

# State Space Time Domain AR Signal Processing for Kalman Filter

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## Abstract

**Objective:** Stochastic process have been performed useful in applications of signal and image processing in varied applications. Kalman filters are examples of such processing in state space time domain AR signals, AR process can be used as models of natural phenomena. **Methods/Analysis:** This paper explores the applications of Kalman filter AR signal processing using LMS in second algorithm, convergence speed is studied. RLS algorithm ensures fast convergences. **Findings:** Predictor - connector algorithm is used for mathematical modeling estimation of constant or random constant having process clatter in AR process has been done by discrete Kalman filter. It is formed that where covariance and dimensions clatter are invariable, the evaluation over covariance and Kalman gain stabilized quickly. These limitations can be pre work out by running to filter off size. **Novelty/Improvement:** Estimation of true state by implement of discrete Kalman filter has shown that results are satisfied. Further extension can be done to estimate other stochastic parameters.

**Keywords:** AR Signals, Discrete Kalman Filter, RLS Algorithm, Stochastic Process, Time Domain

## 1. Introduction

Stochastic progressions are essential designed tools for enlargement and study in Image & Signal Processing, Automatic Control, Environ metrics, Oceanography, Climatology, Econometrics, and numerous other parts of science and engineering. Stochastic models are used to describe experiments, dimensions, or more general phenomena for which the outcomes are more or less random and unpredictable<sup>1</sup>. The basic ingredients in time series analysis of both stationary and non-stationary sequences, including model identification and parameter estimation. The first to use AR-processes, when models for natural phenomena<sup>2</sup> George Udny Yule. Yule in the 1920 put it to somebody the AR (2) - developed an alternative to the Fourier technique as a means to explain periodicities and explain correlation in the sunspot cycle<sup>3</sup>. The trial space  $\Omega$  for an experiment contains everything that can happen and is therefore very complex and detailed. Each outcome is  $\Omega$  unique and we

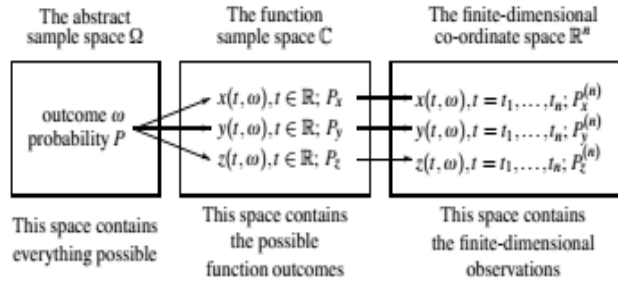
need only one comprehensive probability measure  $P$  to describe every outcome of the experiment. The function (sequence) sample space  $C(R^\infty)$  is simple. It can be used as sample space for a specified experiment for which the result is a function or a sequence of numbers figure 1.

Recursive Least Squares (RLS) and Least Mean Squares (LMS) algorithm is a generalization of gradient vector calculation is made ensures swift junction even while the eigenvalue. The All these improvements come with the cost raised complexity and some permanence problems, which are not as critical in LMS based algorithms<sup>5</sup>.

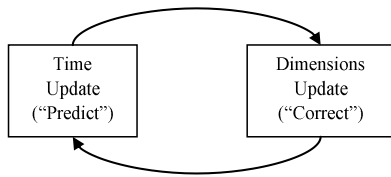
## 2. Mathematical Modeling

The process estimation can be done by means of a form of feedback control in such a technique that the filter guesstimates the process state at some get hold clatter extent. Second, the feedback<sup>6</sup> for the time bring up to date

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**Figure 1.** Represent Overview of the three types of worlds in which our processes live.



**Figure 2.** The ongoing discrete Kalman filter cycle.

equations can also be present considered as predictor equations<sup>7</sup> designed for the bottom of mathematical problems as shown in Figure 2.

Consider coefficient distinction equation

$$a(r) = \sum_{k=1}^D b(k)a(r-k) \quad (1)$$

For which the direct form II realization is shown in Figure 3. Here  $w(r)$  is the procedure for clutter and  $\zeta(r)$  is the dimensions clutter.

