

Performance Analysis of Static and Dynamic State Estimation Incorporating Synchro Phasor Measurements

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Abstract

Objective: State estimation is used as a necessary tool for online monitoring, analysis and control of power systems. The main objective of this paper is to deal with static state estimation using the snap shot data and dynamic state estimation which accounts for the time varying behaviour of power system states in to consideration. **Methods/Analysis:** Effects of inclusion of PMU measurements along with the available metered data have been explored using the weighted least square state estimation technique in this paper. A comparative analysis of static state estimation with and without bad data has been carried out and the bad data has been identified and eliminated by using largest normalized residue test. **Findings:** To investigate the time varying nature of the system states linear state estimator with second order approximation and kalman filter techniques has been proposed in this paper. Case studies are conducted on IEEE 14 bus test system and the test results obtained from non linear, linear first order, second order and kalman filter techniques have been compared. **Application/Improvements:** Correction of state variables are obtained using linear state estimation with first order and second order approximation and kalman filter techniques has been compared to get the better state estimation algorithm.

Keywords: Bad Data, Kalman Filter, PMU, State Estimation, Static, Dynamic, Weighted Least Squares

Nomenclature:

z	=	Measurement vector.
x	=	State vector.
$h(x)$	=	Coefficient vector of non linear function w r to x .
m	=	Number of measurements.
N	=	Number of buses.
n	=	Number of states.
r	=	Residual vector.
W	=	Weight matrix.
σ^2	=	Covariance of measurements.
R	=	Covariance matrix of measurement errors.
J	=	Objective function.
H	=	Jacobian matrix.
Δx	=	Corrections of state variables.

x^k	=	State variables at k^{th} iteration.
h	=	Measurement function.
e_m	=	Real part of the voltage of bus m .
f_m	=	Imaginary part of the voltage of bus m .
P_m	=	Real power injection.
Q_m	=	Reactive power injection.
P_{mk}	=	Real power flow from bus m to k .
Q_{mk}	=	Reactive power flow from bus m to k .
G_{mk}	=	Conductance of mk^{th} element
B_{mk}	=	Susceptance of mk^{th} element.
g_{mk}	=	Conductance of series branch from m^{th} bus to k^{th} bus.
b_{mk}	=	Susceptance of series branch from m^{th} bus to k^{th} bus.
g_{shm}	=	Conductance of shunt element at bus m .
b_{shm}	=	Susceptance of shunt element at bus m .

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$\hat{x}$	=	Estimated state.
r^N	=	Normalised residue.
G	=	Gain matrix.
Ω_{ii}	=	Residue covariance matrix with i^{th} diagonal element.
Δz	=	Change in the measurements.
B	=	Hessian matrix.
X_p	=	Predicted state estimate.
P_p	=	Predicted covariance.
K	=	Kalman filter gain.
X_c	=	Corrected state estimate.
P_c	=	Corrected covariance.
I	=	Identity matrix.

1. Introduction

State estimation is a method which provides the best possible approximation of the states of the power system network by processing the available data. The control centres on consumer side receive the raw measurements which may contain bad data and redundant measurements due to the loss of communication channel failure or equipment failure. So the data should be processed before they are fed to the control centres. This process is called state estimation. The estimation runs at fixed intervals of time by the measurements taken from the snapshot of the entire power system network, at a particular point of time, is called static state estimation. Dynamic state estimation is developed to account time varying behaviour of the power system¹. State estimation techniques such as Weighted Least Square method (WLS), Least Absolute Value (LAV), Iteratively Reweighted Least Squares (IRLS) implementation of Weighted Least Absolute Value (WLAV) with scattered and clustered loss of data is compared based on data redundancy and convergence time. The results shows that at higher level of data redundancy, LAV (Least Absolute Value) has less convergence time for both scattered and clustered loss of data compared to other methods and at lower level of data redundancy, WLS method converges faster for scattered loss of data and WLAV (Weighted Least Absolute Value) has less convergence time for clustered loss of data². State estimators are programmed in languages fort on and MATLAB based on different measurement redundancies by using traditional Weighted Least Squares method. It is observed that the MATLAB estimator was best due to its less pro-

