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Wavelet Coefficients Thresholding Techniques for Denoising MRI Images

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Abstract

Background: Image denoising is one of the primary challenges in the field of image processing. The objective is to derive the original image by suppressing noise from a noisy the image. The need of image denoising techniques is still in high demand. De-noising of medical images is degraded by various noises. Multiresolution techniques are very efficient for medical image denoising. **Methodology:** In this work, it is proposed to examine the efficiency of different wavelet shrinkage thresholding techniques and to determine the best one. **Findings:** The metric used for analysis are PSNR, VSNR and WSNR. The experimental results show that third level decomposition of Symlet in association with Neigh Shrink threshold outperforms all other approaches. **Applications/Improvements:** In this paper, denoising is applied to MRI images. Gaussian Noise, Salt and Pepper Noise, and Speckle Noise can be removed using the methods mentioned. The methods can also be extended to denoising other medical images like CT scan, X-RAY, and Ultra Sound etc.

Keywords: Multiresolution Techniques, Shrinkage Thereshold, Wavelet Bases, Wavelet Transform

1. Introduction

Medical imaging is a set of techniques that produce images of the internal aspect of the body. Medical images are affected by the noises either during its acquisition or transmission. The preprocessing task is to eliminate the noises in the medical images. This problem still remains and there is no good solution for it. The results of image segmentation, feature extraction and image recognition is depending on the removal of noise. So it is essential to remove the noise from the medical images.

Discrete wavelet transform is used in many fields of image processing like image compression, noise removal, and pattern recognition. On decomposition of images, wavelet transform provide a large number of small coefficients and a small number of large coefficients. Recently many methods have been proposed which are mainly based on thresholding these coefficients for noise removal. Hence, in this paper we focus on denoising MRI Images in Wavelet domain using coefficients shrinkage method.

1.1 Related Work

The presence of noise affects the visual quality and also it has an effect on the visibility of low contrast objects. Image denoising is the process of restoration of an image which has been corrupted by noise. Initial methods created for image denoising were based on statistical filter[1, 2]. Statistical filters may be either low pass or high pass. High pass filter amplify noisy background and low pass filter produces the edges blur during the denoising process. To overcome these restrictions, certain thresholding in wavelet or other transformation in multi scale domain can be employed.

Mallet[3] has introduced the theory of wavelet transform. Wavelets have many merits and no redundant information is stored, as wavelet functions are orthogonal. Wavelet transform is useful in the medical field. Wavelet based denoising using thresholding was done by Donoho and Johnstone[4]. A diagnosis tool is proposed based on wavelet transform to detect breast cancer at the earlier stage[5]. This tool is formed to execute multiscalar contrast enhancement at different wavelet scales. Different forms

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of discrete wavelet transform are Undecimated wavelet transform[6], Dual tree complex wavelet transforms[7] and Double density dual tree complex wavelet transforms[8]. Performance comparison of the Wavelet, Bandelet, and Contourlet Transforms for Image Denoising is presented[9]. A comparative analysis of JPEG, Wavelet, Bandelet, and Ridgelet whch are applied to the images of chromosomes is presented[10]. A general framework is presented[11] for constructing bi-orthogonal wavelets based on Bernstein bases.

A review is done[12] at improving the standard method using soft-thresholding denoising techniques based on DWT. A new framework called Complex Gaussian Scale Mixture (CGSM) in complex wavelet domain[13] is proposed for noise reduction. A new method named statistically optimum adaptive Wavelet Packet (WP)[14] thresholding function is proposed for image denoising. In paper[15], the authors introduced a new Poisson-Gaussian Unbiased Risk Estimator (PG-URE) for removal of mixed Poisson-Gaussian noise in bio-imaging applications. JPEG error analysis method[16] is proposed[16] to identify the duplicated and distorted areas in a JPEG digital image.

