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Sub-sampling Techniques for Sampling a Sparse Wideband Signal in Wireless Communications

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Abstract

Background/Objectives: The popularity of Wireless technologies,for applications that urgeshigh bandwidth, increases the demand of Radio frequency (RF) spectrum which leads to scarsity in the spectrum resources. Hence it is required to sense the spectrum of a wideband signal at low sampling rates compared to the Nyquist rate. **Methods/Statistical analysis:** If the wide band signal is Sparse in frequency domain, then it is used to get over the problem of high sampling rates under the compressive sensing framework. Two different efficient schemes called Modulated Wideband Converter (MWC) and Asynchronous Multi-rate (AMR) are proposed here for wideband spectrum sensing at sub-Nyquist rates. The key features of these methods are also compared with the traditional sub-Nyquist sampling approach called Multi-Coset(MC).**Findings:** The MC sampling is capable of sampling at different time offsets, but unable to poses channel synchronisation.In MWC sampling,the sparse multiband signal is multiplied by a periodic waveform to shift the spectral components of the band of interest tothe origin, then filtered and sampled atlow rate. This also makes the computational time to reduce greatly. In AMR sampling, the sparse signal is under sampled at different sampling frequencies to produce sample sets with different aliasing spectra. Later these spectra are compared on a common frequency grid to detect the active bands. This mechanism takes fewer samples there by reduces thee computational cost.The simulation results exhibit these advantages over the traditional sampling techniques for wide band spectrum sensing. **Application/Improvements:** The trade-off between the selection of MWC for less computational time and MR for low complexity in design makes these sub Nyquist schemes suitable for wide band spectrum sensing in cognitive radio like applications.

Keywords: Cognitive Radio, Compressive Sensing, Sensing Algorithms, Wideband Signal

1. Introduction

The communication over primary users (television, mobiles, radio stations and military communication) is carried out over the specified frequency bands. In the past, government agencies assigned majority of the spectrum to primary users by reserving specific frequency interval for each owner. Due to drastic rise in communication users, this type of allocation of spectrum leads to spectrum crowding, which makes the permanent user fail in satisfying the increasing demand of transmission bands. This low frequency utilization laid paths towards CR technology[1]. The goal of CR is to improve the usage ofradio frequency (RF) spectrum via dynamic spectrum access. The main advantage of cognitive radio and networks is allocation of available frequency spectrum to themobile users who are in demand. Because of present spectrum allocation scheme to primary users, spectrum holes and spectrum demands emerged. Spectrum holes are defined as frequency bands which are allotted to authorised users, yet in few locations and at times are not used by them. Hence, they can be used by unlicensed users when the spectrum is not being used[2].

The task of spectrum sensing in multiband signals results considerable reduction in sampling rates.Efficient sampling of wide band signals is a challenging problem, since normal Analog-to-Digital Converters (ADCs) can-

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not be affordable. Wideband Spectrum sensing[1,3] aims to sense the frequency bandwidth which surpasses the rationality bandwidth of the channel.The sampling frequency required is very much less than the Nyquist frequency ifa wideband signal is sampled at sub-Nyquist rates.The wideband spectrum is naturally sparse because of the low spectrum utilization;hence compressive sensing plays a vital role in sensing a wideband signalat sub-Nyquist sampling rates. Compressive sensing is an efficient technique of acquiring a signal withcomparativelya small number of measurements further a particular representation of the signal was achieved based on the signal's sparseness or compressibility.

In this article, different approaches of sub-Nyquist sampling schemes were proposed.They are (i) Multi-coset sampling, (ii) Modulated Wideband Converter and (iii) Multi-rate Sub-Nyquist Sampling.Multi-cosetscheme requires exact delay synchronization over differentchannels, moreoverthey are responsive to variations in the sampling frequency. Multi-rate scheme was categorized into synchronous and asynchronous multi-rate sampling. Stage synchronization among the channelsis not required for asynchronous method, which unwinds the related equipment outline cost and lessens the calibration effort.

