ISSN (Print) : 0974-6846
Indian Journal of Science and Technology, Vol 9(26), DOI: 10.17485/ijst/2016/v9i26/93169, July 2016 ISSN (Online) : 0974-5645

Wavelet Transform of Modal Data of Beam in Damage Detection Exercise

Naresh Jaiswal[1*] and Deepak Pande[2]

[1]PVG's College of Engineering and Technology, Pune – 411009, Maharashtra, India; nareshjaiswal@yahoo.co.in
[2]College of Engineering, Pune – 411005, Maharashtra, India; dwp.mech@coep.ac.in

Abstract

Most of the classical methods in damage detection technologies are based on natural frequency, mode shapes, curvature, strain energy etc. In recent years wavelet based damage detection has acquired considerable space in the related literature. The purpose of this study is to develop the wavelet coefficients based damage detection. The damage here is simulated as transverse cuts in the cantilever model. Finite element modal analysis is carried out to extract ten bending modes by Block Lanczos extraction algorithm. The modal data obtained is decomposed by Discrete Wavelet Transform into approximate coefficients and detail coefficients. The higher order curve obtained as detail coefficients contains much information about damage. The details are studied across many other wavelets types. The study focused to explore the effectiveness of different mother wavelets. The findings of the study are useful to select the suitable mother wavelet based on discrete wavelet transform. Higher modes have shown better sensitivity towards small level of damage. This established the necessity of higher order modes in damage detection methodologies. The work has benefitted in deciding the preferred wavelets in such damage detection problems. The results of various damage configurations fall in the same line pointing towards single mother wavelet selection, thus reconfirming the benefits of the method.

Keywords: Damage Detection, Mode Shape, Signal Processing, Wavelet Transform

1. Introduction

Vibration based structural damage identification techniques has sought great attention by researchers.

The greater part of vibration based damage detection strategies in the past literature discuss changes in natural frequencies. A prior examination carried by[1] noticed that the stress distribution through a vibrating structure was non-uniform and was distinctive for every natural frequency. The authors expressed that any localized damage would influence every mode in different way, depending upon the specific location of the damage. Hence, the estimation of the natural frequencies of a structure at two or more phases of its lifetime offered the likelihood of identifying any decay that may have occurred in the interval. The numerical results acquired in this study, utilizing finite element method, are confirmed by experimental results.

Natural frequencies represent overall characteristics of the structure and are captured very easily by simple experimentation. [2]Has looked into the issue of estimation of crack location from changes in natural frequency of the first mode of vibration. Both numerical and experimental studies are given.[3]Developed numerical method for determining the location of crack in a beam when first three natural frequencies are known. The crack is simulated as a rotational spring. The location of crack is confirmed with the help of various plots of spring stiffness versus crack location plotted for each natural frequency. Location of crack is revealed at the intersection of the three curves.

Mode shapes take better place in damage detection exercises as being said to be more sensitive to local perturbation. [4]Investigated an aluminum thin plate with a surface crack. Experimental modal analysis is performed on the plate to obtain the mode shapes before and after damage. The displacements data of each mode shape are

*Author for correspondence

then used to compute the modal strain energy. For all measured mode shapes under consideration, a damage index is defined by using the ratio of modal strain energies of the plate before and after damage. [5]Studied the relation between fundamental mode shape and static deflection. The sensitivity of these parameters to damage in cantilever beams is comprehensively investigated in this study. FEM analysis is carried to establish the sensitivity index regarding the use of these features in damage detection exercise in cantilever beams.

Delamination in composites which is also treated as damage is widely being studied by the researchers.[6]Developed the procedure to analyze the vibration response of laminated composite plates which takes care of the sudden changes of properties from layer to layer. Examination of different damage detection methods consisting of parameters like mode shapes, modal assurance criteria, mode shape curvature etc. is etc. is done by[7]. [8]Conducted modal tests on a beam with saw cuts. Soft springs are used to simulate the boundary conditions during the tests. Finite element analysis is also carried out for the test specimen. Damage is estimated using the relative changes in modal displacements at frequencies which are shifted because of the damage. [9]Used an evolutionary algorithm to detect multiple cracks in cantilever beam. The method based on the changes of the Eigen frequencies and some strain energy parameters is discussed to detect the cracks. The crack is modeled as a rotational spring. Natural frequencies changes were connected to the damage index vector due to the presence of the cracks.

