

Solving Fuzzy Assignment Problem using Ranking of Generalized Trapezoidal Fuzzy Numbers

P. Malini^{1*} and M. Ananthanarayanan²

¹Department of Mathematics, Jeppiaar Engineering College, Chennai - 600119, India;
malinigeethas@gmail.com

²Department of Mathematics, A. M. Jain College, Chennai - 600114, India;
ananthanarayanan47@yahoo.com

Abstract

Background/Objectives: The fuzzy optimal solution is totally based on ranking or comparing fuzzy numbers. Ranking fuzzy numbers play an vital role in decision making problems, data analysis and socio economics systems. The aim is to optimize the total cost of assigning all the jobs to the available persons. Ranking fuzzy number offers an powerful tool for handling fuzzy assignment problems. **Methods/Statistical analysis:** In this paper we used Hungarian method for solving fuzzy assignment problems using generalized trapezoidal fuzzy numbers. By using the ranking procedure we convert the fuzzy assignment problem to a crisp value assignment problem which then can be solved using Hungarian Method to find the fuzzy optimal solution. We presented the method which is not only simple in calculation but also gives better approximation and satisfactory results which is illustrated through the numerical examples. **Findings:** We propose a new ranking method which discriminates the fuzzy numbers where as few of the existing method fails to discriminates the fuzzy numbers. This method ranks all types of fuzzy numbers i.e. normal and generalized fuzzy numbers. Both triangular and trapezoidal fuzzy numbers). It is evident from the numerical examples that the proposed ranking measure for a fuzzy assignment problem is easy to compute and cost effective and gives much more optimal value. **Applications/Improvements:** The proposed ranking procedure can be applied in various decision making problems. This ranking method could also be used to solve other types of problems like game theory, project schedules, transportation problems.

Keywords: Fuzzy Assignment Problem, Ranking Function, Trapezoidal Fuzzy Numbers

1. Introduction

In this work we investigate a more realistic problem namely the fuzzy assignment problem with fuzzy parameters. In an assignment problem, n jobs are to be performed by n persons depending on their efficiency to do the job. In assignment problem C_{ij} denotes the cost of assigning the j^{th} job to the i^{th} person. We assume that one person can be assigned exactly one job also each person can do at most one job. The problem is to find a minimum assignment so that the total cost of performing all jobs is minimum or the total profit is maximum. The idea of fuzzy sets was first introduced by¹ as a mathematical model of representing impreciseness of vagueness. There onwards many authors

presented various approaches for solving the FLP problems. Few of these ranking approaches have been reviewed and compared by Bortolan and Degani². Presently Chen and H Wang³ reviewed the existing method for ranking fuzzy numbers and each approach has drawbacks in some aspects such as indiscrimination and finding not so easy to interpretate. As of now none of them is completely accepted.

Chan (1985) stated that in many situations it is not possible to restrict the membership function to the normal form and proposed the concept of generalized fuzzy numbers. Since then remarkable efforts are made on the development of numerous methodologies for the comparison of generalized fuzzy numbers. And generalized fuzzy

*Author for correspondence

numbers are used in different areas of fuzzy optimization. The development in ordering fuzzy numbers can be found in⁴⁻⁸. Moreover Crisp numbers can be linearly ordered by the relation $\leq$ or $\geq$ and this kind of inequality is not existing in fuzzy number.

In this paper a new method is proposed for the ranking of generalized fuzzy trapezoidal numbers. To illustrate this proposed method, examples are discussed. As the proposed ranking method is very direct and simple it is very easy to understand and using which it is easy to find the fuzzy optimal solution of fuzzy assignment problems occurring in the real life situations.

This paper is organized as follows: Section 2 briefly introduced the basic definition of fuzzy numbers. In Section 3, a new ranking method is proposed. In Section 4, Hungarian method is adopted to solve Fuzzy Assignment problems. To illustrate the proposed method a numerical example is solved. Finally the paper ends with a conclusion.

2. Preliminaries

In this section we define some basic definitions.

2.1 Definition

If x is a set of objects denoted by X , then a fuzzy set A in X is defined as a set of ordered pairs $A = \{(x, \mu_A(x)) / x \in X\}$ where $\mu_A(x)$ is called the membership function for the fuzzy set A . The membership function maps each element of X to a membership value between 0 and 1.