The state variables of the system are the numerical quantities memorized by the system that comprise the state. In Figure 2,  $v_1(r)$ , .....,  $v_M(r)$  are the internal variables which comprise the state variables for this system<sup>8</sup>

We have,

$$v_i(r+1) = v_{i+1}(r) \quad (2)$$

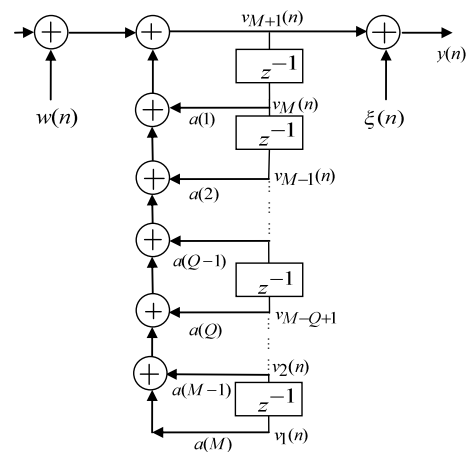
$$v_q(r+1) = v_{q+1}(r)$$

$$= x(r) + w(r) + a(1)v_q(r) + a(2)v_{q-1}(r) + \dots + a(q)v_1(r) = x(r) + w(r) + \sum_{i=1}^q a(i)v_{q-i+1}(r) \quad (3)$$

Equation (2) and Equation (3) are the state equations for the system.

$$\begin{bmatrix} v_1(r+1) \\ v_2(r+1) \\ v_3(r+1) \\ \vdots \\ v_{M-1}(r+1) \\ v_M(r+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 \\ a(q) & a(q-1) & a(q-2) & a(q-3) & a(q-4) & \dots & a(1) \end{bmatrix} \begin{bmatrix} v_1(r) \\ v_2(r) \\ v_3(r) \\ \vdots \\ v_{M-1}(r) \\ v_M(r) \end{bmatrix} + [0 \ 0 \ 0 \ \dots \ 0 \ 1]^T (x(r) + w(r)) \quad (4)$$

State Space Time Domain Modeling of Auto Regressive (AR) Process



**Figure 3.** Direct form II realization of the discrete time system with input-output.

$$\Rightarrow \mathbf{v}(e+1) = \mathbf{A}\mathbf{v}(e) + \mathbf{c}(x(e) + \mathbf{w}(e)) \quad (5)$$

Where

$$\mathbf{E} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 \\ a(q) & a(q-1) & a(q-2) & a(q-3) & a(q-4) & \dots & a(1) \end{bmatrix} \quad (6)$$

and

$$\mathbf{f} = [0 \ 0 \ 0 \ \dots \ 0 \ 1]^T \quad (7)$$

The output can be computed from the state variables at time  $n$  using

$$\begin{aligned} y(e) &= v_{q+1}(e) + \zeta(e) = x(e) + w(e) + [a(q) \ a(q-1) \ \dots \ a(1)] \begin{bmatrix} v_1(e) \\ v_2(e) \\ \vdots \\ v_M(e) \end{bmatrix} + \zeta(e) \\ &= x(e) + w(e) + [a(q) \ a(q-1) \ \dots \ a(1)] \begin{bmatrix} v_1(e) \\ v_2(e) \\ \vdots \\ v_M(e) \end{bmatrix} + \zeta(e) \end{aligned}$$

$$= \mathbf{b}^T \mathbf{v}(e) + d(x(e) + w(e)) + \zeta(e) \quad (8)$$

where

$$\mathbf{b} = [a(Q) \ a(Q-1) \ \dots \ a(1)]^T \quad (9)$$

and

$$d = 1 \quad (10)$$

## 2.1 The Discrete Kalman Filter Algorithm

State Equation

$$\mathbf{v}(e+1) = \mathbf{A}(e)\mathbf{v}(e) + \mathbf{C}(e)(\mathbf{x}(e) + \mathbf{w}(e))$$

Observation Equation

$$\mathbf{y}(e) = \mathbf{B}(e)\mathbf{v}(e) + d(\mathbf{x}(e) + \mathbf{w}(e)) + \mathbf{z}(e) \quad (11)$$

