Indian Journal of Science and Technology, *Vol 9(30), DOI: 10.17485/ijst/2016/v9i30/99023, August 2016*
ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645

Least Square based Image Denoising using Wavelet Filters

P. V. Veena*, G. Remmiya Devi, V. Sowmya and K. P. Soman

Centre for Computational Engineering and Networking (CEN), Amrita School of Engineering, Amrita Vishwa Vidyapeetham, Amrita University, Coimbatore, Tamil Nadu - 641112, India; veenakrt27@gmail.com; remmiyanair@gmail.com, v_sowmya@cb.amrita.edu; kp_soman@amrita.edu

Abstract

Background/Objectives: Noise in a digital image, is unwanted information that degrades the quality of an image. The main aim of the proposed method is to denoise a noisy image based on least square approach using wavelet filters. **Methods/Statistical Analysis:** One dimensional least square approach proposed by Selesnick is extended to two dimensional image denoising. In our proposed technique of least square problem formulation for image denoising, the matrix constructed using second order filter coefficients is replaced by wavelet filter coefficients. **Findings:** The method is experimented on standard digital images namely Lena, Cameraman, Barbara, Peppers and House. The images are subjected to different noise types such as Gaussian, Salt and Pepper and Speckle with varying noise level ranging from 0.01db to 0.5db. The wavelet filters used in the proposed approach of denoising are Haar, Daubechies, Symlet, Coiflet, Biorthogonal and Reverse biorthogonal. The outcome of the experiment is evaluated in terms of Peak Signal to Noise Ratio (PSNR). The analysis of the experiment results reveals that performance of the proposed method of least square based image denoising by wavelet filters are comparable to denoising using existing second order sparse matrix. **Applications/Improvements**: Digital images are often prone to noise; hence, proceeding with further processing of such an image requires denoising. This work can be extended in future to m-band wavelet filters.

Keywords: Image Denoising, Least Square, Peak Signal to Noise Ratio (PSNR), Wavelet Filters

1. Introduction

Digital images are referred as electronically recorded depiction that allows transmission and reception. These digital images are often prone to get corrupted by additive or multiplicative noise in the process of digitization and transmission. The reason for the noise to occur is the unwanted fluctuations that arises while capturing the image in electronic devices. When the noise is added, the original image information is appended with extraneous information. In order to overcome the intervention of noise in pixel information, we approach the image denoising methods. Denoising has always been the most important concept in digital image processing. In prior to any image processing technique (segmentation, feature extraction, texture analysis etc.), image denoising serves as an essential pre-processing stage[1]. The major challenge in image denoising technique is to remove the noise with the preservation of edges[2].

Image denoising techniques using the higher order Singular Value Decomposition (SVD) and based on overcomplete dictionary is proposed[3,4]. A denoising technique based on fourth order partial differential equations is implemented[5]. Image denoising with a sparse band matrix is proposed[6]. A nonlocal Bayesian algorithm is proposed in the domain of image denoising[7].

Wavelet filters that satisfies invertible characteristic is used for image denoising. The significance of invertible property lies in the fact that, the original image can be recovered, after it has been filtered using wavelet filters. Usually, this kind of wavelet filters is used for noise reduction in image processing. The most fundamental form of wavelet filters is Haar filters that encapsulates much of the

**Author for correspondence*

recent applications in denoising using wavelets. The other forms of wavelet filters are Daubechies, Symlet, Coiflets, Discrete Meyer wavelet, Biorthogonal and Reverse biorthogonal.

Wavelets based denoising has emerged as a trending method of denoising standard digital images. Several researches were made in the domain of denoising using wavelets. Image denoising using wavelet transform and median filtering techniques is done[8]. Image denoising based on wavelet thresholding methods is proposed[9]. Wavelets are used to learn the nonlocal hierarchical dictionary for image denoising[10].

In this paper, we have extended the one-dimensional approach proposed by Selesnick to two-dimensional image denoising using the concept of least squares weighted regularization. The sparse matrix of second order differentiation is replaced with a matrix formed by the high pass decomposition coefficients of wavelet filters[11]. The accuracy in denoising is measured through the image quality metric, PSNR. The proposed method is found to be comparable with the methodology involving second order sparse matrix. Also, advantage of the proposed method is that it involves low mathematical complexity.