cessing time, less number of iterations and to produce accurate results³. Comparative study of Weighted Least Squares method, Hachtel's augmented matrix method and fast decoupled method with and without coupling are discussed based on the maximum, mean and standard deviation of estimated state variables and the execution time. The result shows that the objective function of Weighted Least Squares method becomes minimal. Therefore it is concluded that Weighted Least Squares is the best method because of its online implementation and performance. Fast decoupled with coupling is also best because it takes less execution time and can get good estimates⁴. A new method Particle Swarm Optimization hybridized with Differential Evolution (PSO-DE) technique has been done and compared with WLS and General Particle Swarm Optimization (GPSO). Here the results shows that PSO-DE minimizes the objective function, frequency occurrence of the minimum value near the mean value is more and it is more efficient and more accurate⁵. Static and dynamic state estimation techniques with SCADA (Supervisory Control and Data Acquisition) measurements are discussed and it is extended to PMU (Phasor Measurement Unit) measurements. Hence it is concluded that the estimated states of PMU measurements are almost equal to the true values i.e. PMU's are more accurate⁶. A new method Weighted Linear Least Square (WLLS) is proposed and compared with WLS and WLAV. Here it is concluded that WLLS method is fast, accurate and it can be applicable for ill conditions⁷. Incorporating a single PMU and multiple PMU's are explained⁸. SCADA (Supervisory Control and Data Acquisition System) provides data to the control centres at which the power system network operates. SCADA is replaced with PMU due to their higher accuracy of measurements and state estimates. PMU provides not only the voltage magnitude but also the phasor angles and the accuracy of PMU measurements are high when compared to the normal measurements due to their higher weight-age. Due to the incorporation of highly accurate phasor measurements, convergence speed increases mainly in the large networks and the estimated values are more sensitive to the phasor measurements than the power flow measurements. As PMU's are expensive, they are placed at particular buses where they can gather more information from other buses by optimal PMU placement⁹. A two stage estimation model is considered using SCADA and PMU measurements. In first stage, using PMU measurements linear state estimation in rectangular form has been

done. In second stage, estimated states of first stage are transformed to polar form and non linear state estimation has been done by incorporating SCADA measurements. The results explain that the first stage is more accurate in terms of speed¹⁰. As installation of PMU's increases, accuracy of system states increases. Data obtained under the failure in the instrument or in the communication channels is named as bad data. When the measurements are free from bad data then the estimated values of bad data suppression algorithm and the Weighted Least Squares estimator are alike. In the presence of bad data, bad data suppression algorithm can adequately detect it and can produce better estimates¹¹. The sensitivity of input uncertainties in WLS estimator and bad data detection using normalised residues are shown¹². A two stage linear state estimation has been done by PMU measurements and bad data detection by using normalised residue test¹³. Estimated states are obtained by WLS and largest residual test is performed to detect and eliminate the bad data¹⁴. Static state estimators cannot capture the dynamic behaviour of the power system network efficiently. So, dynamic state estimation is preferred for reliable operation of power system¹⁵. The system states are monitored continuously at the regular intervals. Dynamic state estimators have the capability of tracking the current system states and also predicting the state vector for next sampling time. An attempt has been made to incorporate PMU in dynamic state estimation¹⁶. A new method to incorporate PMU in static state estimation has been discussed¹⁷. Kalman filter process the set of equations to estimate the states in such a way that the mean of the squared error is minimized. Basic information about kalman filter and Extended Kalman Filter is given¹⁸. The effect of bad data on kalman filter and its overall performance on the system is observed¹⁹. Kalman filter algorithm and properties are illustrated²⁰. As kalman filter is applicable only for linear systems two new methods are proposed namely Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF)²¹. In Extended Kalman Filter technique, it is necessary to find Jacobian matrix which is a very difficult task. In UKF a deterministic sampling technique called unscented transform is used. Here the minimum number of sigma points around the mean is considered. Those sigma points are propagating through the nonlinear functions, mean and covariance of the estimate are recovered in UKF and can get more accurate true mean and variance. There is no need to find Jacobian matrix. Extended Kalman Filter is more desirable for online applications than Unscented

Kalman Filter because of its high computational requirements²². A new method for state estimation using kalman filter with unit time delay is introduced to estimate the system states for linear systems. It is concluded that, for increase in time steps covariance decreases. As covariance decreases difference between estimated states and true states also decreases²³.

This paper presents the effect of phasor measurements, bad data detection and elimination in static state estimation. But in static, dynamic nature of power system network is not considered. To overcome that limitation, dynamic state estimation including phasor measurements with new algorithms namely linear state estimation using first order approximation, second order approximation and kalman filter technique has been done and the results are compared.

The organisation of this paper is as follows: Section 2 illustrates WLS method and its application to power system state estimation. It also describes the incorporation of phasor measurements, bad data detection and elimination techniques. Section 3 presents new algorithms proposed for dynamic state estimation. Usage of second order linear state estimator and kalman filter techniques has been investigated. Results are shown in Section 4. Conclusions from the results are summarized in Section 5.

