

Reactive Power Constrained Locational Marginal Pricing using Loss Distribution Matrix

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Abstract

Objective: The objective of this paper is to obtain LMP with constrained reactive power by using optimal power flows. **Methods:** Loss distribution matrix method has been used to obtain loss distribution matrix. In this paper, by constraining reactive power to Matrix Loss Distribution methodology we obtain LMP. The benefit of Matrix Loss Distribution is illustrated through the case study IEEE 14 bus system and a numerical example on 5 Bus system has been discussed in detail. **Findings:** Matrix Loss Distribution method developed by V. Sarkar has been modified in this paper by considering reactive power.

Keywords: Fictitious Nodal Demand (FND), Locational Marginal Pricing (LMP), Loss Distribution Matrix (LDM), Vector Loss Distribution (VLD)

1. Introduction

Power industry across the world has been experiencing so many changes in its business as well as in operational model. It is a beginning to the new era of power industry. Restructuring of power industry is a big challenge to power system engineers. Engineers encounter several different problems in market designing. In this new structured environment, unbundling the cost is important to ensure that no consumer/buyer has an unfair advantage¹.

So, electricity production cost is split into different components: Energy costs, transportation costs and distribution costs. The process of congestion management is the cost of transmission is a key component to the cost associated with securely maintaining the load flows on the system within the limits.

In deregulated network managing the overloading network plays a crucial role. In congestion management there are several approaches for efficient management of congestion. Locational Marginal Pricing is one of the best approaches for congestion management compared to other approaches. Managing the congestion efficiently is one of the important issue in de-regulated network^{2,3}.

Locational Marginal Prices are being used in several electrical markets for settling energy transactions. Generally, LMP is used to calculate the three components: System energy, losses and transmission line overloading cost⁴. This paper presents the formulas to calculate the LMPs by using loss distribution Matrix⁵.

To calculate LMPs there are two models in optimal power flow: ACOPF and DCOPF. The main goal of optimal power flow is to minimize the cost of power generation by maintaining within the operational limits. ACOPF can give the accurate power flows but compared to DCOPF, it takes large execution time. DCOPF is simple, faster and robust. So DCOPF model is mostly used to calculate LMP^{6,7}. In DCOPF it is difficult to produce the accurate power flow when reactive power compensation is inadequate.

The OPF finds the best solution to an objective function subject to the power flow constraints and other operational imperatives, for example, generator least yield limitations, transmission security and voltage imperatives and limits on exchanging mechanical hardware equipment. The OPF models proposed in⁸⁻¹¹ assume as inelastic loads. Along these lines, the imaginary load is identically

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considered as an independent source while performing the load management.

Depending up on the active power consumption level, load entity differs in reactive power requirement. In OPF formulation in case active power load is adjustable then it is necessary to represent the reactive power load in a suitable way¹².

There are few methods to obtain Locational Marginal Pricing. One of the methods is Matrix Loss Distribution. In Matrix Loss Distribution, losses are distributed to various nodes and an another method which is similar to LDM is VLD. In VLD, it is important to distribute the losses in same ratio. But in Matrix Loss Distribution it is not essential to distribute the losses in fixed ratio¹³. In this paper rather than VLD, LDM is presented. There are another few methods to calculate LMP. But compared to other approaches LDM is the best one.

The organization of this paper is: Section 2 deals with Locational Marginal Pricing and Section 3 Loss Distribution Matrix (LDM). A Numerical example is carried out in next section. The test results are discussed in detail on Section 5. Section 6 is about Conclusion.

2. Locational Marginal Pricing (LMP)

The LMP component has been calculated effectively in numerous power markets around the world in various electricity markets¹⁴. The Locational Marginal Pricing (LMP) component is best in the most generally utilized methods for industry sector settlement in the deregulated power framework environment. The LMP is the total of supplying negligible cost, losses and transmission congestion cost if any congestion exists in transmission line. LMP is the best method for congestion management¹⁵.

