Indian Journal of Science and Technology, *Vol 9(16), DOI: 10.17485/ijst/2016/v9i16/79236, April 2016*
ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645

Graph Theory Matrix Approach – A Review

N. K. Geetha[1, 2*] and P. Sekar[1]

[1]Department of Mathematics, C. Kandaswami Naidu College for Men, Anna Nagar, Chennai - 600102, India; nkgeeth@gmail.com

[2]Department of Mathematics, Saveetha School of Engineering, Saveetha University, Chennai - 602105, India; cicesekar@yahoo.co.in

Abstract

Background: Graph theory matrix approach as a decision making tool. **Methods:** Graph theory matrix approach for determining the permanent index in comparing and ranking the alternatives in decision making. **Findings:** The best combination of attributes that influence the system can be found out using this combinatorial approach. **Application:** Decision making can be done appropriately using this approach.

Keywords: Digraph, GTMA, Matrix, Permanent Function, Permanent Index

1. Introduction

Graph theory is becoming increasingly significant as it is applied to other areas of mathematics, science and technology. It is being actively used in fields as varied as biochemistry, Electrical Engineering (Communication Networks and coding theory), Mechanical Engineering, Computer Science (Algorithms and computation) and Operation research. The powerful combinatorial methods found in graph theory have also been used to prove fundamental results in other areas of pure mathematics.

In many problems dealing with discrete objects and binary relations, a Graphical Representation of the object and the binary relations on them is a very convenient from of representation. This leads us naturally to a study of the theory of Graphs.

1.1 Graph Theory

A Graph G = (V(G), E(G)) consists of v, a non empty set of vertices (nodes or points) and E, a set of edges (also called lines).

A Graph G is an ordered triple (V(G),E(G),φ) consists of a non-empty set V called the set of vertices (nodes or points) of the graph G, E is said to be the set of edges of the Graph G, and φ is mapping from the set of edges E to a set of order or unordered pairs of elements of V.

1.2 Label Graph

A Graph in which each vertex is assigned a unique name or label is called a Label Graph.

1.3 Directed and Undirected Graph

In a Graph G(V,E) an edge which is associated with an ordered pair of vertices is called a directed edge of Graph G, while an edge which is associated with an unordered pair of vertices is called an undirected edge. A graph in which every edge is directed is called a directed graph or simply a digraph. A graph in which every edge is undirected is called an undirected graph. The end vertices of an edge are said to be incident with the edge and vice versa. In Figure 1, edge e_1 is incident with the vertices v_1 and v_2 also the vertex v_1 is incident with e_1 and e_3. The vertices v_1 and v_2 are also called the initial and terminal vertices of the edge e_1.

** Author for correspondence*

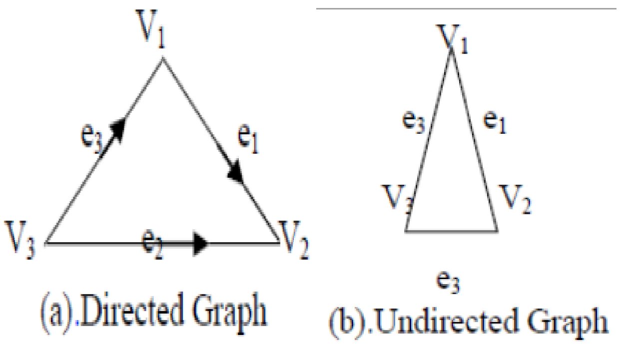

Figure 1. Label Graph.

