

Stability Analysis of Sitr Model and Non Linear Dynamics in Wireless Sensor Network

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Abstract

Background/Objectives: Security is one of the essential concern in wireless sensor network. To find the stability points when worms appears in the wireless sensor network. **Methods/Statistical Analysis:** By using ODE formulate the Sitr model for wireless sensor network. Find the existence of positive equilibrium and perform the stability test with the help of Jacobian matrix. Some theorems are proposed for the analysis of model. **Findings:** The model explains that some infected individuals should move from treated phase to recovered phase even after applying the protection mechanism. The entire dynamics of the transmission of worms can be analyzed by this mathematical model, propagating feat by worms in WSN can be determined by the value R_0 basic reproduction number. Simulation has performed using MATLAB of the proposed model. This shows the validity of proposed model. **Application/Improvements:** Proposed model is useful to reduce the battery overhead, enhance the lifetime of wireless sensor network.

Keywords: Epidemic Model, Equilibrium, Stability, Wireless Sensor Network, Worms

1. Introduction

The growing use of wireless sensor network in past years offers so many facilities and functionalities for data collection and transmission through sensor nodes. The collected data send to sink via neighbor nodes in multi hop¹ Sensors are installed in inaccessible and remote place making them vulnerable than other system². Now a day's WSN is used in various fields such as disaster relief operations, biodiversity mapping, surveillance, defense, medicine and healthcare etc³. Sensor nodes have some limitation like computing power, memory, communication and limited battery capacity. Wireless sensor network is wireless networks having massive potential for many applications, but it also has many challenges like deployment and coverage of nodes, scalability, and quality of services, computational power energy efficiency and security etc. security is one of the major concern among

all above defy requires more attention. Energy saving^{4,5} is also a challenge in WSN. In WSN sensor node works in two modes, sleep and active mode, in sleep mode, the sensor node is in inactive state in which it cannot send or receive the data, while in active mode, sensor nodes is in active state, it means it can send or receive the data. When a node is in active state it may also transmit the worms. Because WSN has weak defense mechanism, a malicious code can be recruited. In case of infection the sensor node may moves from active mode to sleep mode. Figure 1 shows the distribution of sensor nodes and represent how data can be transmitted from source node to sink node. When the source node sends data to sink via neighbor nodes, during the transmission of data⁶ there is some obstruction in WSN such as worms and virus attack.

Some new types of worms are surfacing specially for portable devices like laptops and phones, they have the ability to transmit from device to device directly through

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wireless method for example, as Bluetooth or Wi-Fi⁷. Wireless devices are targeted by malicious signals, for example Cabir worm and Mair worm and spreading behavior of these worms and are epidemic in nature⁸. Therefore, to prevent the malware attacks on sensor nodes security mechanism using epidemic models has been explored by various groups by considering the concept of stability, reproduction number^{9,10}.

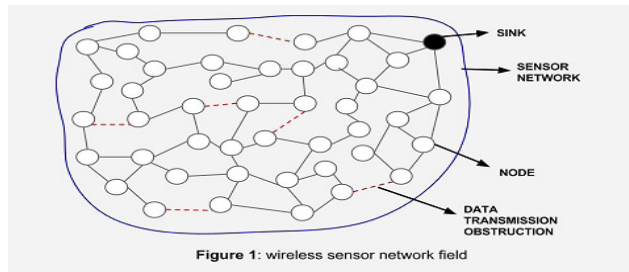


Figure 1.

In the proposed model, initially all sensor nodes are susceptible towards worms attack and becomes infectious with time and transmit the worm with data or may be spread through wireless communication. It is found that some sensor nodes are not aware about the worms attacks, so they are not using any protection mechanism to prevent malicious attacks. When infected nodes are detected, to increase the lifetime of sensor network, infected nodes are sent immediately into sleep mode. So they do not spread worms to the neighbor nodes and using treatment mechanism on infected nodes to removed worms from the network without disturbing network stability, because these infected nodes becomes dangerous for wireless sensor network. Those sensor nodes, which are detected early are sent to the treated compartment and malicious signals can be eradicated by antivirus. It may be possibility that some nodes are not treated properly or it may not detect early its worms becomes active. These types of nodes are sent to the recovered state and after successful recovery again nodes comes into the susceptible state. By using the concept of treatment class that is early detection of worms increase the lifetime of WSN removed worms from the system and prevent loss of data from network. This model explores the rate of treatment and control infection or elongate WSN life.