Research is also being carried on damage detection with modern signal processing techniques like wavelets which use classical modal data.[10]Has reported the wavelet based damage detection technique by using a single notch in different depths of a cantilever beam height. The results presented experimental and numerical analyses of damage detection according to the higher order modes. The analysis of first eight modes and the influence of the mode order on damage detection by the wavelet transform is investigated therein. Different crack depths in cantilever beam are investigated by[11]. Gabor wavelet is used for the investigation. The cantilever beam with artificial crack is subjected to static displacement. When the crack is 26% of the beam thickness, the spatial wavelet transform approved its effectiveness to identify the damage in the beam. [12]Investigated experimental and analytical deflection of cantilever beam with crack. The intensity and the location of the crack is evaluated using wavelet transform. The fundamental mode of vibration is investigated. The crack location is determined taking into consideration the sudden spatial response.

2. Numerical Model and Damage Cases

To show how damage alters mode shape characteristics of the structure, the cantilever structure is modeled in Ansys-14 using solid 185 element. The element is defined by eight nodes having three degrees of freedom at each node: Translations in the nodal x, y and z directions. Beam dimensions are- length = 500 mm, thickness = 5 mm and width = 50 mm. Material characteristics: Young's modulus = 2.068 X 10^{11} Pa, density = 7800 kg/m^3, Poisson's ratio = 0.3. Damage is simulated as a transverse cut of 0.4 mm width. To simulate the intensity of damage level, depth of cut is varied from 1 mm to 4 mm in step of 1 mm at various locations. Finite element modal analysis is carried out. The model is solved for 22 modes out of which 10 bending modes are selected for further analysis. The author/s investigated number of damage cases with varying magnitude and location of cut. The investigated damage cases for this work are given in Table 1. The modal displacement corresponding to each mode is normalized to one. The frequency range used is 0 to 4200 Hz to extract higher modes up to tenth.

3. Spatial Wavelet Transform

Brief background of wavelet analysis is presented here which is used in the present work. Classically the wavelets

Table 1. Damage designations

	at 100 mm from fix end	**at 200 mm from fix end**	**at 300 mm from fix end**	**at 400 mm from fix end**
Cut size of 1mm depth	D1_100	D1_200	D1_300	D1_400
Cut size 2mm depth	D2_100	D2_200	D2_300	D2_400
Cut size 3mm depth	D3_100	D3_200	D3_300	D3_400
Cut size 4mm depth	D4_100	D4_200	D4_300	D4_400

are used to analyze the signals in time domain. Equation (1) represents the continuous wavelet transform, where $f(t)$ is the signal to be analyzed, $\psi(t)$ is the chosen mother wavelet function and $\overline{\psi}$ represents its complex conjugate.

a and b are the parameters representing the extent of dilation and translation respectively.

$$(Wf)(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t)\overline{\psi}(\frac{t-b}{a})dt \tag{1}$$

Inverse wavelet transform enables the reconstruction of the analyzed signal $f(t)$ given by Equation (2).

$$\int\int_{-\infty}^{+\infty} W(f)(a,b)\psi(\frac{t-b}{a})\frac{1}{a^2}dadb \tag{2}$$

The family of wavelet comprises of dilated and translated versions of a basis function $\psi(t)$ i.e. mother wavelet. Dilation parameter and translation parameter of mother wavelets determine how the mother wavelet scales and shifts along the time axis.

By means of translating wavelet along the time axis the signal under consideration is examined in the neighborhood of the present location. A scale factor greater than unity results into a dilation of the mother wavelet along the time axis and a positive translation corresponds to shift to the right of the scaled wavelet. A smaller value of a also means a higher resolution filter, i.e., the signal is analyzed through a narrow wavelet window.

In practice, Discrete Wavelet Transform is often employed where the signal to be analyzed is broken into discrete blocks. The procedure requires less computation time if dyadic values (power of 2) of dilation parameter and translation parameters are used. Equations (3) to (7) explain such signal wavelet analysis and its subsequent reconstruction.

$$a = 2^j;\ b = 2^j k\ j,k \in Z \tag{3}$$

Where Z = set of positive integers.