Thresholding eliminates certain coefficient which falls below a certain value. Selected threshold method is applied on the obtained coefficients for further processing. The recovery of coefficients and application of threshold at each level is used to recognize noise clearly and effectively. Selecting the appropriate threshold is main concerned issue. Careful balance of threshold cut-off is an important aspect as one cannot discard too many coefficients leading to smoothing and neither very few coefficients leading to under smoothed estimate[17]. Data adaptive thresholds[18] were formed to achieve optimum value of threshold. In the recent years there has been many researches done on wavelet domain and the selection of thresholding for image de-noising[19,20],

Application of universal threshold in wavelet transform for denoising an image is VisuShrink which is automatic and fast thresholding method[21]. It is a very simple technique where a simple threshold function is applied to obtained coefficients of the image. SureShrink provides more detailed image, hence giving better results than Visushrink[22]. This method is best suited for images inculcate with Gaussian noise[23]. The drawback of SureShrink method that is consideration of sparcity where local neighborhood of each coefficient is neglected resulting in biased estimator hence removing many terms from derived coefficients. To overcome this and increase precision of estimation, NeighBlock approach came in the picture that utilizes information of neighboring pixels. Consideration of neighboring pixels helps in deciding the threshold value. This method is best in case of Doppler signal. In this method, min-max or principle of minimum value and maximum value is considered. A fixed threshold is used for estimating mean square error of coefficients. Heursure is a method that is made by combining SURE and global thresholding method. The drawback of SURE method when applied to signal-to noise ratio being very small resulting in more noises is overcome by heursure method that accounts for a fixed threshold selection by global thresholding method.

1.2 Motivation Justification

Most of the standard techniques use a defined filter window to compute the local noise variance of a noise image. As a result, in the homogeneous region of the image, the noise level is greatly reduced. But in others areas like in the edges or lines, the image is either blurred or over smoothed. Hence, in this paper a method for denoising of medical images are carried out based on the combination of Wavelet bases in association Shrinkage Thresholding technique.

The main advantage of the Discrete Wavelet Transform (DWT) is that it can preserve the edges and fine details of the image while denoising. As there are numerous types of wavelet bases function it is necessary to identify the best base that is best fit for medical image noise removal. Further it is observed that when the level of decomposition changes, there is a variation in the performance of the denoising. Motivated by these facts, in this paper wavelet thresholding technique is employed for medical noise removal. Though the wavelet techniques on reconstruction removes some of the noise present in the image, it cannot completely denoise. Hence co-efficient which are resultant of decomposing using wavelet are to be threshoulded.

The threshoulding procedure is, in which small coefficients are removed while others are left unchanged. During the last decay, plenty of new thresholding techniques for wavelet coefficients have been emerged for noise removal. Hence it is essential to identify the suitable thresholding technique that best suits either the particular noise or a particular wavelet technique. Further it is also observed that whenever there is a variance in the noise level parameters, the performance of thresholding techniques are also varies. Justified by these facts, in this paper a comparative analysis of wavelet techniques in association with different threshoulding techniques is carried out.

1.3 Organization of the Paper

The rest of the paper is organized as follows. Methodology which includes outline of the proposed work, Discrete Wavelet transform, and Wavelet Shrinkage Thresholding techniques are presented in Section II. Experimental results are shown in Section III. Performance evaluation is discussed in Section IV. Finally conclusion is presented in Section V.

2. Methodology

2.1 Outline of the Proposed Method

Basic de-noising algorithms that use Discrete Wavelet Transform consist of three steps as shown in Figure 1. Discrete wavelet transform is used to decompose the noisy image and as a result wavelet coefficients are obtained. These coefficients are threshold using standard techniques to denoise the image. Inverse DWT is applied to the modified coefficients to get denoised image. In this study we focus on finding the best wavelet bases and the suitable coefficient shrinkage thresholding technique.

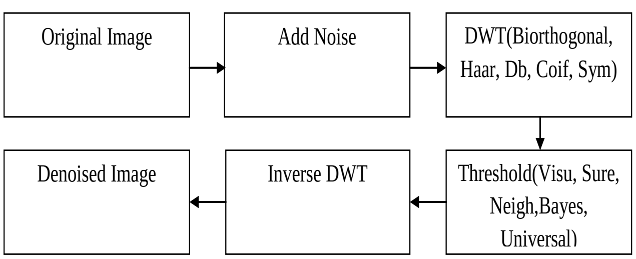


Figure 1. Block diagram of Image Denoising using Wavelet Transform.

2.2 Noise Models

2.2.1 Poisson Noise Model

It is signal-dependent, occurred in photon images. The noise model is defined as in the formulae (1)

$$\mathbf{d(m,n)} \sim \frac{\mathbf{1}}{\boldsymbol{\lambda}}\ \textbf{Poisson}\ \{\boldsymbol{\lambda} * \mathbf{o(m,n)}\} \tag{1}$$

Where o(m, n) and d(m, n) are the pixel values in the original and degraded images respectively. The amount of noise depends on λ.