2.1 Lossless Model Formulation

$$\text{Minimize} \sum_{i=1}^{ng} C_i(P_{gi}) \dots \dots \dots (1)$$

$$\sum_{i=1}^{ng} (P_{gi}) - \sum_{i=1}^{ng} (P_{di}) = 0 \dots \dots \dots (2)$$

$$flow^{max} \leq SF_{k-i} \times [P_{gi} - P_{di}] \leq -flow^{max} \dots \dots (3)$$

$$P_{gi}^{min} \leq P_{gi} \leq P_{gi}^{max} \dots \dots \dots (4)$$

Equation (1) is the objective function. (2) Is the global power balance equation. (3) Gives us an assurance that the transmission line flows don't exceed the limits. (4) Is the constraint on our individual generator¹⁶⁻¹⁹.

The Lagrangian function for optimization problem can be written as:

$$L = \sum_{i=1}^{ng} C_i(P_{gi}) - \lambda \left(\sum_{i=1}^{ng} P_{gi} - \sum_{i=1}^{ng} P_{di} \right) - (\mu' (SF_{k-i} \times (P_{gi} - P_{di}) - flow^{max}) \dots (5)$$

Now to calculate LMP at any bus i, partially derivative the Lagrangian function with respect to the demand at the bus i as follows:

$$LMP_i = \frac{\partial L}{\partial p_{di}} = \lambda + \left(\sum_{i=1}^n \mu_k \times SF_{k-i} \right) \dots \dots \dots (6)$$

We can see from Equation (6) the LMP we obtained is a Loss less which consists of 2 components. The 1st term is referred as system energy. Lagrangian multiplier is a global wide power balance equation. The next term is overloading segment as it is the capacity of the Lagrangian multiplier of the in-equality limitation that relates to the power.

3. Loss Distribution Matrix (LDM)

In this paper, we have been presented loss distribution matrix instead of a VLD. Losses has been distributed to the ends in a ratio by using LDM. Unlike the Loss Distribution Vector (VLD), there are no specifications about maintaining a stable ratio on the total network distribution loss in LDM. In a line, losses have been distributed in a ratio over its 'start' and 'end' nodes. By representing power flow accurately we can obtain design flexibility.

$$\text{Minimize} \sum_{i=1}^{ng} C_i(P_{gi}) \dots \dots \dots (7)$$

$$\sum_{i=1}^{ng} (P_{gi}) - \sum_{i=1}^{ng} (P_{di}) = 1_l P_{loss} \dots \dots \dots (8)$$

$$P_{loss} = \sum_{i=1}^n LFi \times (P_{gi} - P_{di}) + offset \dots \dots \dots (9)$$

$$Q_{loss} = \sum_{i=1}^n LFi \times (Q_{gi} - Q_{di}) + offset \dots \dots \dots (10)$$

$$flow^{max} \leq SF_{k-i} \times [P_{gi} - P_{di}] \leq -flow^{max} \dots \dots (11)$$

$$flow^{max} \leq SF_{k-i} \times [Q_{gi} - Q_{di}] \leq -flow^{max} \dots (12)$$

$$P_{gi}^{min} \leq P_{gi} \leq P_{gi}^{max} \dots (13)$$

$$Q_{gi}^{min} \leq Q_{gi} \leq Q_{gi}^{max} \dots (14)$$

The Lagrangian function can be written as shown in below:

$$L = \sum_{i=1}^n C_i(P_{gi}) - \lambda_p (\sum_{i=1}^n P_{gi} - \sum_{i=1}^n P_{di}) - P_{loss}^L - \lambda_q (\sum_{i=1}^n Q_{gi} - \sum_{i=1}^n Q_{di}) - \tau_p \times (P_{loss}^L - LF(\sum_{i=1}^n P_{gi} + \sum_{i=1}^n P_{di}) - offset) - (\mu_p (SF \times (\sum_{i=1}^n P_{gi} + \sum_{i=1}^n P_{di}) - k \times P_{loss}^L) - flow^{max}) - \tau_q \times (Q_{loss}^L - LF(\sum_{i=1}^n Q_{gi} + \sum_{i=1}^n Q_{di}) - offset) - (\mu_q (SF \times (\sum_{i=1}^n Q_{gi} + \sum_{i=1}^n Q_{di}) - k \times Q_{loss}^L) - flow^{max}) \dots (15)$$

Now to calculate LMP at any bus i, partially derivative the Lagrangian function with respect to the demand at the bus i as follows⁹.