In section II, the mathematical representation of least square weighted regularization and wavelet is discussed. Section III discusses the methodology used with an overview of the purpose of this paper. In section IV, the outcome of experiments and their observations are given. Conclusion of this paper is given in section V.

2. Mathematical Background

2.1 Least Square Weighted Regularization

One-dimensional approach for signal denoising using the concept of least squares weighted regularization is proposed by Ivan. W. Selesnick[12]. The approach is to obtain a smoother signal which is similar to the noisy one. Let y(p) be the input noisy signal and x(p) be the desired output, then the problem formulation is given by,

$$\min_x \| y - x \|_2^2 + \lambda \| Dx \|_2^2 \quad (1)$$

For any signal x(p), the first order difference is given by,

$$y(p) = x(p) - x(p-1) \quad (2)$$

Then the second order differential of this signal is derived by taking the differential of Equation (2). Then the equation becomes,

$$y(p) = x(p-1) - 2x(p) + x(p-1) \quad (3)$$

D is defined as the second order differential square matrix,and from Equation (3) it is defined as,

$$D = \begin{bmatrix} 1 & -2 & 1 & & & \\ & 1 & -2 & 1 & & \\ & & 0 & & 0 & \\ & & & 1 & -2 & 1 \end{bmatrix}$$

λ is a control parameter and is always greater than zero. When the first term in Equation (1) is minimized it makes the signal x(p) to be similar to that of y(p). When the second term i.e. $\| Dx \|_2^2$ is minimized it makes the signal x (p) smooth. Minimizing the sum of Equation (1) makes the signal x to be smoothened similar to y. The smoothness depends on the parameter λ. When the value of λ is zero, then the input signal will be same as that of the output signal. When λ is high, the signal will be smoother. The mathematical solution for signal denoising using least square is given by,

$$x = (I + \lambda D^T D)^{-1} y \quad (4)$$

where I, is an identity matrix of size same as that of D.

2.2 Wavelet

Wavelets are functions used to localize a given function in both time and frequency domain. This is done by scaling and translation of a function. The scaling and translation of a function ψ (x), is represented as $\psi_{a,b}$ (x) where a is the scaling coefficient and b is the translation coefficient[13].

Mathematically we denote a wavelet as,

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right) \quad (5)$$

The function ψ (t) corresponding to Equation (5) is called the wavelet function or the mother function. The mother function is localized i.e., as time tends to infinity it will decrease to zero. By scaling and translating the wavelet function, we can create a whole family of wavelets.

All the properties of the original signal will also be held by the wavelet coefficients without increasing the mathematical complexity. Normalizing the mother wavelet gives the function g(t) which is represented as,

$$g(t) = \frac{1}{\sqrt{2^j}}\psi\left(\frac{-t}{2^j}\right) \tag{6}$$

Where j denotes j^{th} level.For example the normalized version of Haar filter whose mother wavelet is $\psi =$ [1; 1]is $g[n] = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ which is the high pass decomposition coefficients of Haar filter.

3. Proposed Method

Among all other wavelet filters, Haar is the most fundamental form of wavelet filters. The bases of Haar filters provide information that is sufficient to reconstruct the original signal. The reconstructed signal is found to be more efficient in the way that, much of the noise is removed from the signal. The proposed system is validated using the metrics such as Signal to Noise Ratio (SNR) and Peak Signal to Noise Ratio (PSNR). The wavelet function of the filters used in this paper is generated using 'pywavelets' as shown in Figure 1. Haar and Daubechies share same properties like asymmetric, orthogonality and biorthogonality. Symlet and Coiflet have common properties such as near symmetric, orthogonality and biorthogonality. Finally, Biorthogonal and Reverse biorthogonal filters have properties such as symmetric, non-orthogonality and biorthogonality.

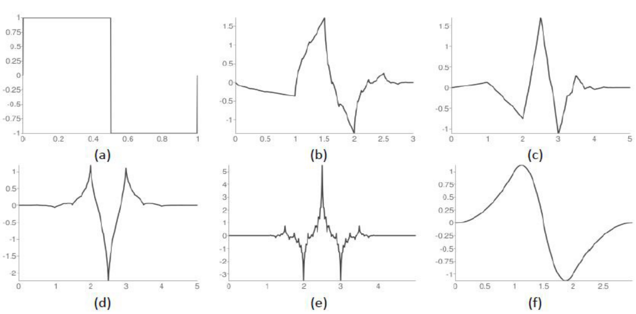

Figure 1. Wavelet functions of (a) Haar, (b) Daubechies, (c) Symlet, (d) Coiflet, (e) Biorthogonal, (f) Reverse Biorthogonal.