2.2 Definition

A fuzzy set A is defined on universal set of real numbers is known as generalized fuzzy number provided the membership function satisfies the following conditions:

- $\mu_A(x) = R \rightarrow [0, 1]$ is continuous.
- $\mu_A(x) = 0$ for all $x \in A (-\infty, a] \cup [d, \infty)$.
- $\mu_A(x)$ is strictly increasing on $[a_1, a_2]$ and strictly decreasing on $[a_3, a_4]$.
- $\mu_A(x) = \omega$ for all $x \in [a_2, a_3]$, where $0 < \omega \leq 1$

2.3 Definition

A generalized fuzzy number $A = (a_1, a_2, a_3, a_4; \omega)$ is called as generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_A(X) = \begin{cases} \frac{\omega(x - a_1)}{a_2 - a_1}; & a_1 < x < a_2 \\ \omega; & a_2 < x < a_3 \\ \frac{\omega(x - a_3)}{a_4 - a_3}; & a_3 < x < a_4 \\ 0; & \text{otherwise} \end{cases}$$

If $\omega = 1$, then $A = (a_1, a_2, a_3, a_4; 1)$ is a normalized trapezoidal fuzzy number and A is a generalized or non normal trapezoidal fuzzy number if $0 < \omega < 1$

As a particular case if $a_3 = a_2$, the trapezoidal fuzzy number reduces to a triangular fuzzy number given by $A = (a_1, a_3, a_4; 1)$.

3. Ranking of Trapezoidal Fuzzy Numbers

In this section, a new approach for ranking of generalized trapezoidal number is proposed using trapezoid as reference point. Ranking methods map fuzzy number directly in to the real line. Let A be a generalized trapezoidal fuzzy number. The ranking of A i.e. $R(A)$ is calculated as follows :

$$R(A) = \frac{\omega}{2} \left[\frac{\alpha}{2} (a_1 + a_4) + \frac{1-\alpha}{2} (2a_2 + 2a_3) \right] \\ = \frac{\omega}{4} [\alpha(a_1 + a_4) + 2(1-\alpha)(a_2 + a_3)],$$

where $\alpha \in (0, 1)$

3.1 Remark

If $a_3 = a_2$ then the trapezoidal fuzzy number is reduced to a triangular fuzzy number. In such case $R(A)$ is given by

$$R(A) = \frac{\omega}{4} [\alpha(a_1 + a_4) + 4a_3(1-\alpha)]$$

3.2 Remark

We have a pessimistic decision maker's view point, provided $\alpha = 0$.

We have an optimistic decision maker's view point, provided $\alpha = 1$.

We have the neutral decision maker's view point, provided $\alpha = 0.5$.

4. Numerical Examples

4.1 Example

Consider the following cost of minimization Assignment problem. Here the cost (C_{ij}) involved in executing a given job is considered as fuzzy quantifiers. The problem is then solved by proposed method to find an optimal solution.

$$C_{ij} = \begin{matrix} & J_1 & J_2 & J_3 & J_4 \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{pmatrix} (3,5,6,7) \\ (7,8,10,11) \\ (2,4,5,6) \\ (6,8,10,12) \end{pmatrix} & \begin{pmatrix} (5,8,11,12) \\ (3,5,6,7) \\ (5,7,10,11) \\ (2,5,6,7) \end{pmatrix} & \begin{pmatrix} (9,10,11,15) \\ (6,8,10,12) \\ (8,11,13,55) \\ (5,7,10,11) \end{pmatrix} & \begin{pmatrix} (5,8,10,11) \\ (5,8,9,10) \\ (4,6,7,10) \\ (2,4,5,7) \end{pmatrix} \end{matrix}$$

4.1.1 Solution

Now by using the ranking technique convert the given fuzzy assignment problem in to a crisp valued assignment problem. The problem is done by taking the value of ω as 1 and α as 0.5.

$$\begin{matrix} & J_1 & J_2 & J_3 & J_4 \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{pmatrix} 8 & 7 & 8 & 8 \\ 14 & 8 & 8 & 8 \\ 11 & 8 & 6 & 11 \\ 14 & 15 & 7 & 4 \end{pmatrix} \end{matrix}$$

After applying Assignment Method the optimal solution is

$$B_1 \rightarrow J_1, B_2 \rightarrow J_4, B_3 \rightarrow J_2, B_4 \rightarrow J_3 \text{ and} \\ \text{the minimum cost is} = Rs(8+8+8+7) = Rs 31.$$

4.1.1.1 Remark

If this problem is solved as in⁹ we would get the minimum assignment cost is Rs 40. On the other hand if the problem is solved using the proposed method we get the minimum assignment cost is Rs 31.