2. Model Formulation

In this paper we proposed an epidemic model consist of subclass of sensor node at any time t are susceptible infec-

tious $I(t)$, Treated $T(t)$ and recovered $R(t)$ of total size $N(t)$, i.e., $N(t) = S(t) + I(t) + T(t) + R(t)$, for any time $t \geq 0$. Our model consist of four differential equation

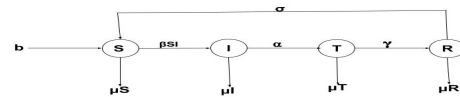


Figure 2. Describe the dynamical transfer of sub class. The SITS model is given by:

$$\left. \begin{aligned} \dot{S} &= b - \mu S + \sigma R - \beta SI, \\ \dot{I} &= \beta SI - (\mu + \alpha) I, \\ \dot{T} &= \alpha I - (\gamma + \mu) T, \\ \dot{R} &= \gamma T - (\sigma + \mu) R, \end{aligned} \right\} \quad (2.1)$$

where b considered to be constant rate for new nodes which connected to the WSN, μ is the crashing rate of sensor nodes, β is coefficient of transmission, σ is the transmission rate from class R to S class, the rate of treatment of infected node is α and γ is the recovery rate. We will discuss the system in the domain $\Gamma = \{(S, I, T, R) \in \mathbb{R}_+^4\}$. Since the model monitors sensor nodes of different class, so all the state variables remain non negative for all t greater than or equal to zero.

3. Existence of Positive Equilibrium

For equilibrium points, we have

$$\dot{S} = 0; \dot{I} = 0; \dot{T} = 0; \dot{R} = 0; \text{ and on a simple calculation.}$$

The equilibrium points are given as:

$$P_0 = (S, I, T, R) = \left(\frac{b}{\mu}, 0, 0, 0\right) \text{ for worm free state}$$

and $P^* = (S^*, I^*, T^*, R^*)$ for endemic state with

$$S^* = \frac{b}{R_0 \mu}, \quad I^* = \frac{b(\gamma + \mu)(R_0 - 1)(\sigma + \mu)}{R_0 \{(a + \mu)(\sigma + \mu)(\gamma + \mu) - \gamma a \sigma\}},$$

$$T^* = \frac{b(R_0 - 1)(\sigma + \mu)a}{R_0 \{(a + \mu)(\sigma + \mu)(\gamma + \mu) - \gamma a \sigma\}},$$

$$R^* = \frac{b(R_0 - 1)a\gamma}{R_0 \{(a + \mu)(\sigma + \mu)(\gamma + \mu) - \gamma a \sigma\}};$$

Where, R_0^{10} is the basic reproduction number given

$$\text{by } R_0 = \frac{b\beta}{\mu(a + \mu)}.$$

It is clear that P^* exist and unique if and only if $R_0 > 1$

4. The Stability Analysis

Theorem 1: The system (2.1) is locally asymptotically stable if its all eigenvalues are less than zero at worm free equilibrium P_0 .

Proof. At worm free equilibrium point P_0 , the Jacobian matrix is

$$J(P_0) = \begin{pmatrix} -\mu & -\frac{\beta b}{\mu} & 0 & \sigma \\ 0 & -(a + \mu) & 0 & 0 \\ 0 & a & -(\gamma + \mu) & 0 \\ 0 & 0 & \gamma & -(\sigma + \mu) \end{pmatrix} \quad (4.1)$$

Eigenvalues of (5.1) are:

$$\omega_1 = -\mu, \omega_2 = -(a + \mu), \omega_3 = -(\gamma + \mu), \omega_4 = -(\sigma + \mu).$$

It is clear that $\omega_1 < 0, \omega_2 < 0, \omega_3 < 0, \omega_4 < 0$, therefore the system is locally asymptotically stable at P_0 .

Theorem 2: The system (2.1) is globally asymptotically stable if $R_0 \leq 1$ at worm free equilibrium P_0 .

Proof. Consider the Lyapunov function $L(t): \mathcal{R}^4 \rightarrow \mathcal{R}^+$ defined by $\frac{dL(t)}{dt} = \mathcal{L}(t)$

$$\Rightarrow \frac{dL(t)}{dt} = \beta SI - (a + \mu)I$$

$$\leq \left[\frac{\beta b}{\mu} - (a + \mu) \right] I = \left[\frac{\beta b}{\mu(a + \mu)} - 1 \right] I = (R_0 - 1)I$$

It is clear that $\frac{dL(t)}{dt} = 0$ only when $I = 0$. Therefore the maximum invariant set $\Gamma = \{(S, I, T, R) \in \mathcal{R}_+^4\}$ is

the singleton set. Therefore the global stability of worm free equilibrium P_0 when $R_0 \leq 1$ from Lasalle invariance principle¹¹.

Theorem 3. The endemic equilibrium is locally asymptotic stable if all eigen values are negative.