2. WLS Method

General measurement model used for mathematical formulation of WLS method for finding 'n' states with 'm' measurements is as follows:

$$z = h(x) + r \quad (1)$$

The objective function is to minimize the sum of the squares of the weighted deviations of estimated measurements from the actual measurements. It is defined as:

$$J = \sum_{i=1}^m W_{ii} r_i^2$$

where,

$$W = R^{-1} = \begin{bmatrix} \frac{1}{\sigma_1^2} & & & \\ & \frac{1}{\sigma_2^2} & & \\ & & \ddots & \\ & & & \frac{1}{\sigma_m^2} \end{bmatrix} \quad (3)$$

where, σ_m^2 is the covariance of measurements.

R is the covariance matrix of ($m \times m$).

Here, errors are assumed to be independent random variables with zero mean and known variance. Measurement errors are assumed to be uncorrelated.

Using Equation 1 and Equation 3, Equation 2 can be written as:

$$J = \sum_{i=1}^m \frac{(z - h(x))^2}{R} \quad (4)$$

$$J = (z - h(x))^T R^{-1} (z - h(x)) \quad (5)$$

The corrections of state variables Δx are obtained by expanding Equation 5 using Taylor series expansion and equating it to zero.

$$\Delta x^k = [G]^{-1} H^T \left(\hat{x}^k \right) R^{-1} r \left(\hat{x}^k \right) \quad (6)$$

where,

$$G = H^T \left(\hat{x}^k \right) R^{-1} H \left(\hat{x}^k \right) \quad (7)$$

$$r \left(\hat{x}^k \right) = z - h \left(\hat{x}^k \right) \quad (8)$$

$$H \left(\hat{x}^k \right) = \frac{\partial h \left(\hat{x}^k \right)}{\partial x}, \quad (9)$$

is the Jacobian matrix consisting of first order derivatives of the measurement function $h(x)$.

Using the iterative process state variables are obtained with the equation,

$$\hat{x}^{k+1} = \hat{x}^k + \Delta x^k \quad (10)$$

In power system state estimation, real and imaginary parts of voltage at each bus are considered as state variables and metered data like real and imaginary parts of voltage, real and reactive power injections and real and reactive power flows are taken as measurements. Application of Weighted Least Squares method to power system state estimation is as follows:

2.1 Base Case Algorithm for Non Linear WLS Static State Estimation

Step 1: Read the line data, bus data and measurement data.

Step 2: Form bus admittance matrix (Y bus).

Step 3: Start the iterative process.

Step 4: Calculate the measurement function h ,

$$h = [e_m; f_m; P_m; Q_m; P_{mk}; Q_{mk}] \quad (11)$$

where,

$$P_m = \sum_{k=1}^n [G_{mk} (e_m e_k + f_m f_k) + B_{mk} (f_m e_k - e_m f_k)] \quad (12)$$

$$Q_m = \sum_{k=1}^n [G_{mk} (f_m e_k - e_m f_k) - B_{mk} (e_m e_k + f_m f_k)] \quad (13)$$

$$P_{mk} = -g_{mk} (e_m e_k + f_m f_k) - b_{mk} (f_m e_k - e_m f_k) + (e_m^2 + f_m^2) (g_{mk} + g_{shm}) \quad (14)$$

$$Q_{mk} = -g_{mk} (f_m e_k - e_m f_k) + b_{mk} (e_m e_k + f_m f_k) - (e_m^2 + f_m^2) (b_{mk} + b_{shm}) \quad (15)$$

Step 5: Calculate residual matrix from Equation 8.

Step 6: Calculate the Jacobian matrix $H(\hat{x}^k)$ can be written as H.

$$H = \begin{bmatrix} \frac{\partial e_m}{\partial e} & 0 \\ 0 & \frac{\partial f_m}{\partial f} \\ \frac{\partial P_m}{\partial e} & \frac{\partial P_m}{\partial f} \\ \frac{\partial Q_m}{\partial e} & \frac{\partial Q_m}{\partial f} \\ \frac{\partial P_{mk}}{\partial e} & \frac{\partial P_{mk}}{\partial f} \\ \frac{\partial Q_{mk}}{\partial e} & \frac{\partial Q_{mk}}{\partial f} \end{bmatrix} \quad (16)$$

Step 7: Calculate the objective function from Equation 2.

Step 8: Calculate the gain matrix from Equation 7.

Step 9: Calculate the correction of state variables from Equation 6.

Step 10: Update the state variables from Equation 10.

Step 11: States obtained in Cartesian form are converted in to polar form.