2. Graph Theory Matrix Approach

Graph theory matrix approach is a logical and systematical approach which was originated from combinatorial mathematics. Graph theory matrix approach helps to analyze and understand the system as a whole by identifying system and sub-system up to the component level. It helps in selecting the most suitable choice from among a large number of alternatives for a given problem. The graphs are used to represent almost any physical situation involving discrete objects and relationship among them. A digraph is used to represent the elements and their interdependencies in terms of nodes and edges. Matrices provide a simple and very elegant means of representing digraph, as it gives one-to-one representation. The permanent of a matrix is used to characterize configurations of a system or the structure of a graph and to develop a unique representation that is independent of labeling. The methodology of Graph theory matrix approach addressed by various researchers in various fields of science and technology is given below:

In assessing the failure mode and effects analysis of mechanical systems, GTMA was adopted by Gandhi et. al.[1] considering the structural and functional interactions. An index called IFME was deduced to measure the failure mode of the system. Venkatasamy et. al.[2] adopted GTMA to the spatial mechanisms for the structural analysis of automobile. The analysis was extended to the subsystem and to component level for characterization, analysis, identification and comparison of automobile. A permanent function was introduced to carrying out the overall structural analysis thoroughly. Singh and Sekhon[3] proposed GTMA in strip-layout selection in component manufacturing. The proposed method minimizes the overall cost of the component by minimizing the wastage. An index called, strip-layout selectivity index was deduced to evaluate and rank the various alternatives.

GTMA is a versatile tool applied to the field of tribology by Wani et. al.[4] to assess the maintainability of mechanical systems. The design alternatives of mechanical systems were compared using maintainability index for selecting and enhancing the maintainability of a system. Selecting the component suitable for a system plays a vital role in reliability of the system. Sehgal et. al.[5] used GTMA method in selecting a rolling element bearing by evaluation of reliability. A reliability digraph was constructed and reliability matrix was formed to assess the reliability index.

GTMA paved its way in selecting a cutting fluid for a given machining operation as proposed by Rao et. al.[6]. A digraph called cutting fluid attributes digraph was developed and a cutting fluid index was deduced to select the right cutting fluid. Rao and Gandhi[7] adopted Graph theoretic approach in the field of machining and manufacturing to evaluate the machinability of work materials. Machinability attributes were considered in developing the machinability attributes digraph. To evaluate and rank the work materials, a universal machinability index was proposed. In minimizing operation and maintenance cost in a steam power plant,

Mohan et. al.[8] used Graph theoretic approach. The attributes considered in developing a system structure graph were performance, availability, safety and security. Permanent function was evaluated from structure matrix for complete analysis of the system. Despite various quality assessment tools present, Grover et. al.[9] proposed GTMA in improving the quality of the product, comparing and selecting the optimum combination. The attributes were modeled as digraph and the quality index was deduced. In selecting the plant location Rao[10] adopted GTMA. Plant location digraph was constructed with attributes and their interdependencies. Plant location selection index was determined for selecting the plant location from among different alternatives. Robots are often used to perform duties which are unsuitable for human workers. Selection of robot plays a vital role in doing the specified job in an industry.

Rao et. al.[11] used GTMA in selecting a robot that best suits the requirements in industrial applications. In logistics and supply chain management, Kaur et. al.[12] adopted GTMA to achieve the objectives and reduce supply chain costs besides lead times. The co-ordination capability among the organizations was compared using the composite index of co-ordination. In assessing the use of quality tools and techniques, Grover et. al.[13] used graph theoretic approach. The quality initiatives were converted into a digraph and a unique numerical index was deduced to evaluate the organization itself in utilizing the quality tools and techniques. In the field of manufacturing, Rao[14] proposed GTMA to assess the environmental impact of manufacturing processes. Considering the assessment factors and their interdependencies, a digraph was constructed. The assessment index was evaluated and the manufacturing processes were ranked. In the field of non-traditional machining,

Chakladar et. al.[15] used graph theoretic approach to select the most suitable non-traditional machining process. Pair wise comparison matrices were used to find the interdependencies among the attributes to construct the digraph and the permanent index was used to segregate and shortlist the non-traditional machining processes. In the field of supply chain management, Wagner and Neshat[16] adopted graph theoretic approach in measuring and managing supply chain vulnerability. The vulnerability index was evaluated to compare the effectiveness of various risk mitigation strategies. They concluded that the graph theoretic approach simplifies the analysis and comparison to reduce the supply chain disruptions. Jangra et. al.[17] used graph theoretic approach to evaluate the performance of carbide compacting die. Five factors and the interactions between them were considered in constructing a digraph. The permanent index value was deduced using permanent function to compare and rank the factors.