In the discrete wavelet analysis, a signal is assumed to be constituted by its approximations and details. The detail at level j is defined as:

$$D_j = \sum_{k \in z}(\int_{-\infty}^{+\infty} f(t)\overline{\psi}_{j,k}(t)dt)\psi_{j,k}(t) \tag{4}$$

and the approximation at level J is defined as

$$A_J = \sum_{j>J} D_j \tag{5}$$

It means,

$$A_{J-1} = A_J + D_J \tag{6}$$

and

$$f(t) = A_J + \sum_{j \le J} D_j \tag{7}$$

Equations (6) and (7) can be put together to form a tree structure of a signal. Procedure of reconstruction of the original signal can be understood from these Equations. With selection of different dyadic scales, a signal can be separated into many low and high resolution segments, popularly called as wavelet decomposition tree. The wavelet tree structure with details and approximations components at various levels may recover important data pertaining to signal characteristics that may not be clearly observed in the original data or the results from different other methods.

The analysis of signal in spatial domain goes on similar lines as that of time domain data. One may simply consider space signal $f(x)$ instead of $f(t)$. In this work, Discrete Wavelet Transform (DWT) is used to investigate the spatial signals i.e. mode shapes of the beam. The spatial version of wavelet transform analyzes the signal by using the wavelet filter of particular spatial frequency band to translate along a length of beam axis. It is possible to locally examine the signal by adopting multiple level of decomposition.

4. Wavelet Analysis of Spatial Response of Cantilever Beam

Mode shapes extracted from finite element modal analysis are processed with variety of mother wavelets up to third level. The transform results into low spatial frequency approximate part and the high spatial frequency detail parts. Each detail part i.e. d1, d2, d3, is scanned for maximum value of coefficient along the beam axis. The procedure is repeated for all damage cases mentioned in Table 1. The method is extended up to ten mode shapes. Figures 1 and 2 show first and tenth mode shape with mother wavelet as db4 and sym4 for illustration purpose. The absolute values of peaks emerged in d1, d2, d3 are picked for all other types of wavelets and for all damage cases. Total 26 commonly used mother wavelets viz. db2, db3, db4, db5, sym2, sym3, sym4, sym5, coif1, coif2, coif3, coif4, bior2.2, bior2.4, bior2.6, bior2.8, bior3.1, bior3.3, bior3.5, bior3.7, rbio1.3, rbio1.5, rbio2.2, rbio2.4, rbio2.6, rbio2.8 are considered.

The plot of wavelet coefficients in detail part (i.e. d1, d2, d3) conspicuously suggests the damage location by developing the sudden peak in spite of the fact that there is no such trace of damage seen in the corresponding modal data (mode shape). For each mother wavelet there exists a maximum coefficient at the damage location. The subsequent analysis is to pick the best mother wavelet amongst all analyzed wavelets on the basis of Maximum Detail Coefficient for the same damage case. Maximum values at the damage location for each wavelet are plotted in Figure 3 to Figure 12. Matlab program is developed to select the best mother wavelet for the said exercise.

Investigation of the individual plot (Figure 3 to Figure 12) gives us the relative sensitivities of various wavelets for the same intensity of damage and at the same location. For illustration purpose the results of damage case of D1_100

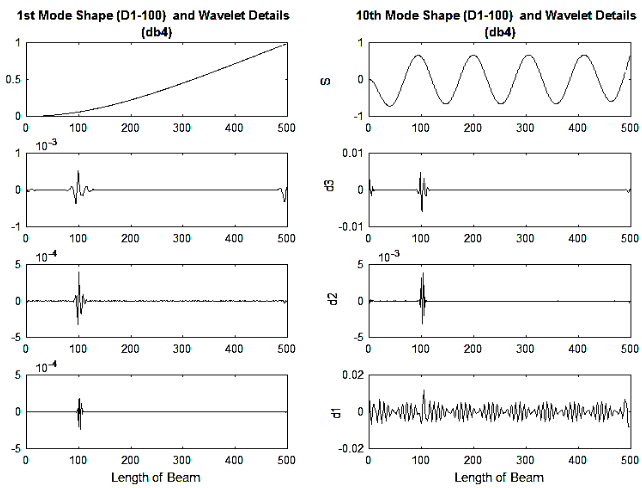

Figure 1. 1st and 10th mode shapes and wavelet details for damage case D1_100; db4, Level 3.