1.1 Compressive Sensing

Compressed sensing (or) Compressive Sampling is one of the efficient signal acquisition techniques that take few measurements about the signal. Later, the original signal is reconstructed from these incomplete set of measurements using optimization techniques. The special attraction towards compressive sensing is that it is a technique for sensing and compressing data simultaneously. The CS mainly depends on two fundamental principles: the first of it is the sparse representation of the signal of interest in some basis, called the Sparse basis and the second one is designing of the Sensing(Measurement) matrix and Incoherence between the sensing matrix and the Sparse basis[4].

Representations of Sparse and Measurement matrices are given as follows. Let $\{\psi_i\}_{i=1}^{n}$ be a set of n orthonor mal basis vectors for the space $\mathbb{R}^n$ and $\Psi \in \mathbb{R}^{n\times n}$ be an orthonormal matrix where the i^{th} column is the i^{th} basis vector ψ_i. Then any signal $x \in \mathbb{R}^n$ can be expressed as a linear combination of these basis vectors5 by

$$x = \sum_{i=1}^{n} s_i \psi_i \quad \text{otherwise} \qquad x = \Psi s$$

where 's' is a sparse vector. If the input signal x is not sparse, then it is made sparse under a known basis.

A measurement matrix $\Phi \in \mathbb{R}^{m\times n}$ gives the sampled measurements of the signal of interest $x(t)$ based on Random demodulation.The relation between the reduced measurements y and the input x is given as

$$y = \Phi x$$

The sparse signal x(t) can be perfectly reconstructed from its reduced measurements **y** satisfying the condition that *m* is less than *n* where the rows of Φ range $\mathbb{R}^n$. Thus substituting ,

$$y = \Phi x \quad = \Phi \Psi s \quad = \Theta s$$

where $\Theta = \Phi\Psi$[4].Therefore solving the above equation for $\mathbf{s}$ is equivalent to solving for $\mathbf{x}$, as Ø is a known predetermined basis[6].

2. Signal Model

In Cognitive Radio applications, the received signal in the wideband receiverrepresentsa multiband model as shown in Figure 1. In such application are conversionof the input widebandsignal which is spread across the spectrum is to be converted todigital. Thendepending on the application some of the transmissions are to be sampled at low rates. Finally, the digital signals are perfectly reconvertedto analog. Suppose a wideband signal $x(t)$ is band limited to $[0, f_{max}]$, then its Fourier transform is $X(f)$, and this spectrum contains at most N frequency bands, each of width $B\ MHz$. To sample these wide band signals, Sub-Nyquist sampling is used. In this paper, different sub-nuquist sampling schemes like Multi-coset sampling, Modulated Wideband Converter and Multi rate sampling are proposed.

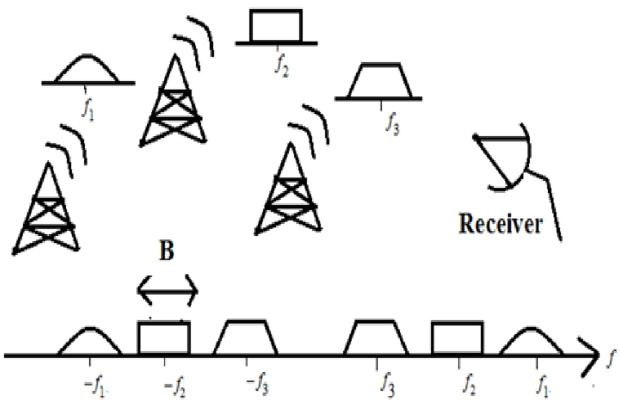

Figure 1. A Wide band signal transmission.

3. System Model

3.1 Multi-coset Sampling

For sampling sparse multiband signals, a sub-Nyquist sampling technique calledMCsampling is used which is a periodic andnon-uniform scheme. In this scheme, awide-band input signal $x(t)$ is sampled with a sampling period T (such that uniform sampling at rate $1/T$ causes no aliasing), anda suitable integer $L > 0$, non-uniformly at the instants $t_i(n) = (nL + C_i)T$ for $1 \leq i \leq p$ and $n \in \square$.

