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Sliding Mode Controlfor Robust Regulationof Chemical Processes

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Abstract

Objective: This paper implies a comparison between sliding mode control and linear state feedback controller. When the linear systems are given bounded disturbance their robustness properties are studied. **Method:** Pole placement, classical sliding mode control using constant rate reaching law and constant plus proportional rate reaching law, super-twisting controller. **Findings:** Finite time asymptotic convergence is not achieved in linear feedback controller, Sliding Mode Control when used produced finite time asymptotic convergence. The measure of the control variable is obtained by taking norm of the control input. **Applications:** Process Control, Aerodynamics.

Keywords: Discontinuous Control, Full State Feedback, Linear Feedback Control, Linear System, Pole Placement, Sliding Mode Control

1. Introduction

1.1 Sliding Mode Control

The sliding mode control is a form of Variable Structure Control System (VSCS)[1-2]. This is a discontinuous control and insensitive to perturbations. Even in the presence of disturbances, the sliding mode control reaches zero in finite time[3-5].

1.2 Linear State Feedback

Supposing all the state variables are measurable and completely controllable, and then the poles can be placed at any desired location by means of state feedback through an approximate state feedback gain matrix.

A first order integral plus dead time system, an unstable first order plus dead time system, a system with dead time and integrator, a third order system is taken and their robust properties are studied with the help of the linear state feedback control and the Sliding Mode Control[6].

1.3 Problem Statement:

Consider the system

$$\frac{e^{-ST}}{(S+1)^n} \tag{1}$$

Approximating e^{-sT} to $\frac{1}{sT+1}$

Writing it in the form

$$\frac{1}{(ST+1)(S+1)^n} \tag{2}$$

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and obtaining the A,B matrices

$$\dot{x}(t)=Ax(t)+Bu(t)+d(t,x) \qquad (3)$$

where $x \epsilon R^n$ is the state vector

$u \epsilon R^n$ is the vector of the control input

$A \epsilon R^{n\times n}$ is the system matrix

$B \epsilon R^{n\times m}$ is the matrix of control gains and the function

$d:R\times R^n \rightarrow R^n$ pronounces the system uncertainties and disturbances

It is supposed that the matrices A and B are identified(3) , the twosome(A,B)is controllable and the whole state vector x can be measured and then used for feedback control design.

2. Control Strategy

2.1 Linear State Feedback Controller

Full state feedback (FSF), or pole placement, is a method engaged in feedback control system theory to place the closed-loop poles of a plant in pre-determined sites in the s-plane. Placing poles is anticipated because the place of the poles corresponds directly to the Eigen values of the system, which governs the characteristics of the response of the system.

The system must be considered controllable in order to implement this method.

2.2 Principle

If the closed-loop input-output transfer function can be represented by a state space equation

$$\dot{X}=Ax+Bu$$

$$Y=Cx+Du$$

then the poles of the system are the roots of the characteristic equation given by

$$|SI-A|=0$$

Full state feedback is utilized by commanding the input vector u. Consider an input proportional (in the matrix sense) to the state vector, System with state feedback (closed-loop)

$$u=-Kx$$

Substituting into the state space equations above,

$$\dot{X}=(A-BK)x$$

$$Y=(C-DK)x$$

The roots of the FSF system are given by the characteristic equation, Det[SI-(A-BK)] Paralleling the terms of this equation with those of the anticipated characteristic equation yields the values of the feedback matrix K which power the closed-loop eigenvalues to the pole places specified by the desired characteristic equation[1]. When the linear state feedback control is used the state variables do not converge to zero in finite time in the presence of disturbance as in Figures 1,11,21,31.

2.2 Sliding Mode Control

Sliding mode control is used to study the robustness properties of descriptor systems with bounded external disturbances. When the transfer function G(S) is converted into state space many continuous control methods such as the PID controllers can be used. The sliding mode control (SMC) is a discontinuous control method for all the non-linear processes[7,8].

The main aim of the SMC is

- To design a stable surface
- To design a control law to force the system states onto the chosen surface in finite time.

The two sliding mode controllers that are used in this paper are

- Classical sliding mode control
- Constant rate reaching law
- Constant plus proportional rate reaching law
- Super twisting controller

2.2.1 Classical Sliding Mode Control

The objective of sliding mode control is to ensure sliding motion in finite time from an arbitrary initial condition. If sliding surface is s(x, t) = x(t), the sign of s(x, t) and (x, t) should be opposite to ensure finite time reaching. A control law can be obtained by the so-called reaching law approach in which the switching function dynamics are specified a priori. In the classical sliding mode control the sliding surface variables are linear[9]. The phase when x(t) approaches zero is called the reaching phase and the phase when x(t) reaches and slides along zero is called the sliding phase[7,8].

The following reaching laws were proposed in order to force x(t) to zero:

The Constant Rate Reaching Law:

$$\dot{s} = -Q\ \mathrm{sign}(s)$$

This law forces the state variables to zero as in Figures 2,12,22,32.