- **For Active Power**

$$LMP_i = \lambda_i^e + \lambda_i^l + \lambda_i^c \dots (16)$$

$$\lambda_i^e = \tau \dots (17)$$

$$\lambda_i^l = -\tau \times LF_i \dots (18)$$

$$\lambda_i^c = (\sum_{k=1}^{\mu} \mu_k \times SF_{k-i}) - K' \times SF_{\mu} \dots (19)$$

- **For Reactive Power**

$$LMP_i = \lambda_i^e + \lambda_i^l + \lambda_i^c \dots (20)$$

$$\lambda_i^e = \tau \dots (21)$$

$$\lambda_i^l = -\tau \times LF_i \dots (22)$$

$$\lambda_i^c = (\sum_{k=1}^{\mu} \mu_k \times SF_{k-i}) - K' \times SF_{\mu} \dots (23)$$

3.1 Algorithm for LDM

Step 1: Read the base case data i.e., line information, bus data and bids.

Step 2: Calculate the Y-Bus.

Step 3: Calculate the initial values of and by using:

$$Pg(i) = \sum_{i=1}^n (v(i) \times v(j) \times G \times \cos(\theta(i, j) - \delta(i) + \delta(j)))$$

$$Qg(i) = \sum_{i=1}^n (v(i) \times v(j) \times B \times \sin(\theta(i, j) - \delta(i) + \delta(j)))$$

Step 4: Calculate the Jacobian and Hessian matrix.

Step 5: Check correction of vector elements and optimality limits.

If limits are not satisfied then go to step 6 else step 8.

Step 6: Modify the variables Pg, Qg, V.

Step 7: Check the limits.

Step 8: Calculate the total cost.

Step 9: Minimizing the cost of power generation.

Step 10: Calculate the power loss as in Equation (9).

Step 11: Calculate the Lagrangian function to obtain the Locational Marginal Pricing as in Equation (15).

Step 12: Obtain Locational Marginal Pricing partially derivatives the Lagrangian function as in Equations (16)-(23).

3.2 Algorithm for FND

Step 1: Read the data load, generator and bus data for the power system network.

Step 2: Calculate the Y-Bus using the algorithm of Y-Bus.

Step 3: Calculate the initial values of and by using:

$$Pg(i) = \sum_{i=1}^n (v(i) \times v(j) \times G \times \cos(\theta(i, j) - \delta(i) + \delta(j)))$$

$$Qg(i) = \sum_{i=1}^n (v(i) \times v(j) \times B \times \sin(\theta(i, j) - \delta(i) + \delta(j)))$$

Step 4: Calculate the Jacobian and Hessian matrix.

Step 5: Check correction of vector elements and optimality limits.

If limits are not satisfied then go to step 6 else step 8.

Step 6: Modify the variables Pg, Qg, V.

Step 7: Check the limits.

Step 8: Calculate the total cost.

Step 9: Minimizing the cost of power generation.

Such that:

$$\sum_{i=1}^{ng} (P_{gi}) - \sum_{i=1}^{ng} (P_{di}) = P_{loss}$$

Step 10: Calculate the power loss same as in Equation (9).

Step 11: Calculate FND by using:

$$FND_i = \sum_l \frac{1}{2} \times flow_k^2 \times R_k^2$$

Step 12: Calculate the Lagrangian function to obtain the locational marginal pricing.

$$L = \sum_{i=1}^n C_i(P_{gi}) - \lambda ((\sum_{i=1}^n P_{gi} - \sum_{i=1}^n P_{di}) - P_{loss}^L) - offset - (\mu (SF \times (\sum_{i=1}^n P_{gi} + \sum_{i=1}^n P_{di}) - flow^{max}))$$

Obtain locational marginal pricing,

LMP is given by:

$$LMP_i = \frac{\partial L}{\partial p_{di}} = \lambda - \lambda \times LF_i + \left(\sum_{k=1}^n \mu_k \times GSF_{k-i} \right)$$

4. Numerical Example

An example on IEEE 5 bus system with five-node and six-line network. In Table 1 line information represented. The generation and load data including bids are shown in Table 2. Figure 1 represents 5 bus system.

In a power system, power flow will be shared among other lines of the system when a line is corrupted. This will cause overloading on some lines¹⁹. Overloading also referred as congestion.