It composed of several parallel channels as shown in Figure 2. A time shift is introduced between the channels and are operated at low sampling rates compared to that of the Nyquist rate. Then the Nyquist grid is divided into different segments[7] ofL sampleseach. Each coset $\{C_i\}$ provides p data sequences from the L samples which represents the multi-coset sampling pattern.

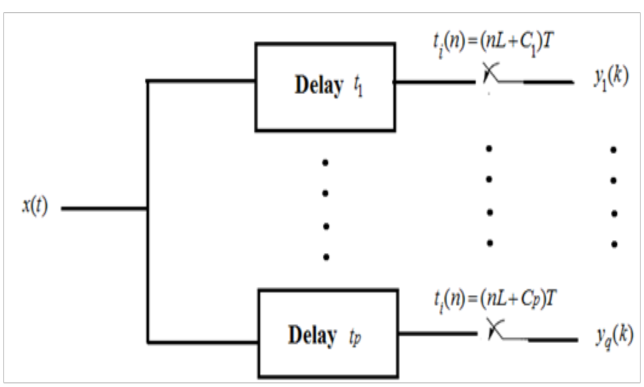

Figure 2. Multi-coset sampling operation.

3.1.1 Multi-coset SamplingPractical Considerations

Let x(n) be a sampled sequence where its nyquist rate can be given as T_{NYQ} .In multi-coset sampling, the spectrum is divided in to L blocks, whereby the i^{th} coset takes the i^{th} sample from each block. Thus each coset gives p sequences. Hence the sampling rate of MC can be considered as p/LT_{NYQ}, is less than the Nyquist rate.The rate at which practical ADCs work cannot meet the sampling rate of multi-coset scheme. Also the time delays of ADCs for various cosets cannot be maintained accurately. This deviates the reconstruction of the signal from unpredictability of the delays. These limitations makemulti-coset sampling impracticable for wide band applications. The simulation[8] results are shown in Figures 3 and 4and various parameters considered in MC sampling are shown in Table 1.

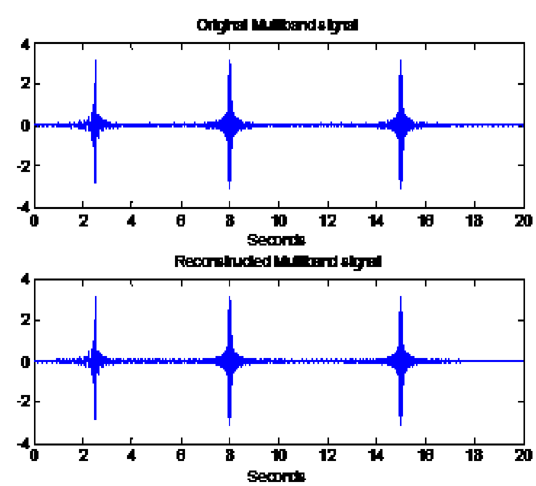

Figure 3. Original and Reconstructed Multiband signal.

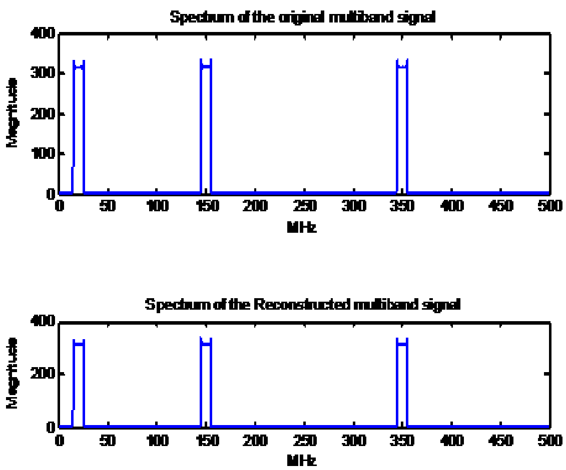

Figure 4. Spectrum of Original and Reconstructed Multiband signal.

