \in S_w^* M(\xi, \gamma)\,(j=1,2)$,

we promptly see that

$$\sum_{n=2}^{\infty} \left[\begin{bmatrix} ((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k \\ \times\left(n\left[n-\xi(n+2\gamma-1)+1\right]\right) \end{bmatrix} \atop \div\left[2(\xi\gamma-1)\right] \right] a_{n,j} \le 1$$

$$(j=1,2). \tag{7.6}$$

Therefore, by use the inequality (Cauchy-Schwarz), we follow

$$\sum_{n=2}^{\infty} \left[\begin{bmatrix} ((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k \\ \times\left(n\left[n-\xi(n+2\gamma-1)+1\right]\right) \end{bmatrix} \atop \div\left[2(\xi\gamma-1)\right] \right] \sqrt{a_{n,1}a_{n,2}} \le 1. \tag{7.7}$$

This infers we require just demonstrating that

$$\frac{a_{n,1}a_{n,2}}{2(\xi\tilde{n}-1)} \le \frac{\sqrt{a_{n,1}a_{n,2}}}{2(\xi\gamma-1)} (n \ge 2) \tag{7.8}$$

or, equivalently, that

$$\sqrt{a_{n,1}a_{n,2}} \le \frac{2(\xi\tilde{n}-1)}{2(\xi\gamma-1)} (\mathrm{n} \ge 2). \tag{7.9}$$

Hence, by the inequality (7.7), it can prove that

$$\frac{2(\xi\gamma-1)}{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .n\left[n-\xi(n+2\gamma-1)+1\right]} \le \frac{2(\xi\tilde{n}-1)}{2(\xi\gamma-1)} (\mathrm{n} \ge 2). \tag{7.10}$$

It follows from (7.10) that

$$\tilde{n} \le \frac{1}{\xi} + \frac{\left(2(\xi\gamma-1)\right)^2}{(2\xi)((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .n\left[n-\xi(n+2\gamma-1)+1\right]}. \tag{7.11}$$

Now, defining the function $\Psi(n)$ by

$$\Psi(n) = \frac{1}{\xi} + \frac{\left(2(\xi\gamma-1)\right)^2}{(2\xi)((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k .n\left[n-\xi(n+2\gamma-1)+1\right]} (n \ge 2). \tag{7.12}$$

$\Psi(n)$ is an increasing of n.

Therefore,

$$\tilde{n} \le \Psi(2) = \frac{1}{\xi} + \frac{\left(2(\xi\gamma-1)\right)^2}{(2\xi)((\lambda-\delta)(\beta-\alpha)+1)^k .2\left[2-\xi(1+2\gamma)+1\right]}, \tag{7.13}$$

completes the proof (7.2).
by theorem (7.2) following.

Theorem 7.3. *Let* $f_j(z)\,(\mathrm{j}=1,2)$ *given by* (7.1) *in* $S_w^* M(\xi, \gamma)$, $0 < \xi \le 1$, *where* $0 \le \gamma \le 1$, *and* $n \in \square_0$.

Then $h(z)$ *defined by*

$$h(z) = (z-w) + \sum_{n=2}^{\infty} (a_{n,1}^2 + a_{n,1}^2) z^n \tag{7.14}$$

Then $h(z)\in S_w^*M(\xi,\gamma)$,

$$\upsilon=\frac{2\left[2(\xi\gamma-1)\right]^2}{(2\xi)((\lambda-\delta)(\beta-\alpha)+1)^k.2\left[2-\xi(1+2\gamma)+1\right]}. \tag{7.15}$$

The sharp for the functions *by* (7.4).

Proof. see that

$$\sum_{n=2}^{\infty}\frac{\left[((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k.n\left[n-\xi(n+2\gamma-1)+1\right]\right]^2}{\left[2(\xi\gamma-1)\right]^2}a_{n,j}^2\le$$

$$\left(\sum_{n=2}^{\infty}\frac{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k.n\left[n-\xi(n+2\gamma-1)+1\right]}{\left[2(\xi\gamma-1)\right]^2}a_{n,j}\right)^2\le 1$$
$$(j=1,2) \tag{7.16}$$

For $f_j(z)\in S_w^*M(\xi,\gamma),(j=1,2)$, we have

$$\sum_{n=2}^{\infty}\frac{1}{2}\frac{\left[((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k.n\left[n-\xi(n+2\gamma-1)+1\right]\right]}{\left[2(\xi\gamma-1)\right]^2}$$
$$.(a_{n,1}^2+a_{n,2}^2)\le 1. \tag{7.17}$$

the largest υ

$$\frac{1}{2(\xi\upsilon-1)}$$
$$\le\frac{((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k.n\left[n-\xi(n+2\gamma-1)+1\right]}{2\left[2(\xi\gamma-1)\right]^2},$$
$$(n\ge 2). \tag{7.18}$$

From the above inequality, we obtain

$$\upsilon\le\frac{1}{\xi}+\frac{2\left[2(\xi\gamma-1)\right]^2}{(2\xi)((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k.n\left[n-\xi(n+2\gamma-1)+1\right]}. \tag{7.19}$$

Now, defining a function $\Phi(n)$ by

$$\Phi(n)=$$
$$\frac{1}{\xi}+\frac{2\left[2(\xi\gamma-1)\right]^2}{(2\xi)((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k.n\left[n-\xi(n+2\gamma-1)+1\right]}$$
$$(n\ge 2). \tag{7.20}$$

$\Phi(n)=$ is an increasing function of n.

$$\upsilon\le\Phi(2)$$
$$=\frac{2\left[2(\xi\gamma-1)\right]^2}{(2\xi)((\lambda-\delta)(\beta-\alpha)+1)^k.2\left[2-\xi(1+2\gamma)+1\right]}, \tag{7.21}$$

which completes the proof.

10. Integral Operators

In the this part in article presented this is properties in $S_w^*M(\xi,\gamma)$.

Theorem 8.1 *If* $f(z)$ *in (1.12) for* $S_w^*M(\xi,\gamma)$, *where*

$0<\xi\le 1, 0\le\gamma\le 1,$

and $n\in\square_0$ *.and let c be a real number such that* $c>-1$

. Then the function $G(z)$

$$G(z)=\frac{c+1}{(z-w)^c}\int_0^{(z-w)}(u)^{c-1}f(u)du\ (c>-1) \tag{8.1}$$

also belongs to the class $S_w^* M(\xi,\gamma)$.

Proof. by (8.1),

$$G(z) = (z - w) + \sum_{n=2}^{\infty} b_n z^n,$$

where $b_n = \left(\frac{c+1}{n+c}\right) a_n$.

Then we have

$$\sum_{n=2}^{\infty} ((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k . n\left[n - \xi(n+2\gamma-1)+1\right] b_n$$

$$\sum_{n=2}^{\infty} ((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k . n\left[n - \xi(n+2\gamma-1)+1\right]\left(\frac{c+1}{n+c}\right) a_n$$

$$\leq \sum_{n=2}^{\infty} ((\lambda-\delta)(\beta-\alpha)(n-1)+1)^k . n\left[n - \xi(n+2\gamma-1)+1\right] a_n \leq 2(\xi\gamma-1)$$

since $f(z) \in S_w^* M(\xi,\gamma)$.

Hence by Theorem (2.1), $F(z) \in S_w^* M(\xi,\gamma)$ □

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12. References

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