2.2.2 Gaussian Noise Model

This most common type of noise results as contributions from many independent signals. This is a consequence of the central limit theorem which states that the sum of many random variables with various PDFs results in a signal with a Gaussian PDF.

2.2.3 Speckle Noise Model

Speckle noise mainly affects the natural characteristics of imaging, including medical ultra sound imaging. It is produced by the coherency of signals coming from multiple distributed targets. In the medical field, Speckle noise is referred to as 'texture' which contains diagnostic details of information

2.3 Discrete Wavelet Transform

The Discrete Wavelet Transform is used as signal decomposition in a set of independent spatially oriented frequency channels which represents spatial and spectral localization of image. The input image is passed through set of filters which is referred as decomposition. During the inverse transform, the components can be regained into the original signal without loss of information called as reconstruction. The decomposition on an image by DWT is processed in different levels. Figure 2 shows DWT decomposition of different bands at level 3. On decomposition, DWT yields bands like LL, LH, HL and HH. The LL band holds the most important information of an image. The next level of wavelet transform is employed only to the sub band image LL.

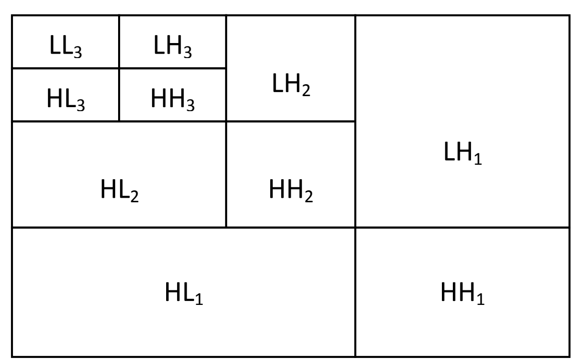


Figure 2.

2.4 Wavelet Coefficients Shrinkage Thresholding Techniques

The threshold approaches for image de-noising based on wavelet transform is explained below.

2.4.1 Visushrink

This approach is used in many applications. This algorithm offers the advantages of smoothness and adaptation. The drawback is, it exhibits visual artifacts.

Threshold T is computed using the formulae,

$$\mathbf{T} = \sigma\sqrt{2} * \log n2 \tag{2}$$

2.4.2 SureShrink

Donoho and Johnstone proposed Sure Shrink, a threshold chooser based on Stein"s Unbiased Risk Estimator (SURE)[24]. The Sure Shrink threshold t is defined as

$$t = \min(t, (\sqrt{(2 [*\log] n)})) \tag{3}$$

Where 't' is a value that minimizes Steins Unbiased Risk Estimator. (is the noise variance computed. 'n'is the size of the image. The aim of Sure Shrink is to reduce the mean squared error

2.4.3 Neighshrink

Let d(i, j) denotes is the wavelet coefficients. Let B(i, j) is a neighborhood window around d(i, j). Then the wavelet coefficient to be threshoulded is shrinked according to the formulae,

$$d(i,j) = d(i,j)^{*} B(i,j) \tag{4}$$

2.4.4 Bayes Shrink

Chang, Yu and Vetterli proposed Bayes Shrink[25] to minimize the Bayesian risk. It is defined as

$$t_n = \frac{\sigma^2}{\sigma_s} \tag{5}$$

Where σ^2, σ_s are the signal and noise variance level.

2.4.5 Normal Shrink

The value of the threshold is adaptive to the characteristics of different sub band and normal shrink is defined as

$$\sigma^2/\sigma_y \tag{6}$$

Where σ^2 Means the noise variance and β is the scale parameter and computed using the following equation.

$$\beta = \sqrt{\log\left(\frac{L_k}{J}\right)} \tag{7}$$

L_K means the length of the sub band at K^{th} scale.

3. Experimental Results

Experiments were conducted to denoise a MRI image of a skull which is shown in Figure 3. Gaussian, Salt and Pepper, and Speckle noises were considered. The denoised output images for different wavelet bases are presented in Figure 4. It is identified that Sym base provides better results. Hence it is subjected to different level of decomposition and the output is presented in Figure 5. Better results were obtained at level 3. Keeping this parameter, the effects on applying different thresholds were studied and results are shown in Figure 6. The effect on applying different noise variance is presented in Figure 7.

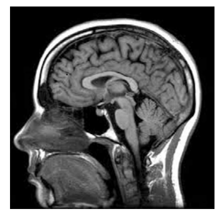


Figure 3. Original Image.