Step 12: If $\max|\Delta x^k| \leq \text{tolerance}$, then stop the iteration process. Otherwise increment the iteration $k = k+1$ and return to step 4.

In this paper non linear state estimation has been done by WLS method including PMU measurements is as follows

2.2 Incorporation of PMU Measurements

PMU is an instrument which can receive synchronized sampling clocks from global positioning system satellite signals, to calculate the voltage magnitudes and phase angles at each bus accurately. Accuracy of these measurements is very high. In measurements, when variance (σ^2) is high then its weightage will be low. As PMU measurements are more accurate they should be given higher weightage. In this paper variance is taken low for some of the measurements and those measurements are treated as PMU measurements. Some of the existing measurements have been converted as PMU measurements. In this paper, detection and elimination of bad data also has been done after including the PMU measurements.

2.3 Bad Data Detection and Elimination

Errors in measurements are detected by the normalised residue (r^N) and are compared with threshold. If the normalised residue is more than threshold value then the measurement of that particular residue is said to be bad.

Step 1: Run base case algorithm and obtain state

estimates ($\hat{x}$).

Step 2: Find the residual vector, $r = z - h(\hat{x})$.

Step 3: For normalization of a residue, calculation of residue covariance matrix is necessary.

$$\Omega_{ii} = R - [HG^{-1}H^T] \quad (17)$$

Where, R is the covariance matrix Equation 3.

Step 4: Calculate the normalised residue

$$r_i^N = \frac{|r_i|}{\sqrt{\Omega_{ii}}} \quad (18)$$

Step 5: Compare the residues with threshold value.

Step 6: Identify the maximum normalised residue which is more than the threshold value.

Step 7: Eliminate that particular measurement with maximum normalised residue and recalculate the new state vectors.

So far static data has been considered. But, to account for the time varying nature of measurements dynamic state estimation has been carried out. New algorithms such as second order linear approximation and kalman filter techniques have been proposed in the following Section 3.

3. Proposed Methodology

To start with, first order linear approximation has been presented, followed by the second order linear approximation and kalman filter techniques.

3.1 Linear State Estimation using First Order Approximation

In this method non linear state equations are converted to linear equations by using Taylor series expansion using first order derivatives and neglecting the higher order terms.

If the system is linear $h(x) = H(x)$.

Algorithm:

Step 1: Run base case algorithm with original values of measurements $[z]$.

Step 2: Store the original Jacobian matrix $[H]$, estimated state vector $[x_{\text{actual}}]$ and the actual measurement vector $[z]$.

Step 3: Modify the actual measurement values for the next time instant as $z_{\text{modified}} = z + \Delta z$.

Step 4: With these modified values find the modified state vectors $[x_{\text{modified}}]$ by running the WLS state estimation algorithm.

Step 5: Find the changes in the state vector values Δx for a corresponding change in the measurement Δz .

Step 6: Using the values Δx , Δz and H , now linearize the non linear equations by using Taylor series expansion using first order derivatives and neglect the higher order terms. If $h(x) = H(x)$ then system is said to be linear.

Step 7: Calculate the residual matrix,

$$r = \Delta z - H \cdot \Delta x \quad (19)$$

Step 8: Calculate the objective function from Equation 2.

Step 9: Calculate the correction of state variables using Equation 6.

Step 10: Update the state variables using Equation 10.

Step 11: states obtained in Cartesian form are converted in to polar form.

Step 12: If $\max |\Delta x^k| \leq \text{tolerance}$, then stop the iterative procedure. Else update the iteration $k = k+1$ and return to step 7.

To get the actual change in states, new algorithms using second order approximation and kalman filter techniques are presented as follows

3.2 Using Second Order Approximation

In this method non linear state equations are converted to linear equations by using Taylor series expansion using first order and second order terms. Second order differentiation matrix is also called as Hessian matrix.

$$z = (H + BX)x + r$$

$$z = Yx + r \quad (20)$$

Where,

$Y = H + BX$, $X = \text{diag}(x)$ and B is the Hessian matrix which is the derivative of J the Jacobian at the initial operating point. (21)

Calculate the error matrix,

$$r = \Delta z - Y \Delta x \quad (22)$$

For correction of state variables, If Y rows are linearly independent, then,

$$\Delta x^k = Y^T [Y^T R^{-1} Y]^{-1} R^{-1} r \quad (23)$$

If Y columns are linearly independent, then

$$\Delta x^k = [Y^T R^{-1} Y]^{-1} Y^T R^{-1} r \quad (24)$$

Update the state variables,

$$\hat{x}^{k+1} = \hat{x}^k + \Delta x^k \quad (25)$$

Algorithm:

Step 1: Run base case algorithm with original values of measurements (z).