Initialization:

$$\hat{\mathbf{v}}(0|0) = E\{\mathbf{v}(0)\}$$

$$\mathbf{P}(0|0) = E\{\mathbf{v}(0)\mathbf{v}^H(0)\}$$

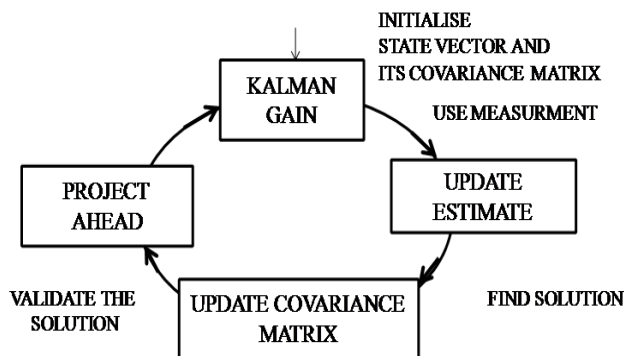
Assuming the estimate of state vector  $\hat{\mathbf{v}}(n|n)$ , the error covariance is  $\mathbf{P}(n|n)$  is get hold of from  $n^{\text{th}}$  iteration.

Calculation error covariance matrix and Eigen state vector at the  $(r+1)^{\text{th}}$  iteration.

$$\begin{aligned} \hat{\mathbf{v}}(r+1|r) &= \mathbf{A}(r)\hat{\mathbf{v}}(r|r) \\ \mathbf{P}(r+1|r) &= \mathbf{A}(r)\mathbf{P}(r|r)\mathbf{A}^H(r) + \mathbf{Q}_w(r+1) \end{aligned} \quad (12)$$

$$\begin{aligned} \hat{\mathbf{v}}(r+1|r+1) &= \hat{\mathbf{v}}(r+1|r) + \mathbf{K}(r+1)[\mathbf{y}(r+1) - \mathbf{B}(r+1)\hat{\mathbf{v}}(r+1|r)] \\ \mathbf{P}(r+1|r+1) &= [\mathbf{I} - \mathbf{K}(r+1)\mathbf{B}(r+1)]\mathbf{P}(r+1|r) \end{aligned} \quad (13)$$

For the period of the dimensions update is to compute Kalman gain<sup>9,10</sup>. Finally, the most recent step is to get hold of the posteriori error covariance estimate. The Kalman filter recursively conditions the up to date guesstimate on all of the past dimensions. The complete picture of the method of Kalman filter<sup>11-14</sup> is shown in Figure 4.



**Figure 4.** A complete picture of the operation of the Kalman filter.

## 3. Implementation and Simulation

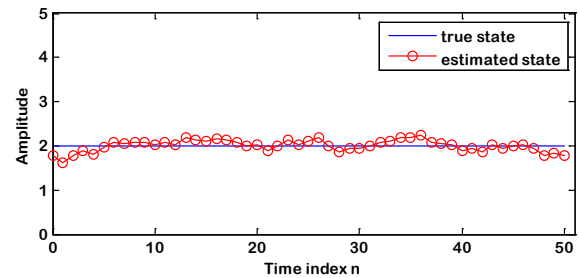
### 3.1 Estimating a Constant using Discrete Kalman Filter

Though the process clatter  $w = 0$ , a very small process variance of the order of  $Q_w = 0.01$  is assumed. Here, the state is nothing but dimensions so  $\mathbf{C} = 1$ . The variance of dimensions clatter is considered as  $Q_z = 0.1$ . Let the initial estimate of  $\mathbf{v}$  and error covariance  $\mathbf{P}$  be 1.50 distinct dimensions  $\mathbf{y}(n)$ . An error normally distributed around zero with a SD of  $\sqrt{0.01}$  is then simulated. Figure 5 depicts the results of this simulation.