Table 1. Line information

	R %	X %	Limits (MW)
1-2	0.281	2.81	999
1-4	0.304	3.04	999
1-5	0.064	0.64	999
2-3	0.108	1.08	999
3-4	0.297	2.97	999
4-5	0.297	2.97	240

Table 2. Bus data

Bus	Pg	Pd	Gen. Cost(\$/MW)
1	110	0	14
2	100	300	15
3	520	300	30
4	300	300	35
5	600	0	10

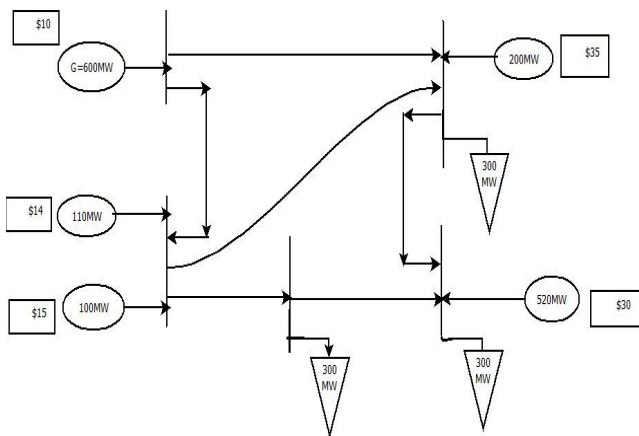


Figure 1. Five bus system

In a line, losses are distributed in 4:8 ratio from “start” - “end” nodes. Based on distribution losses matrix appears as shown in below:

$$K = \begin{bmatrix} 0 & 0.4 & 0 & 0.8 & 0.4 \\ 0.4 & 0 & 0.4 & 0.8 & 0.8 \\ 0 & 0.4 & 0 & 0.4 & 0 \\ 0.8 & 0.8 & 0.4 & 0 & 0.4 \\ 0.4 & 0.8 & 0 & 0.8 & 0 \end{bmatrix}$$

Loss factor is defined as “ratio of line losses to maximum power”. In order to calculate the Loss factors^{20,21}.

$$line_{loss} = \left[\frac{line_{flow}}{MVA_{base}} \right]^2 \times r \dots \dots \dots (21)$$

$$LF = \frac{\partial(line_{loss})}{\partial load} \dots \dots \dots (22)$$

System energy price λ = Price Reference Bus = \$14/ MW

To calculate Shift factor “Ratio of change in line flows to the change in generation” at the slack bus all GSFs are equal to zero. Shift factor can be calculated in terms of Impedances also. (or) Generation shift factor also referred as power transfer distribution factor when generation changes line flow changes at a specific bus respectively. In power system operation, planning and research generation shift factors have been widely used.

Table 3. LMPs by using LDM method without congestion

BUS	LMP P(\$/MW)	LMP Q(\$/MVar)
1	14.000	1.000
2	14.818	1.000
3	30.084	1.000
4	34.924	1.000
5	10.356	1.000

Table 4. LMPs by using FND Method without congestion

BUS	LMP P(\$/MW)	LMP Q(\$/MVar)
1	14.000	2.000
2	14.818	1.976
3	30.084	1.995
4	34.924	1.989
5	10.356	2.000

Table 5. LMPs by using LDM method with congestion

BUS	LMP P(\$/MW)	LMP Q(\$/MVar)
1	13.279	0.531
2	20.838	3.475
3	35.342	0.908
4	25.024	13.331
5	11.355	1.000

Locational Marginal Pricing is defined as the sum of energy pricing and marginal pricing and congestion pricing²².

$$\text{LMP} = \text{LMP}_{\text{energy}} + \text{LMP}_{\text{marginal}} + \text{LMP}_{\text{congestion}}$$

LMPs obtained by using loss distribution matrix as shown in Equations (13)-(20).