3.1 Method Overview

The procedure of denoising an image using wavelet filters is shown in Figure 2. The proposed system consists of the following steps:

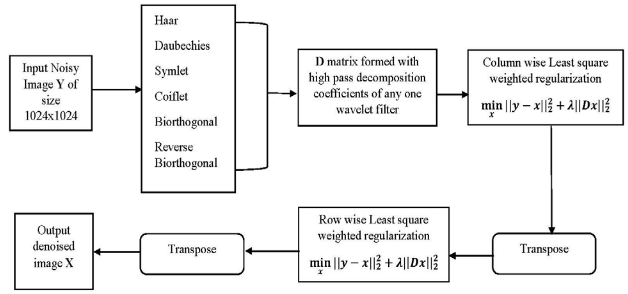

Figure 2. Block diagram of the proposed system.

- Consider an input image (Y) of dimension 1024 x 1024, corrupted with any one of the noises.
- Six filters are used (as given in Figure 2).
- Matrix D is formed using the high pass decomposition coefficients of the particular filter used for denoising the image.
- The noisy image is fed as input to the Equation (2) for performing least square (LS) weighted regularization.
- Column wise LS weighted regularization is performed and the resultant image is transposed.
- The above mentioned resultant image serves as input for performing row wise LS weighted regularization and results in denoised image.
- The final denoised image is obtained by transposing the above denoised image.

4. Experimental Results and Analysis

The high pass decomposition coefficients of different wavelet filters such as Daubechies, Symlets, Haar, Coiflets, Biorthogonal and Reverse biorthogonal is used for image denoising. In particular, from the family of Daubechies, db2 filter is used. Similarly, sym3 from Symlet family, coif1 from Coiflet family, bior2.2 from Biorthogonal family and rbio3.1 from Reverse biorthogonal family. The experiment is implemented on the following images-Lena, Cameraman, Barbara, Pepper and House of size 1024x1024. Three types of noises used in our experiment are Gaussian, Salt and Pepper and Speckle with varying noise levels. Gaussian noise is added to the image for the levels 0.01, 0.025, 0.05, 0.075 and 0.1. Salt and Pepper noise is added to the image for the levels 0.05, 0.075, 0.1, 0.25 and 0.5. Speckle noise is added to the image for the levels 0.05, 0.075, 0.1, 0.25 and 0.5.

In this paper, the matrix D of second order difference is replaced with the matrix formed by the high pass decomposition coefficients of the particular filter used in denoising. The procedure given in Figure 2 is implemented for various lambda values. The input noisy images of Barbara and Cameraman with the three noises (Gaussian, Salt and Pepper, Speckle) with respective maximum noise levels 0.1, 0.5 and 0.5 and output images denoised by the filters Daubechies, Symlet and Coiflets are shown in Figure 3 and Figure 4 respectively.

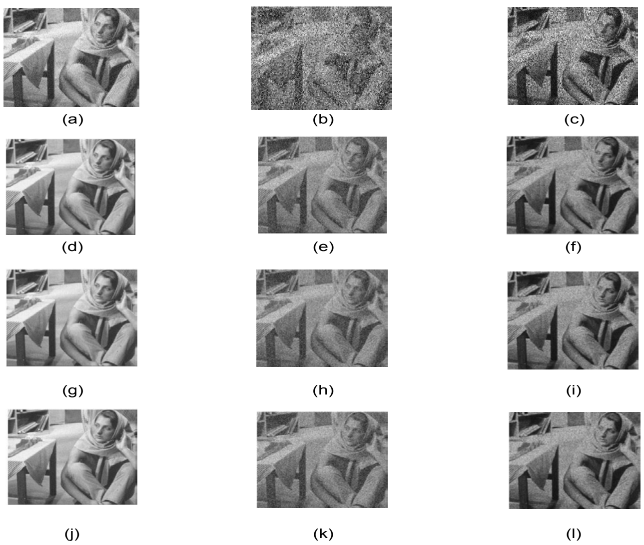

Figure 3. Image of Barbara with noise- (a) Gaussian, (b) Salt and Pepper, (c) Speckle, at noise levels 0.1, 0.5 and 0.5 respectively. Denoised images using (d) (e) (f) Daubechies2, (g) (h) (i) Symlet3, (j) (k) (l) Coiflet1.