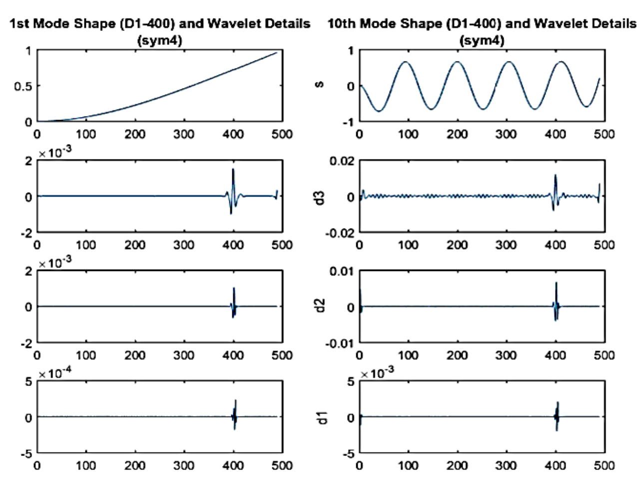

Figure 2. 1st and 10th mode shapes and wavelet details for damage case D1_100; sym4, Level 3.

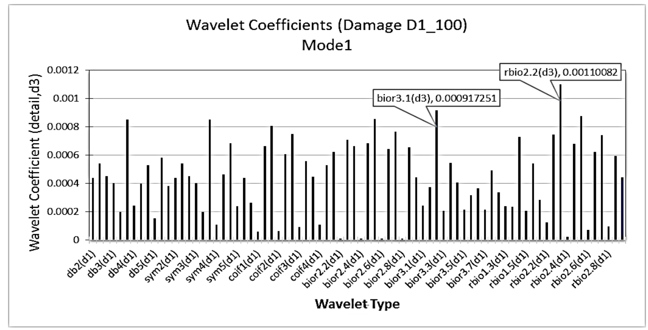

Figure 3. Case:-D1_100_mode 1: Maximum wavelet coefficients for different mother wavelets.

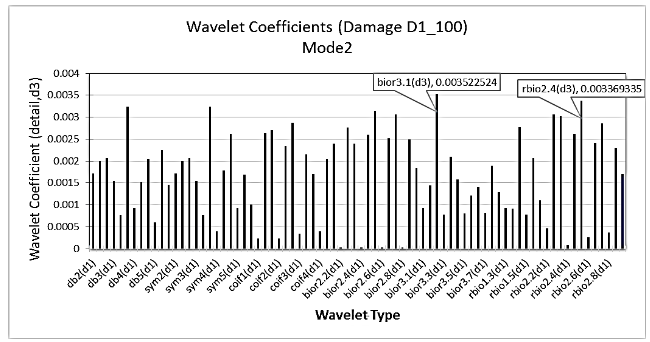

Figure 4. Case:-D1_100_mode 2: Maximum wavelet coefficients for different mother wavelets.

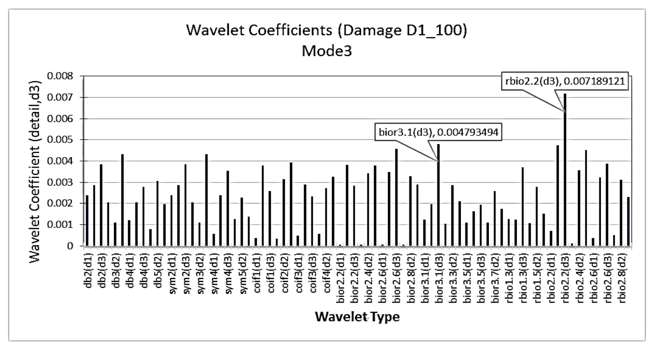

Figure 5. Case:-D1_100_mode 3: Maximum wavelet coefficients for different mother wavelets.

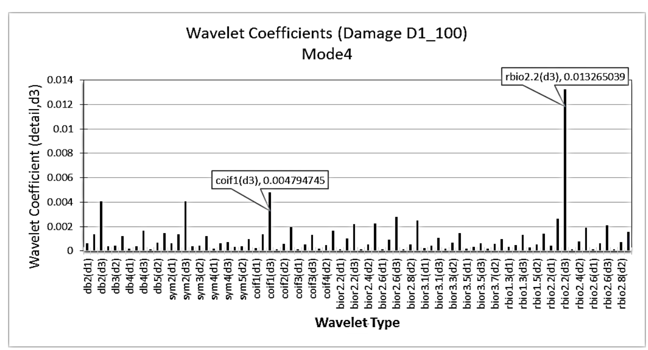

Figure 6. Case:-D1_100_mode 4: Maximum wavelet coefficients for different mother wavelets.