Table 6. LMPs by using FND Method with congestion

BUS	LMP P(\$/MW)	LMP Q(\$/MVar)
1	15.107	3.107
2	14.938	2.279
3	30.214	2.040
4	35.147	2.287
5	12.408	3.695

Table 7. Sensitivity Analysis when load varies u, LMP varies simultaneously

Bus	Load	μ	with congestion	
			P(\$/MW)	Q(\$/MVar)
2	303	-14.21	20.911	3.475
2	306	-14.42	20.983	3.475
2	309	-14.63	21.056	3.475
3	303	15.19	35.283	-0.908
3	306	14.98	35.223	-0.908
3	315	14.35	35.046	-0.908
4	303	-7.21	24.694	-13.331
4	306	-7.42	24.365	-13.331
4	315	-8.05	23.377	-13.331

5. Test Results

IEEE 14 bus data

Table 8: Line Data

SB	EB	R	X	B/2
1	2	.01938	.05917	.0264
1	5	.05403	.22304	.246

2	3	.04699	.19797	.219
2	5	.05695	.17388	.017
2	4	.05811	.17632	.0187
6	11	.09498	.1989	0
6	12	.12291	.25581	0
6	13	.06615	.1327	0
3	4	.06701	.17103	.0173
7	8	0	.17615	0
7	9	0	.1101	0
9	10	.03181	.0845	0
9	14	.12711	.27038	0
4	5	.01335	.04211	.0064
10	11	.08205	.19207	0
12	13	.22092	.19988	0
4	7	0	.20912	0
4	9	0	.55618	0
5	6	0	0.25202	0

Table 9. Generator Data

Bus	Type	Active load	Reactive load	Pg	Qg
1	3	0	0	232.4	-16.9
2	2	21.7	12.7	40	42.4
3	2	94.2	19	0	23.4
4	1	47.8	-3.9	0	0
5	1	7.6	1.6	0	0
6	2	11.2	7.5	0	12.2
7	1	0	0	0	0
8	2	0	0	0	17.4
9	1	29.5	16.6	0	0
10	1	9	5.8	0	0
11	1	3.5	1.8	0	0
12	1	6.1	1.6	0	0
13	1	13.5	5.8	0	0
14	1	14.9	5	0	0

6. Results and Analysis

6.1. IEEE-14 Bus System

Table 8 and 9 represents Line data and generator data of IEEE 14 Bus system. Case study has been performed on IEEE 14 bus system to assess the benefits of loss distribution matrix. Case study has been performed on two methods without congestion and with congestion. Figures 2 and 3 belongs to LMP without congestion and with

congestion. Partially derivate the Lagrangian function we can obtain Locational Marginal Pricing. When load varies Lagrangian multiplier for inequality constraint μ and LMP varies simultaneously as shown in Table 7 on five bus system. Comparison between FND and LMD has been shown in Tables 3, 4, 5 and 6. And Figures 2 and 3 are the results of Locational Marginal Pricing by using LMD.

In this paper LMD results has been compared with FND method to show the difference between this two methods. FND and LMD are similar but the change is an extra term has been added in Loss Distribution Matrix. The term represents Shadow prices for the system losses equation. Either, Marginal pricing of bus increases or decreases. Generally, these two methods are one of the DCOPF models. In DCOPF reactive power is negligible. Whereas, in this paper we are constraining reactive power with active power. So, we used Newton method to calculate power flows and derived the equations to calculate Locational Marginal Pricing by using optimal power flow methodology. To verify whether the LMP reactive power values are correct or wrong, calculations has been performed theoretically on five bus system. This approach handles one of the complex problems, optimal power flow also referred as security constrained program which has been approved by Independent System Operator (ISO). Here, for hourly Locational Marginal Price (LMP) calculations OPF methodology has been implemented. In market planning and simulation this approach can be applied²³.

6.2 Without Congestion

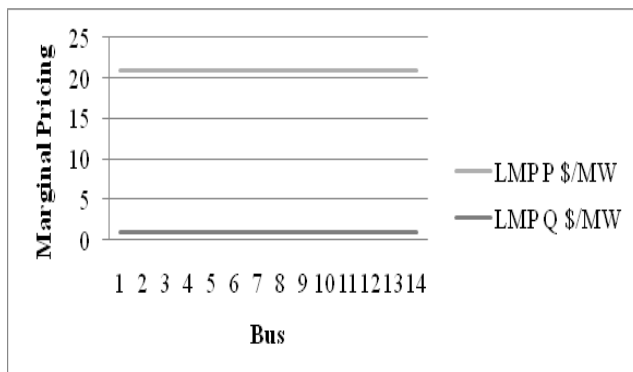


Figure 2. LMP without congestion for active and reactive power.