Proof. The Jacobian matrix associated with endemic equilibrium is

$$J(P^*) = \begin{pmatrix} -\beta I^* - \mu & -\beta S^* & 0 & \sigma \\ \beta I^* & -(a + \mu) & 0 & 0 \\ 0 & a & -(\gamma + \mu) & 0 \\ 0 & 0 & \gamma & -(\sigma + \mu) \end{pmatrix}$$

The eigen values are $\omega_1 = -\mu, \omega_2 = -(\sigma + \mu + \gamma)$, and two eigen values are roots of the equation

$$a_0 \omega^2 + a_1 \omega + a_2 = 0,$$

where

$$a_0 = 1, a_1 = 2\mu + a + \beta I^*,$$

$$a_2 = \mu^2 + \mu a + I^* \beta (\mu + a + \beta S^*).$$

Since all coefficients are nonnegative therefore by Routh-Hurwitz criteria two eigen values ω_3 & ω_4 are negative if $R_0 > 1$, which completes the proof of above theorem.

Theorem 4. The Endemic equilibrium is globally asymptotically stable if $R_0 > 1$

Proof. Consider the suitable Lyapunov function¹¹

$$L(S^*, I^*, T^*, R^*) = \left(S - S^* - S^* \log \frac{S}{S^*} \right) + \left(I - I^* - I^* \log \frac{I}{I^*} \right) + \left(T - T^* - T^* \log \frac{T}{T^*} \right) + \left(R - R^* - R^* \log \frac{R}{R^*} \right)$$

Taking derivative of L with respect to time t , we get

$$\begin{aligned} \mathcal{L} &= \left(\frac{S - S^*}{S} \right) \mathcal{L}_+ \left(\frac{I - I^*}{I} \right) \mathcal{L}_+ \left(\frac{T - T^*}{T} \right) \mathcal{L}_+ \left(\frac{R - R^*}{R} \right) \mathcal{L}_+ \\ &= \left(\frac{S - S^*}{S} \right) \left(b - \beta(S - S^*)(I - I^*) - \mu(S - S^*) + \sigma(R - R^*) \right) + \left(\frac{I - I^*}{I} \right) \left(\beta(S - S^*)(I - I^*) \right. \\ &\quad \left. - (a + \mu)(I - I^*) \right) + \left(\frac{T - T^*}{T} \right) \left(a(I - I^*) - (\gamma + \mu)(T - T^*) \right) + \left(\frac{R - R^*}{R} \right) \left(\gamma(T - T^*) - (\sigma + \mu)(R - R^*) \right) \\ &= b - \frac{bS^*}{S} - \frac{\beta I(S - S^*)^2}{S} + \frac{\beta I^*(S - S^*)^2}{S} - \frac{\mu(S - S^*)^2}{S} + \frac{\beta S(I - I^*)^2}{I} \\ &\quad - \frac{\beta S^*(I - I^*)^2}{I} - (\mu + a) \frac{(I - I^*)^2}{I} - (\mu + \gamma) \frac{(T - T^*)^2}{T} - (\mu + \sigma) \frac{(R - R^*)^2}{R} \\ &\quad + aI - aI^* - \frac{aT^*I}{T} + \frac{aT^*I^*}{T} + \gamma T - \gamma T^* - \frac{\gamma R^*T}{R} + \frac{\gamma R^*T^*}{R}, \end{aligned}$$

$$\Rightarrow \frac{dL}{dt} = P - Q,$$

where,

$$P = b + \frac{\beta I^* (S - S^*)^2}{S} + \frac{\beta S (I - I^*)^2}{I} + a I + \frac{a T^* I^*}{T} + \gamma T + \frac{\gamma R^* T^*}{R},$$

$$Q = \frac{b S^*}{S} + \frac{\beta I (S - S^*)^2}{S} + \frac{\mu (S - S^*)^2}{S} + \frac{\beta S^* (I - I^*)^2}{I} + (\mu + a) \frac{(I - I^*)^2}{I} + (\mu + \gamma) \frac{(T - T^*)^2}{T} + (\mu + \sigma) \frac{(R - R^*)^2}{R} + a I^* + \frac{a T^* I^*}{T} + \gamma T^* + \frac{\gamma R^* T^*}{R}$$

If $P < Q$ then we get $\frac{dL}{dt} \leq 0$, and $\frac{dL}{dt} = 0$ iff $S = S^*, I = I^*, T = T^*, R = R^*$, therefore the largest compact invariant set $\Gamma = \left\{ (S, I, T, R) \in \mathbb{R}_+^4 : \frac{dL}{dt} = 0 \right\}$ is the singleton set $\{P^*\}$. Hence by Lasalle, s invariance principle¹¹ P^* globally asymptotically stable in Γ .