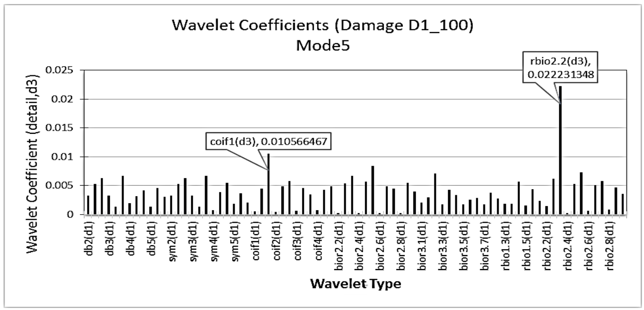

Figure 7. Case:-D1_100_mode 5: Maximum wavelet coefficients for different mother wavelets.

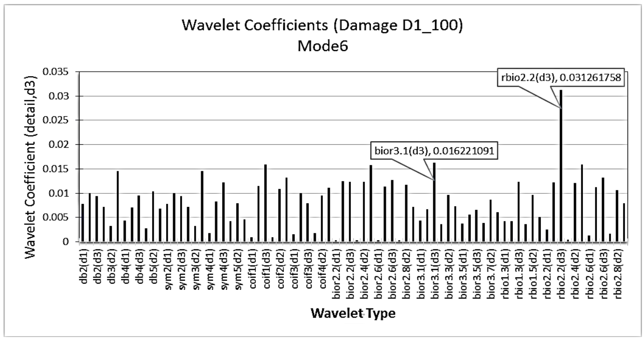

Figure 8. Case:-D1_100_mode 6: Maximum wavelet coefficients for different mother wavelets.

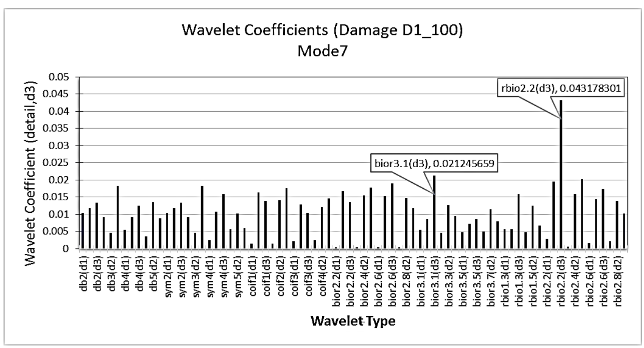

Figure 9. Case:-D1_100_mode 7: Maximum wavelet coefficients for different mother wavelets.

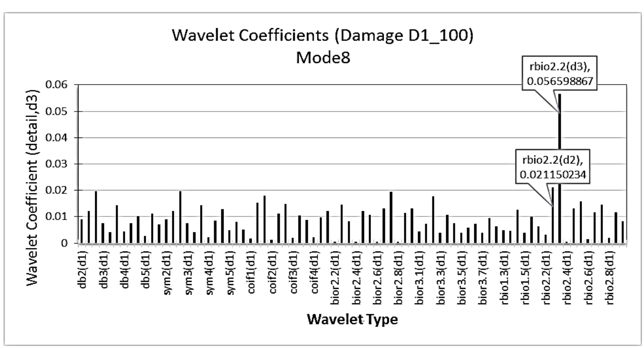

Figure 10. Case:-D1_100_mode 8: Maximum wavelet coefficients for different mother wavelets.

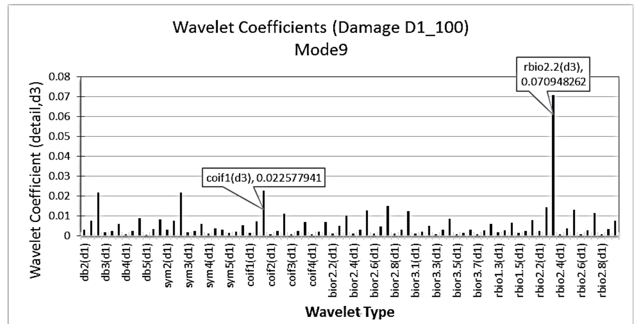

Figure 11. Case:-D1_100_mode 9: Maximum wavelet coefficients for different mother wavelets.