When congestion is absent, Locational Marginal Pricing will be less. And above Figure 2 represents Locational Marginal Pricing without congestion for IEEE

14 bus system. Reactive power consumption depends on active power consumption. So cost of active power will be more than reactive power as shown in Figure 2. When congestion is absent, there will be no overload on transmission lines. So, Cost will remain same. And as shown in numerical example Five bus system when Loss Distribution Matrix results has been compared with fictitious nodal demand there is difference in LMP cost between two methods. Both methods are similar but the difference is one special term added in Loss Distribution Matrix Shadow prices for the system losses equation. Due to this extra added term, whether the LMP cost will raise or will decrease.

6.3 With Congestion

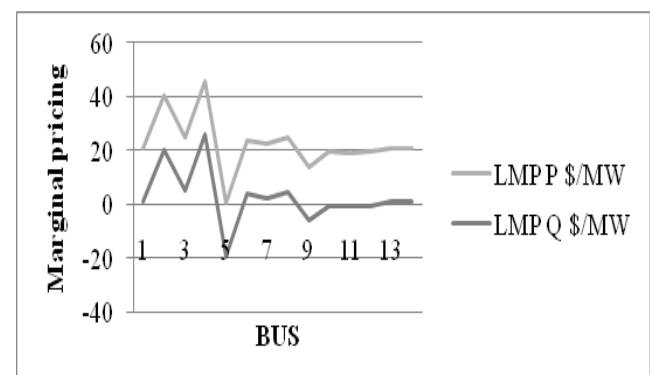


Figure 3. LMP with congestion for active and reactive power.

When congestion is present, Locational Marginal Pricing will be more compared to without congestion. And above Figure 3 represents Locational Marginal Pricing with congestion for IEEE 14 bus system. Reactive power consumption depends on active power consumption. So cost of active power will be more than reactive power as shown in Figure 3. Due to presence of congestion Cost increased as shown in Figure 3 at the buses 2, 4. At various buses, reactive marginal prices change when the objective functions consider reactive power cost. Reactive power pricing has larger impact due to the network losses than on the real power price variations.

7. Conclusion

A LDM methodology has been implemented to obtain Locational Marginal Pricing (LMP) with increased power flow accuracy. The proposed methodology provides the flexibility to assign a various distribution vectors. Case study has been implemented on IEEE 14 bus system to

obtain Locational Marginal Pricing by using loss distribution Matrix methodology with inclusion of reactive power. Two LMPs obtained and it is clearly shown in two graphs. When load changes inequality constraint μ and LMP changes respectively as shown in Table 7. FND and LDM are the two methods which are introduced by researchers to add losses as additional loads to buses. And comparison between two models has been shown in Tables 3, 4, 5 and 6. In LDM as shown in Tables 5 and 6 cost is reducing compares to FND. Here, LMP for active power and reactive power is calculated by using optimal power flow model. By using linear programming method optimal power flow has been solved. At each bus LMP values for active and reactive power has been represented in Figures 2 and 3.

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Nomenclature

δ Phase angle

η Total number of bids and offers

n_g Number of generator buses

n_b Number of buses

n_v Number of voltage controller buses

C_i Cost of generator i

P_{gi} real power generation of bus i	τ Shadow price for the system losses equation
Q_{gi} Reactive power generation of bus i	Δf_l Change in MW Power flow on line
P_{di} real Load on bus i	Offset Representing losses
Q_{di} Reactive Load on bus i	LF_i loss factor at bus i
V_i Voltage Magnitude at bus i	F_k power flow at line k
P_{gi}^{min} Lower limit on	R_k Resistance at line k
P_{gi}^{max} Upper limit on	$GSF_{(k-i)}$ Power transfer distribution factor to line k from bus i
P_{loss} Power loss	S Sensitivity factor to line k from bus i
L Lagrangian function	λ term represents locational marginal pricing vector
μ Lagrangian multiplier for inequality constraint	λ_c Overloading transmission line component of LMP vector
λ Lagrangian multiplier for equality constraint	