Table1. Metrics in Multi-CosetSub-Nyquist Sampling

Parameter	Value
Nyquist rate	1000.000 MHz
System sampling rate	250.000 MHz
Subsampling factor	0.250
Input SNR	100.000 dB
True center frequencies of bands with BW 10.00 MHz	20.00 MHz150.00 MHz 350.00 MHz
Average squared error	0.0000011820
Channel occupancy	0.0600000000
Elapsed time is	0.393580 seconds

3.2 Modulated Wideband Converter

3.2.1 Introduction

The MWC is one of the sub-nyquist uniform sampling method used for obtaining sparse multiband signals. In this scheme, the spectrum of a multiband signal $x(t)$, is spread over N frequency bands each maintaining a bandwidth of B Hz. Its analog front-end is divided into m various channels, as shown in Figure 5, aiming in reduction ofthe sampling rate.

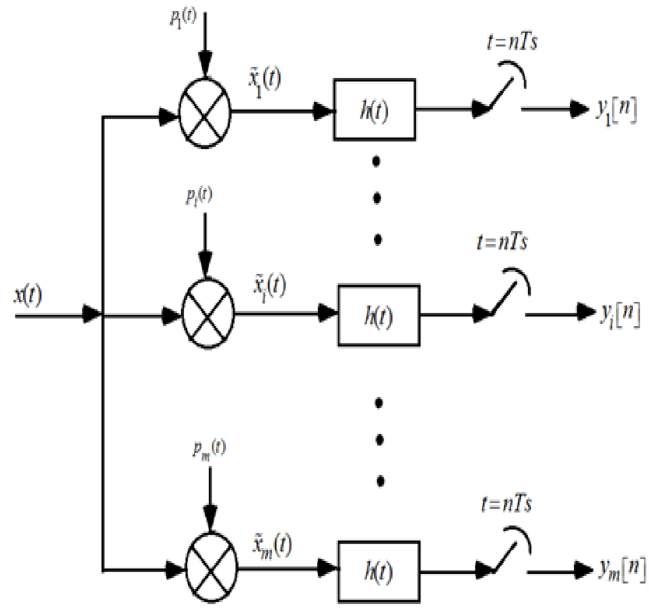

Figure 5. System model in Modulated Wide band Converter.

A pseudo random sequence $p(t)$ is multiplied with the input signal $x(t)$ at each channel. The multiplied signal is passed through a low pass filter, and then sampled at rate of $t = nTs$ results in the sampled sequence $y(n)$. Number of channels chosen are $m \geq 2N$ with filter cut off rate as $1/2T$, also the input sampling rate[1,9]depends on the bandwidth. Thus the sampling rate of the proposed system is $m/T \geq 2NB$, for which the information rate is predicted to be lesser than the Nyquist rate of input signal.

3.2.2 Analysis of the Sampling System

To ease exposition odd M has to be chosen, $T = m / f_{NYQ}$ and $T_s = T_p = T$. Considering the multiband signal is periodic, the signal $p_i(t)$ is expressed in Fourier domain as

$$p_i(f) = \sum_{n=-\infty}^{\infty} C_{in}\delta(f - \frac{n}{T})$$

where, C_{in} is the Fourier co-efficient and δ is the Dirac delta function. In time domain the input signal is multiplied with random square pulse and in frequency domain they are convoluted as follows.

$$X_i(f)=\sum_{n=-\infty}^{\infty} C_{in}X(f-\frac{n}{T})$$

The spectrum of multiplied and low-pass filtered signal lieswithin the range of $F_0=[-1/2T,1/2T]$ and the resultant signal $a_i(t)$ with Fourier transform is

$$A_i(f)=\sum_{n=-n_0}^{n_0} C_{in}X(f-\frac{n}{T})\ ,f\in\mathcal{F}_0$$

Thus $A_i(f)$ is described as a linear combination of shifted replicas of $X(f)$. At each channel, the spectrum is filtered by a low pass filterof $f_c=\frac{1}{2T_s}$ [10]which is then sampled at a rate of $\frac{1}{T_s}$ .The average sampling rate of the system is $\frac{m}{T_s}$ as there are m multi channels, proving it is m times less than the Nyquist rate. Therefore,

$$Y_i(f)=A_i(f)H(f)=\sum_{n=-n_0}^{n_0} c_{in}X(f-\frac{n}{T})H(f) \quad but \quad now \quad f\in\left[-\frac{1}{2T},\ \frac{1}{2T}\right]$$

Thus MWC provides efficient reconstruction of the spectrum with less sampling rates by achieving good reduction in the length of the measurementmatrix. The metrics in Table 2 and the simulation results in Figures6 and 7 replicates the characteristics of MWC sampling.