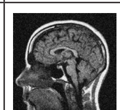
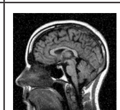
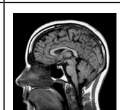
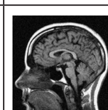
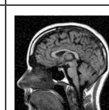
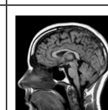
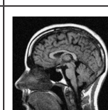
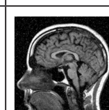
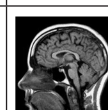
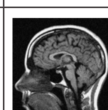
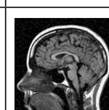
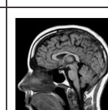
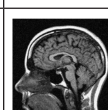
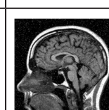
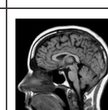
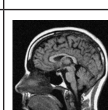
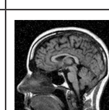
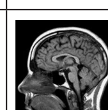
Noise Type		Gaussian	Salt and Pepper	Speckle
Noise Image				
PSNR		19.3592	23.6147	22.1181
WSNR		17.8142	20.4042	10.3213
VSNR		22.4364	12.2694	20.2356
Denoised Image Using Wavelet Bases	Biorthog-onal			
	Reverse Biorthog-onal			
	Daubechies			
	Coiflets			
	Symlets			

Figure 4. Denosing using different wavelet bases.

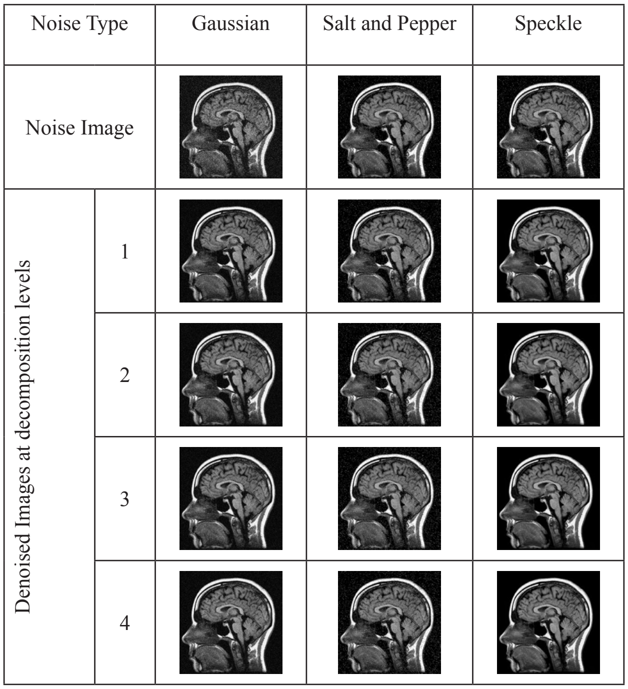


Figure 5. Denosing using different decomposition level.

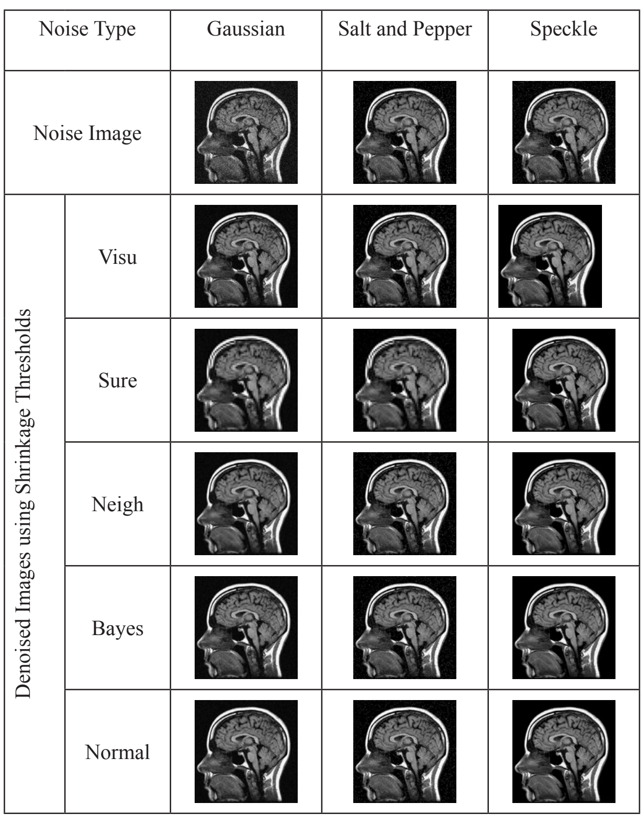


Figure 6. Threshold vs Metric.