The control input for this law is discontinuous as in Figures 3,13,23,33.

The evolution of states are shown in the Figures 4,14,24,34.

The Constant Plus Proportional Rate Reaching Law:

$$\dot{s} = -Ks - Q\ \mathrm{sign}(s)$$

This law forces the state variables to zero as in Figures 5,15,25,35.

The control input for this law is discontinuous as in Figures 6,16,26,36.

The evolution of states are shown in the Figures 7,17,27,37.

Where Q and K are diagonal matrices with positive elements. The reaching laws mentioned above can be used to obtain a control law[10].

2.2.2 Super-Twisting Controller

The discontinuous high frequency switching sliding mode controllers are designed to drive the sliding variable to zero. In many cases high frequency switching control is not possible, and continuous control is a necessity. In order to drive the sliding variables to zero in finite time we try the following continuous control. In the super-twisting controller the sliding surface variables are non-linear[11].

$$u = c|s|^{(1/2)}\mathrm{sign}(s) + w, c>0$$

$$\dot{w} = b\mathrm{sign}(s)$$

Sliding variable dynamics becomes

$$\dot{s} + c|s|^{(1/2)}\ \mathrm{sign}(s) + w = \rho(x,t) \tag{6}$$

The super-twisting controller drives the state variables to zero as in Figures 8,18,28,38.

The control input for the super-twisting control is continuous as in Figures 9,19,29,39.

The evolution of states are shown in the Figures 10,20,30,40.

3. Illustrative Examples

3.1 Considering the Transfer Function[12]

$$\frac{1}{(s+1)^3}$$

as in (1),(2)

Converting to state space

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix} \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

3.1.1 Linear State Feedback

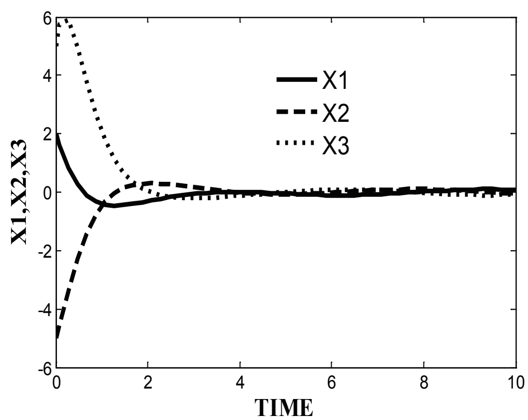

Figure 1. Evolution of state variables.

Table 1. Control energy

$\frac{1}{(s+1)^3}$	WITH DISTURBANCE		WITHOUT DISTURBANCE	
	$\|\|u\|\|_2$	$\|\|u\|\|_\infty$	$\|\|u\|\|_2$	$\|\|u\|\|_\infty$
LINEAR STATE FEEDBACK CONTROLLER	20.0453	15	19.6730	15

3.1.2 Constant Rate Reaching Law

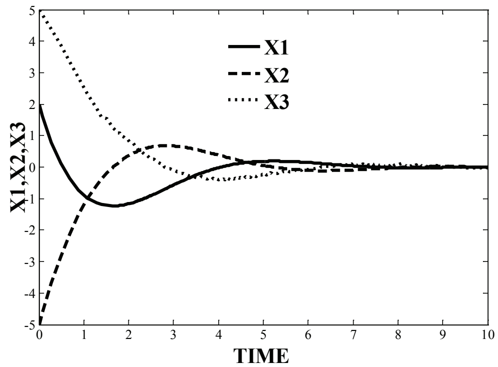

Figure 2. Evolution of state variables.

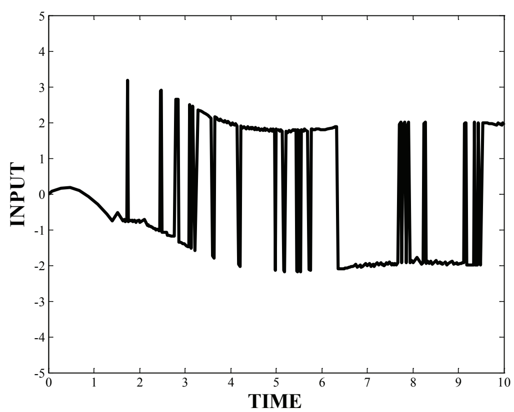

Figure 3. Control input u(t).