Table1. Metrics in modulated wide band converter sub-nyquist sampling

Parameter	Value
Nyquist rate	1000.000 MHz
System sampling rate	200.000 MHz
Subsampling factor	0.200
Input SNR	20.000 dB
True center frequencies of bands with BW 20.00 MHz	20.00 MHz 200.00 MHz 320.00 MHz
Average squared error	0.0038115261
Elapsed time is	0.091950 seconds

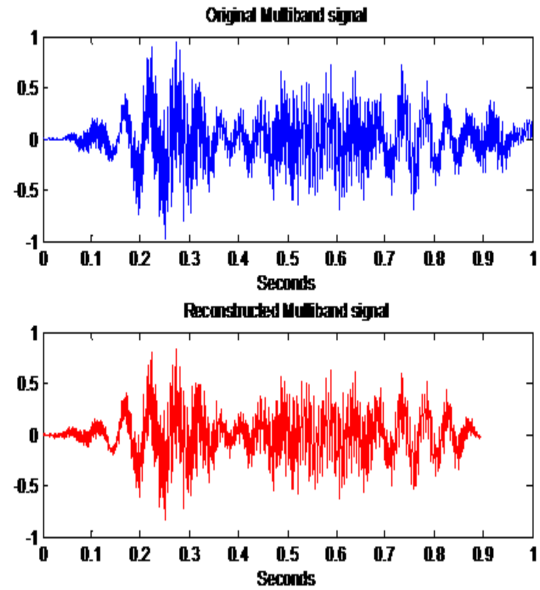

Figure 6. Original and Reconstructed Multiband signal.

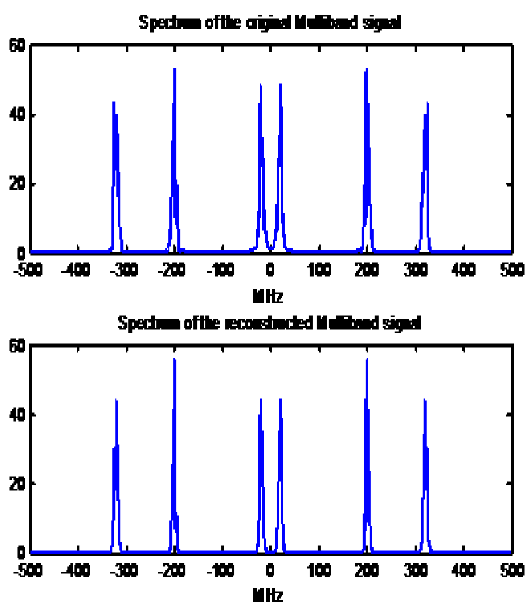

Figure 7. Spectrum of Original and Reconstructed Multiband signal.

3.3 Asynchronous Multi-rate Sampling

The other sub Nyquist sampling technique[11,12] named Asynchronous multi-rate sampling provides low cost in acquiring multiband signals. Identifying the active/ dynamic frequency bands that have more energy than the noise frequencies is the initial step in sensing process. Next is to representthe distinguished active bands.

3.3.1 Detectionof Active Bands usingAMRS

Assume that there are two frequency bands which are active for a baseband signalof bandwidth 'B' Hz. Each are sub-sampled with two different frequencies ofF_{s1} and F_{s2}as shown in Figure 8. Since the sampling frequencies are distinctive, eachset corresponds toa dissimilar frequency spectrum. One can detect the spectrum of the dynamic bandsfrom the information of aliasing spectra. The proposed active band location calculations depend on the idea of spectral examination.