The PSNR values obtained for the output images denoised using proposed technique (least square based image denoising using wavelet filters) is tabulated for the maximum noise levels 0.1, 0.5 and 0.5 respectively. It is evident from Table 1 that the performance of the proposed technique using wavelet filters is comparable to the existing second order filter. For example, let us consider the PSNR value of Barbara image, with respect to Gaussian noise at noise level 0.1. The PSNR value of proposed technique using Symlet filter is 19.6657dB which is comparable to the PSNR value 19.6443dB obtained from existing second order filter. This is visually seen in Figure 3(g). In case of Cameraman, the PSNR value with respect to Gaussian noise at noise level 0.1 obtained using the proposed technique with Daubechies filter is 19.7112dB which is also comparable with the PSNR value 19.7013dB obtained from the second order filter.

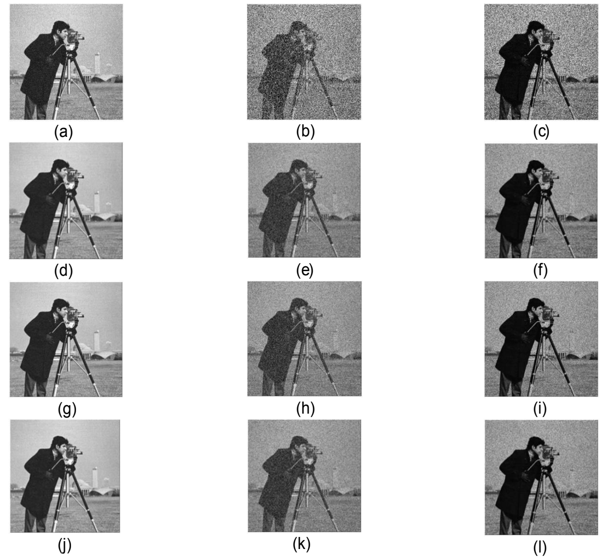

Figure 4. Image of Cameraman with noise- (a) Gaussian, (b) Salt and Pepper, (c) Speckle, at noise levels 0.1, 0.5 and 0.5 respectively. Denoised images using (d) (e) (f) Daubechies2, (g) (h) (i) Symlet3, (j) (k) (l) Coiflet1.

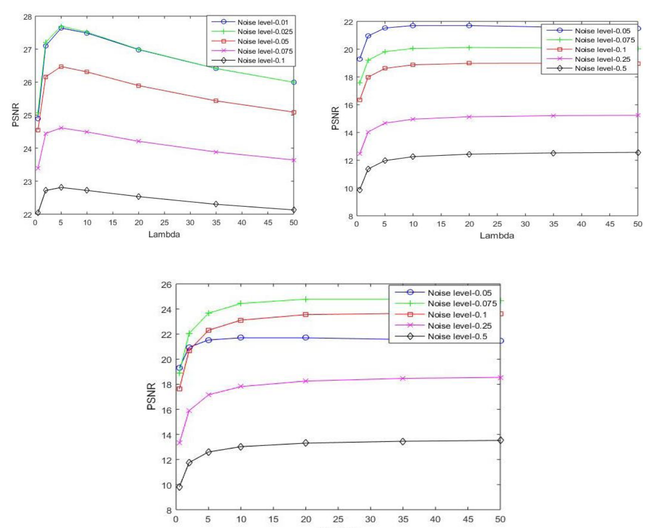

Figure 5. Relation between Lambda and PSNR for (a) Gaussian, (b) Salt and Pepper and (c) Speckle noise with biorthogonal filter of the image house.