Step 2: Store the original Jacobian matrix $[H]$, estimated state vector $[x_{\text{actual}}]$ and the actual measurement vector $[z]$.

Step 3: Modify the actual measurement values for the next time instant as $z_{\text{modified}} = z + \Delta z$.

Step 4: With these modified values find the modified state vectors $[x_{\text{modified}}]$ by running the WLS state estimation algorithm.

Step 5: Find the changes in the state vector values Δx for a corresponding change in the measurement Δz .

Step 6: Using the values Δx , Δz and H , now linearize the non linear equations by using Taylor series expansion using first order and second order derivatives.

Step 7: Calculate Hessian matrix (B) for original values,

$$B(x) = \frac{\partial^2 H(x)}{\partial x^2} \quad (26)$$

$$B = \begin{bmatrix} \frac{\partial^2 e_m}{\partial e^2} & 0 \\ 0 & \frac{\partial^2 f_m}{\partial f^2} \\ \frac{\partial^2 P_m}{\partial e^2} & \frac{\partial^2 P_m}{\partial f^2} \\ \frac{\partial^2 Q_m}{\partial e^2} & \frac{\partial^2 Q_m}{\partial f^2} \\ \frac{\partial^2 P_{mk}}{\partial e^2} & \frac{\partial^2 P_{mk}}{\partial f^2} \\ \frac{\partial^2 Q_{mk}}{\partial e^2} & \frac{\partial^2 Q_{mk}}{\partial f^2} \end{bmatrix} \quad (27)$$

Step 8: Calculate Y matrix from Equation 21.

Step 9: Calculate the residual matrix from Equation 22.

Step 10: Calculate the objective function from Equation 2.

Step 11: Calculate the correction of state variables from Equation 24.

Step 12: Update the state variables using Equation 25.

Step 13: States obtained in Cartesian form are converted in to polar form.

Step 14: If $|\max(\Delta x^k)| < \text{tolerance}$ then stop,

Else update the iteration $k=k+1$ and return to step 8.

3.3 Kalman Filter Technique

Kalman filter is a state estimation technique which produces an optimal state estimate. In this method the mean value of the sum of the estimation errors gets a minimal value.

The methodology starts from the general measurement model given by Equation 1 which is reproduced below:

$$z = h(x) + r$$

Following assumptions are made in this procedure:

- Errors are assumed to be independent i.e. weakly correlated.

$$E[r_i r_j] = 0, i \neq j \quad (28)$$

- Errors have zero mean.

$$E[r_i] = 0 \quad (29)$$

- The initial values of predicted corrections of states (X_p) and predicted covariance (P_p) are used to determine the predicted errors.

$$r = \Delta z - H.X_p \quad (30)$$

Calculate the kalman filter gain,

$$K = P_p H^T [H.P_p.H^T + R]^{-1} \quad (31)$$

Corrected updates to the states and covariance can be obtained by,

$$X_c = X_p + (K.r) \quad (32)$$

$$P_c = [I - K.H]P_p \quad (33)$$

Procedure is continued with updated values until convergence is reached.

Algorithm:

Step 1: Run base case algorithm with original values of measurements (z).

Step 2: Store the original Jacobian matrix [H], estimated state vector [x_{actual}] and the actual measurement vector [z].

Step 3: Modify the actual measurement values for the next time instant as $z_{\text{modified}} = z + \Delta z$.

Step 4: With these modified values find the modified state vectors [x_{modified}] by running the WLS state estimation algorithm.

Step 5: Find the changes in the state vector values Δx for a corresponding change in the measurement Δz .

Step 6: Calculate the Jacobian matrix (H) at initial operating point from Equation 19.

Step 7: Assume the predicted state estimate (X_p) and predicted covariance (P_p).

$$X_p = \Delta x, P_p = I. \quad (34)$$

Step 8: Calculate the residue matrix from Equation 30.

Step 9: Calculate the kalman filter gain using Equation 31.

Step 10: Calculate the corrected state estimate using Equation 32.

Step 11: Calculate the corrected covariance using Equation 33.

Step 12: Update, $X_p = X_c$, $P_p = P_c$ and go to step 8.

Step 13: Iterations will stop when the corrected state becomes minimal or less than tolerance else increment the iteration until it reaches the tolerance value.

Step 14: States obtained in Cartesian form are converted into polar form.

The results obtained from all the algorithms and there comparisons are discussed in the following Section 4.

4. Results and Discussions

IEEE 14 bus system is considered for testing the algorithms. It is shown in Figure 1.