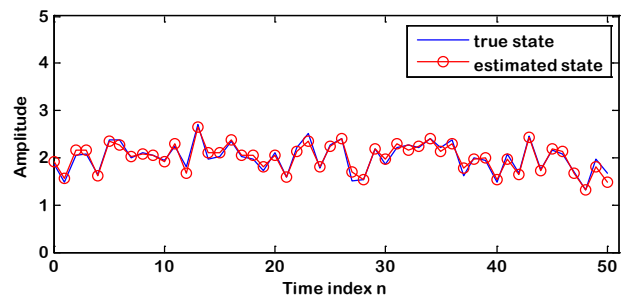
### 3.2 Estimating a Random Constant having Process Clatter using Discrete Kalman Filter

To guesstimate a scalar random constant  $x = 2$  corrupted by  $\sqrt{0.1}$  volt RMS white Gaussian process clatter, a voltage for example. The capacities are tarnished by a  $\sqrt{0.01}$  volt RMS white Gaussian dimension clatter.

The practice is linear dissimilarity (5) dimensions given by equation (8). Here, the progression clatter and



**Figure 5.** Estimating a constant using discrete Kalman filter.



**Figure 6.** Estimating a random constant having process clatter using discrete Kalman filter.

dimensions clatter are well thought-out as white Gaussian clatters with variances 0.1 and 0.01 respectively. The state matrix  $\mathbf{A}$  and the dimensions matrix  $\mathbf{C}$  is both taken as 1. Let the initial estimate of  $\mathbf{v}$  and error covariance  $\mathbf{P}$  be 1.50 distinct dimensions  $y(n)$  standard deviation of  $\sqrt{0.01}$  is then simulated. Figure 5 depicts the results of this simulation.

### 3.3 Estimating an AR (p) Process using Discrete Kalman Filter

$$x(r) = \sum_{k=1}^p a(k)x(r-k) + w(r) \quad (14)$$

where  $w(n)$  the white Gaussian clatter with a variance 0.36, and let

$$y(r) = x(r) + \zeta(r) \quad (15)$$

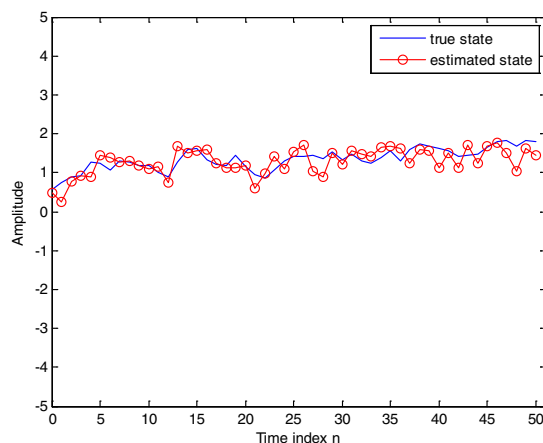
be noisy dimensions of  $x(n)$  and  $\zeta(n)$  be WGN with variance 0.01 that is uncorrelated with  $w(n)$ .

Let  $p = 4$  so the AR (4) process is generated according to the difference equation

$$x(r) = 0.1x(r-1) + 0.2x(r-2) + 0.3x(r-3) + 0.4x(r-4) + w(r) \quad (16)$$

Let the initial estimate of  $\mathbf{v}$  be a zero vector matrix of order  $1 \times p$  and error covariance  $\mathbf{P}$  be identity matrix of regulate  $p$ .

The state matrix  $\mathbf{A}$  is a matrix of order  $1 \times 4$  and the dimension matrix  $\mathbf{C}$  is an identity matrix of order 4. Let the initial estimate of  $\mathbf{v}$  be a zero vector matrix of order  $1 \times 4$  and error covariance  $\mathbf{P}$  be identity matrix of order 4. 50 distinct dimensions  $y(n)$  that had an error in general



**Figure 6.** Estimating an AR (4) process using discrete Kalman filter.

$\sqrt{0.01}$  is simulated. Figure 6 represents the outputs of this recreation. In all the cases, the true value is given by the solid line and the filter estimate by the remaining curve. Under conditions where the covariance of process clatter  $Q_w$  and dimension clatter  $Q_\xi$  are dimensions error remains constant.

## 4. Conclusion

It is tried to estimate the true state by implementing discrete Kalman filter for different cases like a constant, a random constant having process clatter and an AR (p) process using predictor – connector algorithm and observed that the results are satisfactory for different parameters.

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