The relationship between lambda and PSNR value for Gaussian, Salt and Pepper and Speckle at varying noise levels for bior2.2 filter is shown in Figure 5. The observation made from Figure 5 is that for all the three noises, PSNR value increases with increase in lambda. This trend

Table 1. Performance comparison of proposed technique for image denoising against the least square approach for 2D denoising withexisting filter (second order) based on PSNR (DB)

Image	Noise Type	Noise Level	Second order (Existing)			Wavelet Filters (Proposed)			
				Daubechies	Symlets	Haar	Coiflets	Bi-orthogonal	Reverse Bi-orthogonal
Lena	Gaussian	0.1	19.8332	19.8403	19.7735	19.8842	19.8321	19.8195	19.7378
	Salt and pepper	0.5	19.3802	19.2728	18.6159	19.4184	19.2306	18.8069	19.4176
	Speckle	0.5	22.6549	22.6368	21.7005	22.8443	22.4951	21.9591	22.6561
Cameraman	Gaussian	0.1	19.7013	19.7112	19.687	19.6891	19.7035	19.7068	19.5763
	Salt and pepper	0.5	17.317	17.3207	16.907	17.3356	17.3015	17.0147	17.2657
	Speckle	0.5	22.8391	22.8488	22.0253	22.8657	22.7996	22.3235	22.5183
Barbara	Gaussian	0.1	19.6443	19.6021	19.6657	19.4625	19.6111	19.6603	19.1883
	Salt and pepper	0.5	17.7506	17.6657	17.3302	17.7831	17.6779	17.3611	17.7052
	Speckle	0.5	20.909	20.9171	20.6236	20.8662	20.9032	20.904	20.9319
Peppers	Gaussian	0.1	19.6856	19.6826	19.6757	19.6257	19.6909	19.6325	19.5187
	Salt and pepper	0.5	18.0993	17.9922	17.3846	18.1557	17.9547	17.5751	18.0862
	Speckle	0.5	22.2156	22.1415	21.2219	22.4334	22.1355	20.6986	22.4066
House	Gaussian	0.1	22.8241	22.9	22.8356	22.9521	22.8953	22.8089	22.8129
	Salt and pepper	0.5	13.6466	13.6452	13.4267	13.7174	13.6474	13.523	13.6698
	Speckle	0.5	12.5849	12.6187	12.4795	12.6528	12.6107	12.5685	12.6119

is followed till a particular point of lambda after which the PSNR almost remains constant. This is evident in Figure 5(b) and Figure 5(c).

The relationship between average PSNR obtained using wavelet filters (Haar, Daubechies, Symlet, Coiflet, Biorthogonal, Reverse biorthogonal) of five images for three noises at maximum noise level, denoised with second order filter and proposed wavelet filters is represented graphically in Figure 6. The graphical result analysis shows that the proposed technique based on wavelet filter is comparable to the existing second order filter. The performance comparison of existing sparse band filter against the proposed method of wavelet filter is represented as a graph in Figure 7. This is tabulated in Table 2. It can be observed that, in case of all images, the performance of proposed least square based image denoising using wavelet filter is better than sparse band matrix filter. For example, consider the image of Cameraman, with Salt and Pepper noise at noise level 0.1. The PSNR value (20.9dB) obtained using proposed wavelet filter shows 5.89dB improvement than the PSNR (26.79dB) obtained using sparse band matrix filter. Hence, the proposed method has been experimentally proved that the wavelet filters can also be used for denoising.

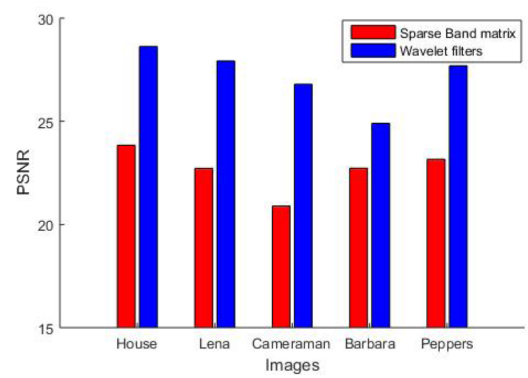

Figure 6. Performance comparison between sparse band filter and proposed wavelet filter.