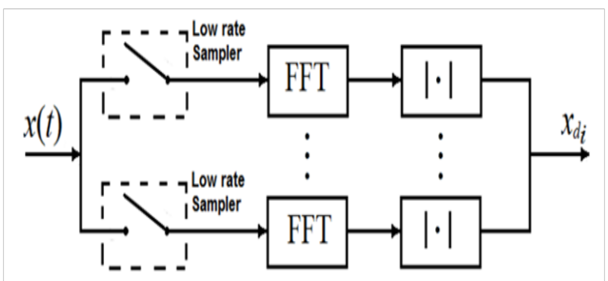

Figure 8. Multi- rate Sub-Nyquist sampling.

The spectral grids of the FFT obtained are different as its corresponding sampling frequencies are not similar. To compare these aliasing[12]spectra, a common frequency grid is essential. By under sampling the signal with two examining frequencies F_{s1} and F_{s2}, M samples are gathered from each case thus correspondingly two sample sets y_1 and $y_2$are framed as shown in Figure 9.

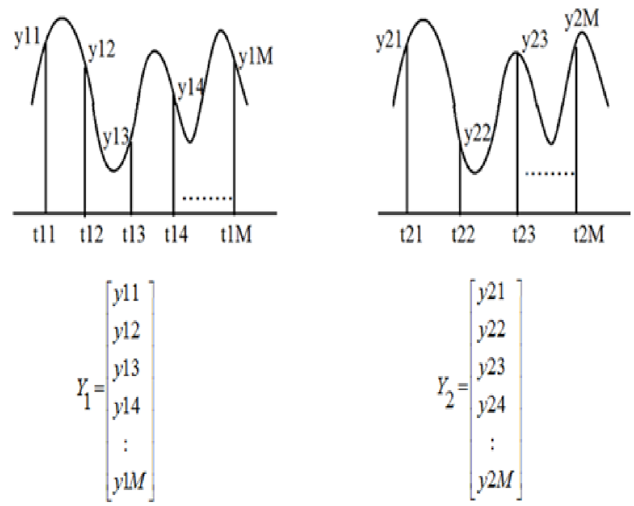

Figure 9. M samples extracted from the signal at under sampling frequencies Fs1 and Fs2.

The FFT spectral grid spacing for testing sets 1 and 2 are F_{s1}/M and F_{s2}/M which are not lined up. Framing a common spectral grid necessitate that the input bandwidth be separated into N uniformly spaced spectral grids, which help to form sensing matrices A_1 and A_2 for each set. The aliasing spectra can be calculated by $A_1^H y_1$ and $A_2^H y_2$ where H represents the Hermitian Transpose[11]. The Spectral grids for two different sets are given as:

for Sample Set 1

$$[0,\frac{F_{s1}}{M},\frac{2F_{s1}}{M},.....\frac{(M-1)F_{s1}}{M}]$$

and

for Sample Set 2

$$[0,\frac{F_{s2}}{M},\frac{2F_{s2}}{M},.....\frac{(M\quad 1)F_{s2}}{M}]$$

Common Spectral Grid

$$[\frac{-NB}{2N},.....\frac{-2B}{2N},\frac{-B}{2N},\frac{B}{2N},\frac{2B}{2N},.....\frac{NB}{2N}]$$

where, B is the input signal bandwidth.The corresponding Sensing matrices A_1 and A_2 obtained from the common spectral grid for sample sets 1 and 2 respectively with a size of $M \times N$ are

$$A_1 = \begin{bmatrix} e^{-i2\pi t_1^1 (N-1)B/N} & \cdots & e^{i2\pi t_1^1 (N-1)B/N} \\ : & : & : \\ e^{-i2\pi t_1^M (N-1)B/N} & \cdots & e^{-i2\pi t_1^M (N-1)B/N} \end{bmatrix}$$

and

$$A_2 = \begin{bmatrix} e^{-i2\pi t_2^1 (N-1)B/N} & \cdots & e^{i2\pi t_2^1 (N-1)B/N} \\ : & : & : \\ e^{-i2\pi t_2^M (N-1)B/N} & \cdots & e^{-i2\pi t_2^M (N-1)B/N} \end{bmatrix}$$

From the aliasing spectra,the frequencies which contain highest energy level from both the spectra is identified as the first dynamic/active band.The components at the frequency supports and the corresponding negative recurrence of the dynamic groups are then subtracted from the two existing sample sets. The remaining[13]waveforms are referred as the residual waveforms r_1 and r_2. With these waveforms if the above procedure is continued the second dynamic/active band is detected. Thus the active frequency bands are detected. The benefit of low sampling rate with high energy-efficiency, and compression capability are achieved by AMRS. Also itis susceptible to noise where CR networks should prove its way.The parameter consideration in Table 3 and the simulation results in Figures10 and 11highlights the complexity reduction in Asynchronous MR sampling.