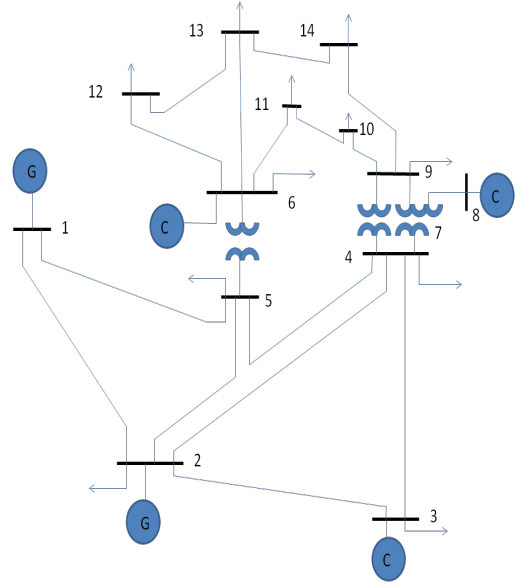


Figure 1. IEEE 14 bus test system.

Base case algorithm has been done for non linear state estimation using WLS method using the line data, bus data and measurement data given in Tables 1, 2 and 3 respectively. Measurement data consists of real and imaginary part of voltage magnitude, real and reactive power injections and real and reactive power flows. In this covariance of metered data is considered as 10^{-4} . Tables 1, 2 and 3 gives the line data, bus data and measurement data of standard bus system.

In Table 3, *Type 1; represents the voltage magnitude, Type 2; represents the phase angle, Type 3; represents the real power injections, Type 4; represents the reactive power injections, Type 5; represents the Real power flows and Type 6; represents reactive power flows.

The tolerance value used for all the case studies is 0.000001.

Table 1. Line data

From bus	To bus	R(pu)	X(pu)	B/2(pu)	Tap
1	2	0.01938	0.05917	0.0264	1
1	5	0.05403	0.22304	0.0246	1
2	3	0.04699	0.19797	0.0219	1
2	4	0.05811	0.17632	0.0187	1
2	5	0.05695	0.17388	0.017	1
3	4	0.06701	0.17103	0.0173	1
4	5	0.01335	0.04211	0.0064	1
4	7	0	0.20912	0	0.978
4	9	0	0.55618	0	0.969
5	6	0	0.25202	0	0.932
6	11	0.09498	0.1989	0	1
6	12	0.12291	0.25581	0	1
6	13	0.06615	0.13027	0	1
7	8	0	0.17615	0	1
7	9	0	0.11001	0	1
9	10	0.03181	0.0845	0	1
9	14	0.12711	0.27038	0	1
10	11	0.08205	0.19207	0	1
12	13	0.22092	0.19988	0	1
13	14	0.17093	0.34802	0	1

Table 2. Bus data

Bus no	B	V(pu)	Theta(deg)
1	0	1.06	0
2	0	1.045	-4.98
3	0	1.01	-12.72
4	0	1.0186	-10.32
5	0	1.0203	-8.78
6	0	1.07	-14.22
7	0	1.062	-13.37
8	0	1.09	-13.37
9	0.19	1.0563	-14.95
10	0	1.0513	-15.1
11	0	1.0571	-14.79
12	0	1.0552	-15.08
13	0	1.0505	-15.16
14	0	1.0358	-16.04

Table 3. Measurement data

Measurement no	Type*	Z(pu)	From bus	To bus	R
1	1	1.06	1	0	0.0001
2	2	0	1	0	0.0001
3	3	-0.942	3	0	0.0001
4	3	-0.478	4	0	0.0001
5	3	0	8	0	0.0001
6	3	-0.09	10	0	0.0001
7	3	-0.035	11	0	0.0001
8	3	-0.061	12	0	0.0001
9	3	-0.149	14	0	0.0001
10	4	0.0439	3	0	0.0001
11	4	0.039	4	0	0.0001
12	4	0.1736	8	0	0.0001
13	4	-0.058	10	0	0.0001
14	4	-0.018	11	0	0.0001
15	4	-0.016	12	0	0.0001
16	4	-0.05	14	0	0.0001
17	5	1.5683	1	2	0.0001
18	5	0.7319	2	3	0.0001
19	5	-0.5446	4	2	0.0001
20	5	0.2809	4	7	0.0001
21	5	-0.4061	5	2	0.0001
22	5	0.1774	6	13	0.0001
23	5	-0.0728	11	6	0.0001
24	5	0.0161	12	13	0.0001
25	6	-0.2039	1	2	0.0001
26	6	0.0357	2	3	0.0001
27	6	0.0339	4	2	0.0001
28	6	-0.0942	4	7	0.0001
29	6	-0.0163	5	2	0.0001
30	6	0.0716	6	13	0.0001
31	6	-0.0335	11	6	0.0001
32	6	0.0074	12	13	0.0001

4.1 PMU Measurements

Out of the metered data 3, 9, 10, 16, 17, 20, 24, 25, 28 and 32 measurements are considered as PMU data. To reflect them as PMU data, their weightage is increased from 10000 to 25000. With these changed measurement data

set which contains PMU measurements, once again state estimation is performed.