3. General Methodology of Graph Theory Matrix Approach

The methodology of GTMA addressed by various authors for analysis, evaluation and selection of optimal combination or parameters is given below:

Step 1. Identifying the attributes or factors that influence the process or system. This is done using the data available in the literature or by conducting survey from the experts.

Step 2. Identifying the sub-factors and their inter-dependencies among the potential alternatives.

Step 3. Graph representation of the attributes as nodes and their relative importance as edges. The digraph consists of a set of nodes N={n_i} with i=1,2,3,…….M and a set of directed edges E={e_{ij}}. A node n_i represents i-th alternative. The number of nodes M is equal to the number of alternatives considered. If a node 'i' has relative importance over another node 'j', a directed edge is drawn from node i to node j (e_{ij}). If node 'j' has relative importance over node 'i' , a directed edge is drawn from node j to node i (e_{ji})[18].

Step 4. Representation of digraph as selection criteria matrix. Matrix representation of selection criteria digraph gives one-to-one representation. This selection criteria matrix is a MxM matrix and considers all of the criteria (A_i) and their relative importance (a_{ij}). Where A_i is the value of the i-th criteria represented by node n_i and a_{ij} is the relative importance of the i-th criteria over the j-th criteria, represented by the edge e_{ij}. The values of A_i should preferably be obtained from the experimental results and are normalized on the same scale. The values of a_{ij} may be obtained from the suitable scale proposed by Rao[18](2007) or on a Fuzzy scale. The attributes matrix, A is shown in Eq. 1.

$$A = \begin{bmatrix} A_1 & a_{12} & a_{13} & . & . & a_{1m} \\ a_{21} & A_2 & a_{23} & . & . & a_{2m} \\ a_{31} & a_{32} & A_3 & . & . & a_{3m} \\ \cdot^{\cdot^{\cdot}} & \ddots & \vdots & . & . & a_{4m} \\ & & & & & a_{5m} \\ a_{m1} & a_{m2} & a_{m3} & . & . & A_M \end{bmatrix} \quad (1)$$

Step 5. The selection criteria function for the matrix is obtained. The permanent of this matrix is defined as the alternative selection criteria function. Application of the concept of permanent function leads to a better appreciation of selection attributes, as there is no negative sign in expression, no information is lost.

Step 6. Evaluation and ranking of the alternatives. The permanent function(Per) values for all alternatives are evaluated and ranked as shown in Eq. 2. A computer program may be developed to calculate the values of permanent function.

$$
\begin{aligned}
\mathrm{Per(A)} = \; & \prod_{i=1}^{M} Si + \sum_i \sum_j \sum_k \ldots \sum_m (a_{ij}a_{ji}) R_k R_l \ldots R_m + \sum_i \sum_j \sum_k \ldots \sum_m (a_{ij}a_{ji}a_{ki} + a_{ik}a_{kj}a_{ji}) R_l R_m \ldots R_m \\
& + \left(\sum \sum \sum \ldots \sum (a_{ij}a_{ji})(a_{kl}a_{lk}) R_m R_n \ldots R_m + \sum_i \sum_j \sum_k \ldots \sum_m (a_{ij}a_{jk}a_{kl}a_{li} + a_{il}a_{lk}a_{kj}a_{ji}) R_m R_n \ldots R_m \right) \\
& + \left(\sum_i \sum_j \sum_k \ldots \sum_m (a_{ij}a_{ji})(a_{kl}a_{lm}a_{mk} + a_{km}a_{ml}a_{lk}) R_n R_o \ldots R_m + \sum_i \sum_j \sum_k \ldots \sum_m (a_{ij}a_{jk}a_{kl}a_{lm}a_{mi} + a_{im}a_{ml}a_{lk}a_{kj}a_{ji}) R_n R_o \ldots R_m + \ldots\ldots \right)
\end{aligned}
\tag{2}
$$

Step 7. Decision making. The alternative with highest value of permanent function is the optimal combination of the process or system. Decision is taken keeping the practical considerations in view.