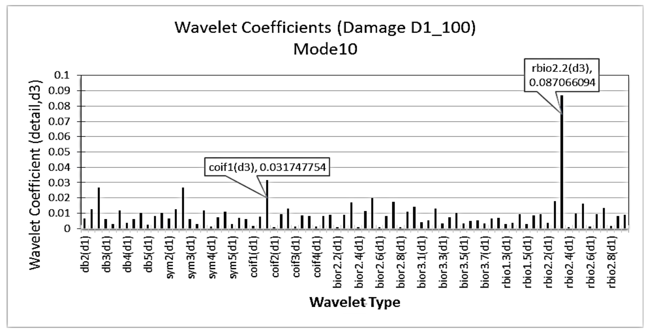

Figure 12. Case:-D1_100_mode 10: Maximum wavelet coefficients for different mother wavelets.

is plotted. In majority damage cases rbior2.2 happens to be most sensitive as it gives largest value of absolute peak at the damage location. Moreover this mother wavelet at higher modes particularly beyond third mode is much more damage sensitive as all other wavelet details coefficients are subdued.

Referring to the Figures 1 and 2, y axis values of details (d1, d2, d3) are at least ten times larger for tenth mode shape as compared to first mode shape for same mother wavelet and same damage case. This points towards high sensitivities of higher mode shapes towards damage. This is exactly expected in damage identification exercises so that smallest level of damage can be detected.

5. Conclusion

The study proved to be useful in deciding the criteria for suitable selection of mother wavelet while working on damage identification exercise with wavelet analysis. The mother wavelet rbio2.2 is found to be the best in the present study as per Maximum Detail Coefficient criterion.

The extraction of higher modes is always beneficial as they have an edge over lower modes in damage identification problem.

Next to rbio2.2 the other sensitive wavelets are bior3.1 in lower modes and coif1 in higher modes.

6. References

1. Cawley P, Adams RD. The location of defects in structures from measurement of natural frequencies. Journal of Strain Analysis for Engineering Design. 1979; 14(2):49–57.
2. Chinchalkar S. Determination of crack location in beams using natural frequencies. Journal of Sound and Vibration. 2001; 247(3):417–29.
3. Maiti SK, Lele SP. Modelling of transverse vibration of short beams for crack detection and measurement of crack extension. Journal of Sound and Vibration. 2002; 257(3):559–83.
4. Hu H, Wu C. Development of scanning damage index for the damage detection of plate structures using modal strain energy method. Mechanical Systems and Signal Processing. 2008; 23(2):274–87.
5. Maosen C, Ye L, Zhou L, Su Z, Bai R. Sensitivity of fundamental mode shape and static deflection for damage identification in cantilever beams. Mechanical Systems and Signal Processing. 2011; 25(2):630–43.
6. Kumar J, Suresh T, Raju D, Reddy VK. Vibration analysis of composite laminated plates using higher-order shear deformation theory with zig-zag function. Indian Journal of Science and Technology. 2011; 4(8):960–6.
7. Elshafey AA, Marzouk H, Haddara MR. Experimental damage identification using modified mode shape difference. Journal of Marine Science and Alication. 2011; 10(2):150–5.
8. Das P, Dutta A, Talukdar S. Assessment of damage in prismatic beams using modal parameters. International Journal on Advanced Science Engineering Information Technology. 2012; 2(2):79–82.
9. Radzienski M, Krawczuk M. Experimental verification and comparison of mode shape-based damage detection methods. Journal of Physics: Conference Series. 2009; 181(1):1–8.
10. Fox CHJ. The location of defects in structures: A comparison of the use of natural frequencies and mode shape data. Proceedings of the 10th International Modal Analysis Conference; San Diego, CA. 1992. p. 522–8.
11. Moradi S, Mohammad HK. On multiple crack detection in beam structures. Journal of Mechanical Science and Technology. 2013; 27(1):47–55.
12. Rucka M. Damage detection in beams using wavelet transform on higher vibration modes. Journal of Theoretical and Allied Mechanics. 2011; 49(2):399–417.