The results obtained are compared with the base case solution without PMU measurements and are given in Table 4.

Table 4. State estimation with PMU measurements

Bus no	without PMU		with PMU	
	V(pu)	Angle(deg)	V(pu)	Angle(deg)
1	1.06	0	1.06	0
2	1.041	-4.98	1.045	-4.98
3	0.9852	-12.72	1.01	-12.72
4	1.0021	-10.32	1.019	-10.32
5	1.0083	-8.78	1.02	-8.78
6	1.037	-14.22	1.07	-14.22
7	1.0332	-13.37	1.062	-13.37
8	1.0605	-13.37	1.09	-13.37
9	1.0204	-14.95	1.056	-14.95
10	1.0148	-15.1	1.051	-15.1
11	1.0218	-14.8	1.057	-14.8
12	1.0187	-15.08	1.055	-15.08
13	1.0137	-15.16	1.05	-15.16
14	0.9953	-16.04	1.032	-16.45

Effect of PMU measurements is speed and accurate. Elapsed time of state estimation by WLS method with metered data is 4.360689 sec. For PMU measurements along with metered data elapsed time is 3.835021 sec.

4.2 Bad Data Detection

A single bad data is induced in real power injection at bus 3 by 5%. Here, largest normalised residue test is used to detect the bad data by calculating the normalised residue and compare it with the threshold value. Threshold value is set to 10^{-12} for bad data identification. Largest residue which is more than threshold value is said to be the bad data.

A graph is plotted between the measurement number and the corresponding residue for different types of measurements and is given in Figure 2.

This graph shows the residues in real and reactive power injection and power flows where the maximum normalised residue said to be the bad. Hence real power injection at bus 3 is said to be the bad data.

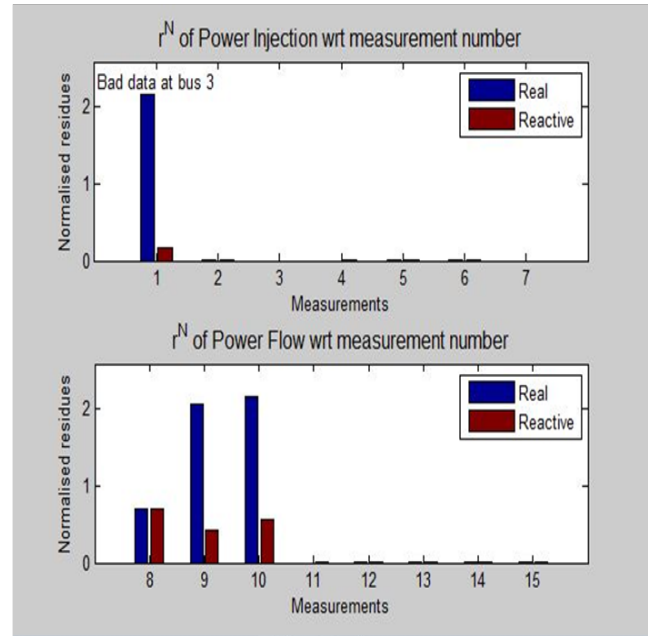


Figure 2. Normalised residues w.r.to measurement numbers.

After detecting the bad data it has been eliminated by nullifying that particular measurement data and again run the state estimation. The results obtained are shown in Table 5.

Table 5 shows the estimated states when single bad data is applied and eliminated. Here, residue obtained in the absence of bad data is 0.0023. After applying single bad data in real power injection at bus 3 is having the residue of 2.149 and it is eliminated by nullifying that particular measurement data and then the residue obtained is 0.0023. In the absence of bad data the voltage at bus 3 is 1.01 pu, after applying a single bad data at real power injection at bus 3, voltage obtained is 0.985 pu and after the elimination of that bad data, voltage at bus 3 is 1.009 pu. This proves that the bad data has been detected and eliminated efficiently.