Table3. Metrics in Multi-rate sub-nyquist sampling

Parameter	Value
Centre frequencies of the multiband signal	75 MHz, 175 MHz
First Under sampled frequency	200 MHz
Second Under sampled frequency	300 MHz
Elapsed time is	0.246623 seconds

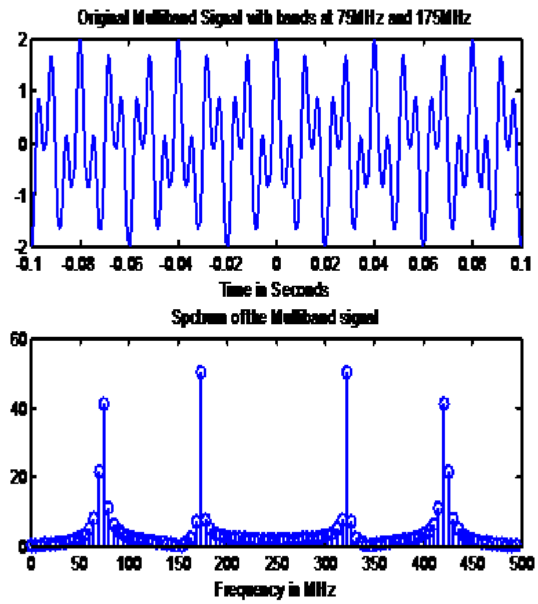

Figure 10. A multiband signal and its spectrum.

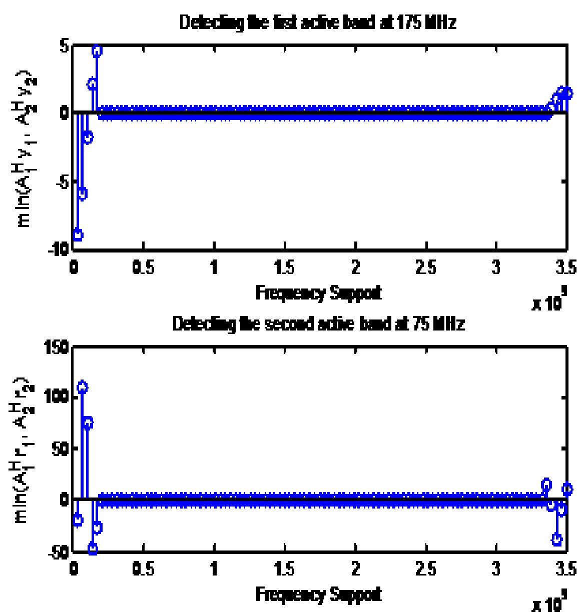

Figure 11. Detection of two active band in two iterations.

4. Conclusions and Future Scope

In the spectrum sensing of wideband signal in Cognitive Radio, the traditional Multi-coset sampling, for specific parameter selection, resulted a computational time of 0.393580 seconds with complexity in its design, which is not feasible. This paves way to one of the efficient sampling technique called MWC that eliminates the mentioned limitations of Multi-coset. The proposed work referred that the computational time of 0.091950 seconds in MWC is the lowest among all the schemes. On the other hand Asynchronous Multi-rate sampling has achieved detection of multiband signal at low cost with a comparable computational time of 0.246623 seconds. So the low cost Asynchronous Multi-rate and the high speed MWC sampling schemes are suitable for Cognitive Radio where fast switching between users is essential.

The future scope of the proposed one is to work optimally in the trade-off between sampling rate, blindness, and practicalimplementation. Design of the proposed algorithms by view of tailoring the hardware complexity is also desired to further minimize the cost is important.

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