4.2 Example

Consider an Fuzzy Assignment Problem of assigning n jobs to n machines (one job to one machine). Let C_{ij} be the cost matrix whose elements are trapezoidal fuzzy numbers.

$$C_{ij} = \begin{matrix} & J_1 J_2 J_3 J_4 \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{pmatrix} (3,5,6,7) & (5,8,11,12) & (9,10,11,15) & (5,8,10,11) \\ (7,8,10,11) & (3,5,6,7) & (6,8,10,12) & (5,8,9,10) \\ (2,4,5,6) & (5,7,10,11) & (8,11,13,55) & (4,6,7,10) \\ (6,8,10,12) & (2,5,6,7) & (5,7,10,11) & (2,4,5,7) \end{pmatrix} \end{matrix}$$

4.2.1 Solution

Now, by applying the proposed ranking method the above fuzzy Assignment Problem becomes a convenient assignment problem. The problem is done by taking the value of ω as 1 and α as 0.5.

$$\begin{matrix} & J_1 & J_2 & J_3 & J_4 \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{pmatrix} 4 & 7 & 8 & 7 \\ 7 & 4 & 7 & 6 \\ 3 & 6 & 9 & 5 \\ 7 & 4 & 6 & 3 \end{pmatrix} \end{matrix}$$

After applying Assignment Method the optimal solution is

$$A \rightarrow J_3, B \rightarrow J_2, C \rightarrow J_1, D \rightarrow J_4$$

The fuzzy optimal total cost is $a_{13} + a_{22} + a_{31} + a_{44}$
 $= (9,10,11,15)x_{13} + R(3,5,6,7)x_{22} + (2,4,5,6)x_{31} + (2,4,5,7)x_{44}$
 $= (16,23,27,35)$. And the crisp value of the optimum fuzzy assignment cost is 19.

4.2.1.1 Remark

If this problem is solved using¹⁰ we would get the minimum assignment cost is Rs 25. On the other hand if the problem is solved using the proposed method we get the minimum assignment cost is Rs 19.

5. Conclusions

In this paper a simple method of solving fuzzy assignment problem were introduced by using ranking of trapezoidal fuzzy numbers. We infer the results found were satisfactory and it has been proved through numerical examples. Each ranking method represents a different point of view on fuzzy numbers. Most of the time choosing a method is a matter of preference. However, believing that the results

obtained in this paper gives us the optimum cost which is much lower than ,when it is done using Yager's Method.

6. References

1. Zadeh LA. Fuzzy sets. *Information and Control*. 1965; 8:338-53.
2. Bortolan G, Degani R. A review of some methods for ranking fuzzy subsets. *Fuzzy Set and Systems*. 1985; 15:1-19.
3. Chen SJ, Hwang CL. *Fuzzy multiple attribute decision making*. Berlin: Springer; 1992.
4. Jain R. Decision making in the presence of fuzzy variables. *IEEE Transactions on Systems. Man and Cybernetics*. 1976; 6:698-703.
5. Lee LW, Chen SM. Fuzzy risk analysis based on fuzzy numbers with different shapes and different deviations. *Experts Systems with Applications*. 2008; 34:2763-71.
6. Phani Bushan Rao P, Ravishankar N. Ranking fuzzy numbers with a distance method using circumcenter of centroids and an Index of Modality. *Advances in Fuzzy System*. 2011; 10:1155-61.
7. Valvis E. A new linear ordering of fuzzy numbers on subsets of $F(R)$. *Fuzzy Optimization and Decision Making*. 2009; 8:141-63.
8. Wang YJ, Lee HS. The revised method of ranking fuzzy numbers with an area between the centroid and original points. *Computer and Mathematics with Applications*. 2008; 55:2033-42.
9. Nirmala G, Anju R. Cost Minimization assignment problem using fuzzy quantifier. *International Journal of Computer Science and Information Technologies*. 2014; 5:7948-50.
10. Yager RR. A procedure for ordering fuzzy subsets of the unit interval. *Information Sciences*. 1981; 24:143-61.