Table 2. Performance comparison of image denoising based on PSNR (DB) using (a) Sparse band matrix [6], (b) Proposed technique

Noise Type		Lena		Cameraman		Barbara		Peppers		House	
	(a)		(b)	(a)	(b)	(a)	(b)	(a)	(b)	(a)	(b)
Gaussian	19.02		19.8842	19.09	19.7112	19.12	19.6657	19.24	19.6909	19.43	22.9521
Salt and Pepper	22.72		27.926	20.9	26.7998	22.73	24.8992	23.16	27.6939	23.84	28.6256
Speckle	24.57		28.8707	25.17	28.548	23.76	24.9133	24.09	29.7223	23.84	28.8671

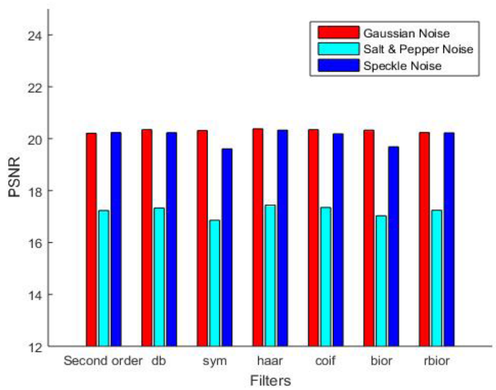

Figure 7. Relation between average PSNR values of six filters for Gaussian, Salt and Pepper, Speckle noise at noise levels 0.1, 0.5 and 0.5 respectively.

5. Conclusion

A methodology to denoise an image based on least square approach using wavelet filters is presented in this paper. This work is the extension of the one dimensional signal denoising approach based on least square (proposed by Selesnick) to two dimensional image denoising. In our proposed work, the matrix constructed using second order filter in the least square problem formulation is replaced with the wavelet filters. The performance of the proposed algorithm is validated through PSNR. From the results of PSNR values, it is evident that the proposed method performs equally well as the existing second order filter. The advantage of the proposed method lies in the fact that it is simple and involves low mathematical complexity.

6. References

1. Singh I, Chand L. Image denoising techniques: A review. International Journal of Engineering Research and Technology. 2013 Apr; 2(4):1-4.
2. Anutam R. Image denoising techniques: An overview. International Journal of Computer Applications. 2014 Jan; 86(16):1-5.
3. Kehua Su, Hongbo Fu, Bo Du, Hong Cheng, Haofeng Wang, Dengyi Zhang. Image denoising based on learning over-complete dictionary. International Conference on Fuzzy Systems and Knowledge Discovery; 2012 May; p. 395–8 .
4. Rajwade J, Rangarajan A, Banerjee A. Image denoising using the higher order singular value decomposition. IEEE Transactions on Pattern Analysis and Machine Intelligence. 2013 Apr; 35(4):849–62.
5. Khare AS, Mohan R, Sharma S. An efficient image denoising method based on fourth-order partial differential equations. IJACR. 2013 Mar; 3(1):1-6.
6. Sowmya V, Neethu Mohan, Soman KP. Sparse banded matrix filter for image denoising. Indian Journal of Science and Technology. 2015 Sep; 8(24):1-6.
7. Lebrun M, Buades A, Morel JM. A nonlocal Bayesian image denoising algorithm. SIAM Journal on Imaging Sciences. 2013 Jun; 6(3):1665–88.
8. Deepa M, Saravanan T. Automatic image registration using 2D discrete wavelet transforms. Indian Journal of Science and Technology. 2016 Feb; 9(5):1-3.
9. Vijay M, Saranya Devi L, Shankaravadivu M, Santhanamari M. Image denoising based on adaptive spatial and wavelet thresholding methods. International Conference on Advances in Engineering, Science and Management (ICAESM); 2012 Mar. p. 161–6.
10. Yan R, Shao L, Liu Y. Nonlocal hierarchical dictionary learning using wavelets for image denoising. IEEE Transactions on Image Processing. 2013 Dec; 22(12):4689-98.
11. Aarthy G, Amitha P, Krishnan T, Pillai GS, Sowmya V, Soman KP. A comparative study of spike and smooth separation from a signal using different over complete dictionary. International Multi-Conference on Automation, Computing, Communication, Control and Compressed Sensing (iMac4s); 2013 Mar. p. 590–5.
12. Selesnick I. Least Squares with Examples in Signal Processing. 2013 Mar.
13. Soman KP, Resmi NG, Ramachandran KI. Insight into Wavelets: From Theory to Practice. 3rd ed. PHI Learning Pvt. Ltd.; 2010.