4. Conclusion

In this paper, the methodology of Graph theory matrix approach and its applications have been reviewed. The potentiality of Graph theory matrix approach is proved in fields of science and technology. Thus this review paper concludes that the Graph theory matrix approach is a versatile tool in decision making and has been adopted to problems in the fields of manufacturing, management, product design, cost analyses, energy sector, materials management, maintenance, reliability, tribology, automotive, hydraulic systems, software engineering, quality, environmental engineering, operational research, supply chain management, education sector and construction management.

5. References

1. Gandhi OP, Agrawal VP. FMEA-a digraph and matrix approach. Reliability Engineering & System Safety. 1992; 35(2):147–58.
2. Venkatasamy R, Agrawal VP. System and structural-analysis of an automobile vehicle—a graph-theoretic approach. International Journal of Vehicle Design. 1995; 16(4/5):477–505.
3. Singh R, Sekhon GS. A computerized diagraph and matrix approach for evaluation of metal stamping layouts. Journal of Materials Processing Technology. 1996; 59(4):285–92. doi: 10.1504/IJVD.1995.061950.
4. Wani MF, Gandhi OP. Maintainability design and evaluation of mechanical systems based on tribology. Reliability Engineering & System Safety. 2002; 77(2):181–88.
5. Sehgal R, Gandhi OP, Angra S. Reliability evaluation and selection of rolling element bearings. Reliability Engineering & System Safety. 2000; 68(1):39–52.
6. Rao RV, Gandhi OP. Digraph and matrix method for the selection, identification and comparison of metal-cutting fluids. Proceedings of the Institution of Mechanical Engineers—Part J. Journal of Engineering Tribology. 2001; 215(1):25–33. doi:10.1177/135065010121500101.
7. Rao RV, Gandhi OP. Digraph and matrix methods for the machinability evaluation of work materials. International Journal of Machine Tools and Manufacture. 2002; 42(3):321–30.
8. Mohan M, Gandhi OP, Agrawal VP. Systems modeling of a coal-based steam power plant. Proceedings of the Institution of Mechanical Engineers—Part A. Journal of Power and Energy. 2003; 217(3):259–77.
9. Grover S, Agrawal VP, Khan IA. A digraph approach to TQM evaluation of an industry. International Journal of Production Research. 2004; 42(19):4031–53.
10. Rao RV. Plant location selection using fuzzy digraph and matrix methods. International Journal of Industrial Engineering. 2006; 13(2):166–76.
11. Rao RV, Padmanabhan KK. Selection, identification and comparison of industrial robots using digraph and matrix methods. Robotics and Computer-Integrated Manufacturing. 2006; 22(4):373–83.
12. Kaur A, Kanda A, Deshmukh SG. A graph theoretic approach for supply chain coordination. International Journal of Logistics Systems and Management. 2006; 2(4):321–41.
13. Grover S, Singh V. A graph theoretic approach to the use of quality tools and techniques. International Journal of Operations and Quantitative Management. 2007; 13(3):199–209.
14. Rao RV. Environmental impact assessment of manufacturing processes using a combinatorial mathematics based decision making method. International Journal of Sustainable Manufacturing. 2008; 1(1):42–50.
15. Chakladar ND, Das R, Chakraborty S. A digraph-based expert system for non- traditional machining process selection. International Journal of Advanced Manufacturing Technology. 2009; 43(3):226–37.
16. Wagner SM, Neshat N. Assessing the vulnerability of supply chains using graph theory. International Journal of Production Economics. 2010; 126(1):121–29.
17. Jangra K, Grover S, Aggarwal A. Digraph and matrix method for the performance evaluation of carbide compacting die manufactured by wire EDM. International Journal of Advanced Manufacturing Technology. 2011; 54(5):579–91.
18. Rao RV. Decision making in the manufacturing environment: using graph theory and fuzzy multiple attribute decision making methods, fourth edn., Springer: London; 2007.