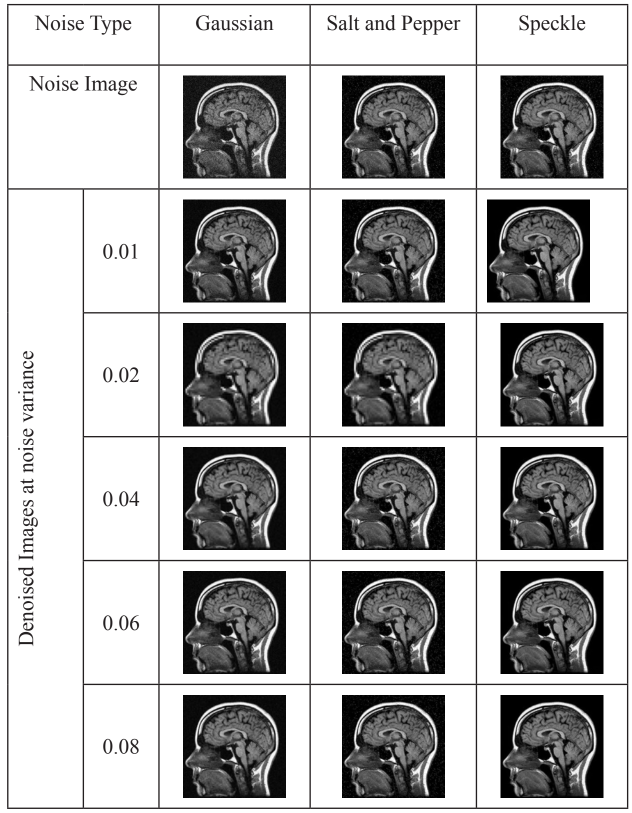


Figure 7. Noise Variance vs Metric.

4. Performance Analysis

4.1 Performance Metrics

4.1.1 PSNR

PSNR is a common measurement to analyse the quality of reconstruction of an image using the following formula.

$$PSNR = 10 \log 10 \left(\frac{255^2}{MSE}\right) \tag{13}$$

4.1.2 WSNR

The CSF was used as a weighting function for noise measurement and the error measurement criterion is the WSNR (weighted SNR):

$$WSNR = 10 \log 10 \Sigma Nn = 1[xn * c(xn)]2 \Sigma Nn \tag{14}$$

Where x_n and y_n denotes the original image and the noisy image

*denotes linear convolution and $c(x_n)$ is CSF in the spatial domain.

4.1.3 VSNR

The VSNR, in decibels, is accordingly given by

$$VSNR = 10 \log 10 \left(\frac{C^2(I)}{VD^2}\right) \quad (15)$$

Where C(I) denotes the RMS contrast of the original image I.

4.2 Performance Evaluation

The performance the Wavelet bases and Thresholding techniques were studied using PSNR, VSNR and WSNR. The first experiment is conducted to estimate the performance of the different wavelet base such as Biorthogonal, Reverse Biorthogonal, Daubechies, Coiflets, and Symlet. Reults are shown in Table 1. Considering all the metrics, it is observed that Symlet base performance is better than other bases. It is expected that the level of decomposition play a crucial role in the quality of the denoised image. Hence second experiment is conducted to identify the right level of decomposition and the metrics are shown in Table 2. Though wavelets by itself can remove noises to some extent, the results can be further enhanced by applying shrinkage thresholding techniques. Hence in experiment 3, Visu, Sure, Neigh, Bayes, and Normal shrink have been tested and their performance is shown in Table 3. Performance of any noise removal technique will deteriorate as the depth of noise level increases. Hence, by varying the noise parameters, results were taken and it is presented in Table 4.

Table 1. Wavelet Type vs Metric

Metric	Wavelet Type	Gaussian	Salt & Pepper	Speckle
PSNR	Biorthogonal	21.183	18.09	29.851
	Reverse Biorthogonal	21.166	18.065	29.867
	Daubechies	21.140	18.088	29.822
	Coiflets	21.158	18.087	29.864
	Symlets	21.177	18.113	29.875
WSNR	Biorthogonal	25.414	22.143	33.823
	Reverse Biorthogonal	25.405	22.111	33.838
	Daubechies	25.374	22.143	33.785
	Coiflets	25.395	22.134	33.851
	Symlets	25.419	22.154	33.871
VSNR	Biorthogonal	28.359	11.149	40.381
	Reverse Biorthogonal	28.463	11.146	40. 595
	Daubechies	28.257	11.139	40.443
	Coiflets	28.221	11.167	40.572
	Symlets	28.409	11.127	40.663