4.3 Linear State Estimation

Linear state estimation using the proposed second order approximation has been done and the obtained results are compared with those of first order approximation and those from kalman filter techniques. The procedure followed for the linear state estimation is as follows:

- Measurement data set has been modified by increasing all measurement values by 2% which makes new measurement data set $z + \Delta z$.

Table 5. Comparison of estimated states with and without bad data

Bus No	Absence of bad data		After applying bad data		After eliminating bad data	
	V(pu)	Angle(deg)	V(pu)	Angle(deg)	V(pu)	Angle(deg)
1	1.06	0	1.058	0	1.058	0
2	1.045	-4.98	1.039	-5	1.043	-5
3	1.01	-12.72	0.985	-12.57	1.009	-12.57
4	1.019	-10.32	0.999	-10.45	1.016	-10.45
5	1.02	-8.78	1.007	-8.81	1.019	-8.81
6	1.07	-14.22	1.017	-16.81	1.062	-16.81
7	1.062	-13.37	1.03	-13.51	1.059	-13.51
8	1.09	-13.37	1.057	-13.51	1.087	-13.51
9	1.056	-14.95	0.999	-17.54	1.048	-17.54
10	1.051	-15.1	0.994	-17.7	1.043	-17.7
11	1.057	-14.8	1.001	-17.39	1.049	-17.39
12	1.055	-15.08	0.998	-17.68	1.047	-17.68
13	1.05	-15.16	0.993	-17.76	1.042	-17.76
14	1.032	-16.45	0.974	-18.65	1.024	-19.07

Table 6. Comparison of results

Bus no	Actual Δx (pu)	Linear 1st order Δx (pu)	Linear 2 nd order Δx (pu)	Kalman filter Δx (pu)	Actual Δx Angle(rad)	Linear 1st order Δx Angle(rad)	Linear 2 nd order Δx Angle(rad)	Kalman filter Δx Angle(rad)
1	0.021	0.021	0.021	0.021	0	0	0	0
2	0.0209	0.0207	0.021	0.021	0	0.0014	0.0007	0.0007
3	0.0214	0.0206	0.0216	0.0216	-0.0002	0.0015	0.0005	0.0004
4	0.0212	0.0205	0.0213	0.0213	-0.0001	0.0014	0.0006	0.0005
5	0.0212	0.0207	0.0213	0.0213	-0.0001	0.0014	0.0006	0.0005
6	0.0193	0.0183	0.0195	0.0193	-0.0006	0	0.0001	0.0001
7	0.0218	0.0209	0.022	0.022	-0.0001	0.0015	0.0007	0.0006
8	0.0219	0.0209	0.0221	0.0221	0.0001	0.0015	0.0008	0.0007
9	0.0193	0.0183	0.0195	0.0193	-0.0007	0	0	0
10	0.0193	0.0183	0.0195	0.0193	-0.0007	0	0	0
11	0.0193	0.0183	0.0195	0.0193	-0.0007	0	0	0
12	0.0193	0.0183	0.0196	0.0193	-0.0007	0	0	0
13	0.0193	0.0183	0.0195	0.0193	-0.0007	0	0	0
14	0.0194	0.0182	0.0196	0.0194	-0.0008	-0.0001	-0.0002	-0.0002

The value of Δz is stored and with the new measurement data set ($z+\Delta z$) non linear state estimation by WLS method is performed yielding new state estimates ($\hat{x}_{new}$).

The actual correction to the states has been calculated as $\Delta x_{actual} = \hat{x}_{new} - \hat{x}$

where, $\hat{x}$ is base case solution.

The above values are used for investigating the accuracy of obtained results from linear state estimation methods and kalman filter technique as given in Table 6.

Correction of states by non linear state estimation, linear state estimation by first order, second order

approximations and kalman filter algorithms are tested and compared.

It is worth noting that the actual change in the state at bus 14 for non linear state estimation is 0.0194 pu. By using the linear state estimation first order, second order and kalman filter techniques change in the state obtained at bus 14 is 0.0182 pu, 0.0196 pu and 0.0194 pu. From these observations, it is proved that the kalman filter technique is more accurate than the linear first order and second order approximations to obtain actual change in the states.

5. Conclusion

This paper compares the accuracy of different algorithms to estimate the system states even when the power system network measurement data contains bad data. IEEE 14 bus test system is considered for case studies. Weighted Least Squares method using metered data and with PMU measurements are compared, it is concluded that states obtained with PMU measurements are more accurate compared to metered data. Bad data has been identified and eliminated. An attempt has been done to linearize the state estimation procedure. Comparing the test results of three algorithms, it is concluded that the kalman filter algorithm is more accurate than the linear state estimation using second order and first order approximations. Further second order approximation has given better results than first order approximation.

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