5. Simulation and Result

Figure 3 explain the behavior of susceptible (S), infectious (I), treated (T) and recovered (R) class with respect to time (t). In the analysis we found that the analyzed system is asymptotically stable and shows that initially worm increases and after some time worm will gradually disappear when reproduction number less than one ($R_0 = 0.711111$).

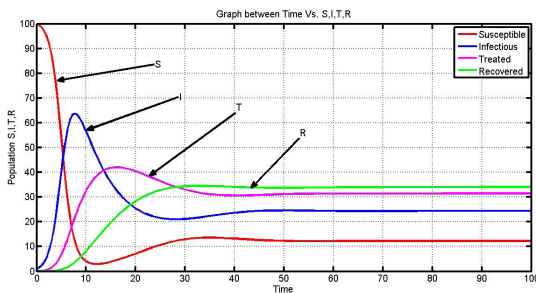


Figure 3. Shows dynamical demeanor of the system for different classes under the condition

when $R_0 < 1$ for $b = 0.32; \beta = 0.01; \mu = 0.03; \sigma = 0.08; \gamma = 0.09; a = 0.012$.

It has been observed from figure 4, the number of susceptible and infected nodes takes non-negative values and approaches to steady state. In this situation the worms persist in the wireless sensor network. In this case $R_0 = 8.672087 > 1$ which shows asymptotic behavior of endemic equilibrium.

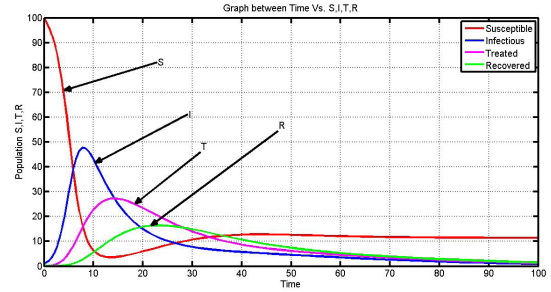


Figure 4. Shows dynamical demeanor of the system for different classes under the condition, when $R_0 > 1$ for $b = 0.32; \beta = 0.01; \mu = 0.003; \sigma = 0.08; \gamma = 0.09; a = 0.012$.

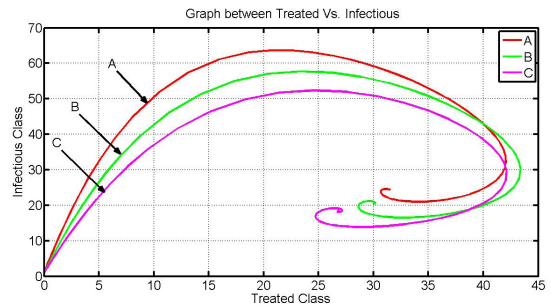


Figure 5. Shows dynamical demeanor of the system Treated Vs. Infectious $b = 0.32; \beta = 0.01; \mu = 0.003; \sigma = 0.08; \gamma = 0.09; a = 0.012$ $b = 0.32; \beta = 0.01; \mu = 0.003; \sigma = 0.08; \gamma = 0.1; a = 0.015$ $b = 0.32; \beta = 0.01; \mu = 0.003; \sigma = 0.08; \gamma = 0.12; a = 0.018$.

It has been observed from figure 5, when treatment rate is high infectious nodes removes worm from the network quickly and elongate the life time of network.

It has been analyzed from figure 6, when treatment class is considered performance of the system is better in comparison to the SIRS model.

6. Conclusion

We developed mathematical model to describe the spreading and controlling activities of malicious signals in wireless sensor network consists of ordinary differential equations to study the effect of treatment dynamics of worm transmission. We derive the expression for basic reproduction R_0 for determining the worm dies out completely. The local stabilities of worm free equilibrium and endemic equilibrium are established by using the Jacobian matrix. It is establish that if R_0 is less than or equal to one, then worm can be eradicated and the system becomes locally and globally asymptotically stable and

when $R_0 > 1$, the endemic equilibrium will be locally and globally asymptotically stable. It is also observed that if the rate of treatment increases the spreading of malicious worms decreases and enhances the life of wireless sensor network. By simulation it is found that performance of proposed model is better in comparison to the SIRS model, infected nodes are quickly removed from the system. Performance by considering the coverage area of wireless sensor nodes should be studied in the future.

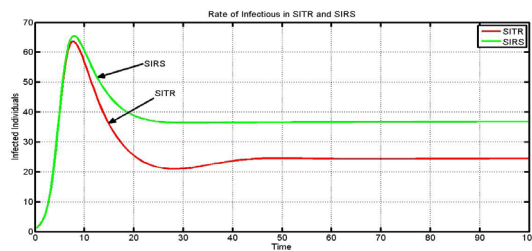


Figure 6. Shows performance of SITR over SIRS.

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