Table 2. Decomposition Level vs Metric

Metric	Level	Gaussian	Salt & Pepper	Speckle
PSNR	1	19.957	18.032	29.854
	2	19.948	17.641	23.871
	3	19.970	17.662	23. 867
	4	19.967	17.636	23.858
WSNR	1	23.274	22.065	33.835
	2	23.001	20.516	25.528
	3	23.292	20.198	25.700
	4	23.282	20.198	25.602
VSNR	1	22.837	11.164	40.485
	2	22. 872	12.261	22.784
	3	22.903	12.154	22.777
	4	22.875	12.116	22.773

Table 3. Threshold Type vs Metric

Metric	Threshold Type	Gaussian	Salt & Pepper	Speckle
PSNR	Visu Shrink	23.013	19.913	30.690
	Sure Shrink	21.081	19.961	33.267
	Neigh Shrink	23.779	21.375	30.788
	Bayes Shrink	23.631	19.956	30.213
	Normal Shrink	23.465	20.028	31.197
WSNR	Visu Shrink	24.433	21.414	33.099
	Sure Shrink	21.687	21.495	30.048
	Neigh Shrink	25.016	21.633	34.493
	Bayes Shrink	25.038	21.414	34.089
	Normal Shrink	24.846	21.110	34. 259
VSNR	Visu Shrink	20.958	14.891	41.876
	Sure Shrink	17.314	17.327	41.275
	Neigh Shrink	21.056	14.891	41.610
	Bayes Shrink	21.206	14.894	41.023
	Normal Shrink	21.684	14.857	41.891

Table 4. Noise Variance vs Metric

Noise Type	Noise Variance	Noisy Image PSNR	Metric PSNR	WSNR	VSNR
Gaussian	0.01	21.4581	23.787	25.246	21.064
	0.02	20.1109	22.980	24.714	20.679
	0.04	18.5884	21.625	23.818	19.791
	0.06	14.1524	20.583	23.022	18.911
	0.08	10.6393	19.749	22.836	18.154
Salt & Pepper	0.01	19.6147	21.760	22.431	17.125
	0.02	17.6249	19.896	21.289	14.914
	0.04	16.0237	17.691	21.114	12.385
	0.06	14.6058	16.295	20.264	10.697
	0.08	12.1539	16.039	19.968	10.247
Speckle	0.01	21.4175	24.086	25.706	22.439
	0.02	21.6285	24.119	25.758	22.516
	0.04	21.9815	24.105	25.878	22.514
	0.06	20.3464	24.118	25.693	22.527
	0.08	20.5781	24.098	25.697	22.501

For Speckle noise removal it is observed that the wavelet based techniques are best suitable as shown in Table 1, All the wavelet bases perform equally well in removing all type of noise. It is identified that the performance of the Symlet is slightly better than other bases. From Table 2, it is observed that all performance metrics of speckle noise are higher when the decomposition level is 1. For Gaussian and poisson noises, the performance metrics are high when the decomposition level is 3. Hence it is suggested that, further level decomposition will only increase the computational complexity without significant performance. From Table 3, it is noted that, irrespective of the noise type, Neigh shrinkage thresholding technique performs well than others. Further the results suggest the importance of using thresholding techniques when compared to results in Table 1 without thresholding. Noise can be better removed only to specified amount of tolerance as shown in Table 4. Considering the Gaussian, Poisson , the performance of the wavelet bases and irrespective of the thresholding technique decreases as noise level increases. The interesting fact regarding the Speckle noise is, it can be removed efficiently by the wavelet bases irrespective of amount of noise present.

5. Conclusion

In this paper, we present a comparative analysis of MRI image denoising using wavelet domain shrinkage thresholding techniques. Experiments were conducted to study the suitability of different wavelet bases like Biorthogonal, Haar, Daubechies, Coiflets, symlets. Noises can removed by thresholding the coefficient of the wavelet bases. Hence, threshold technique like visu shrink, bayes shrink, universal shrink, Neigh shrink, Sure shrink have been applied. Quantitative performance measure such as PSNR, WSNR, and VSNR were used to analyze the denoising effect. It is observed that among all wavelet bases, sym let performs well in association with Neigh shrink at third level of decomposition for Gaussian and also for Poisson noises, and first level of decomposition for speckle noises.

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