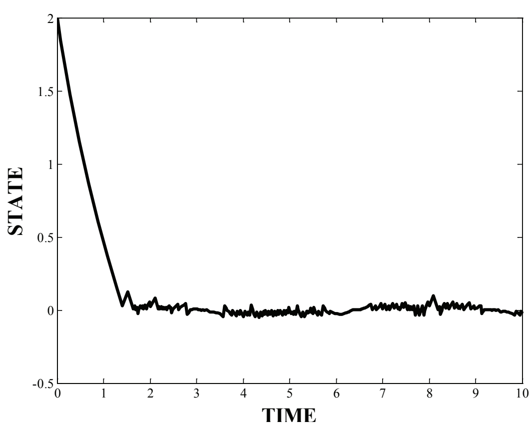

Figure 4. Evolution of sliding surface

3.1.3 Constant Plus Proportional Rate Reaching Law

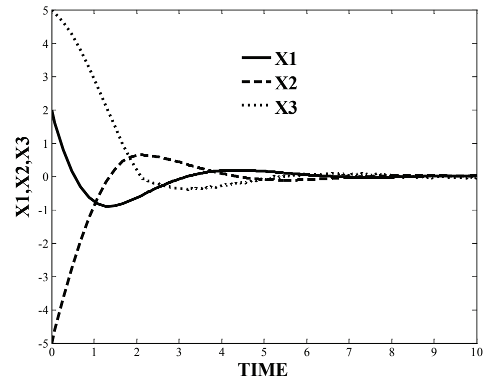

Figure 5. Evolution of state varaibles.

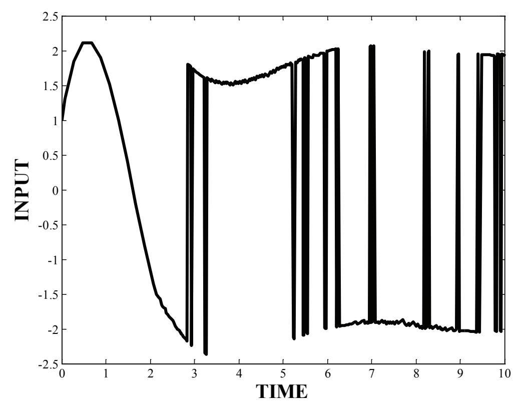

Figure 6. Control input u(t).

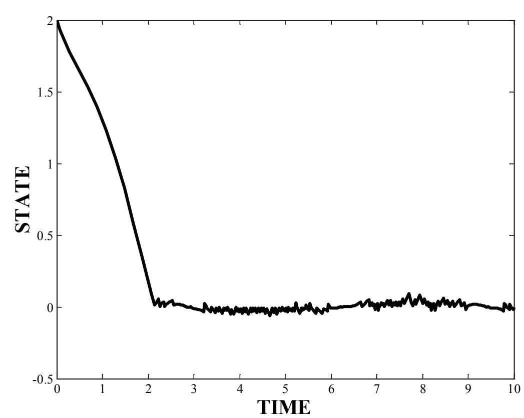

Figure 7. Evolution of sliding surface.

3.1.4 Super-Twisting Controller

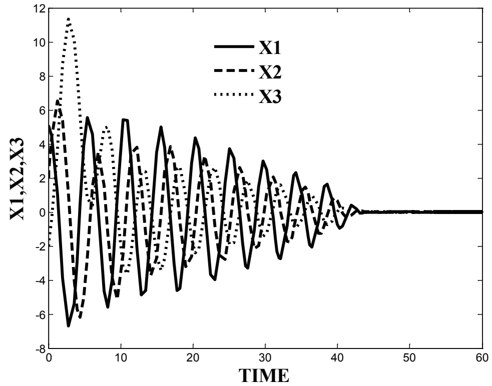

Figure 8. Evolution of state varaibles with time.

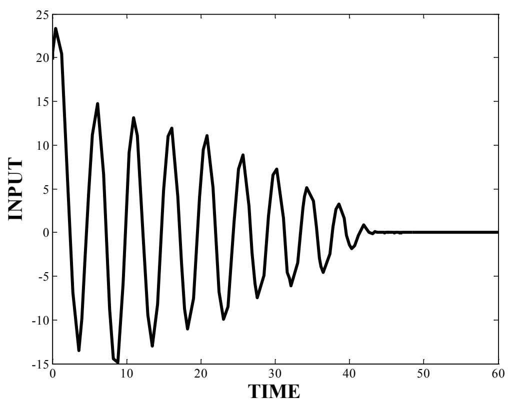

Figure 9. Control input u(t)

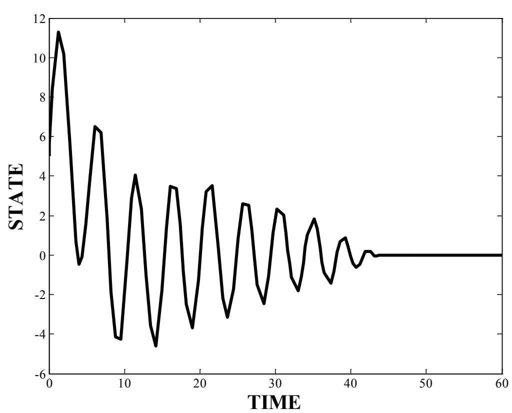

Figure 10. Evolution of sliding surface.

Table 2. Control energy

$\frac{1}{(s+1)^3}$	WITH DISTURBANCE		WITHOUT DISTURBANCE	
	$\|\|u\|\|_2$	$\|\|u\|\|_\infty$	$\|\|u\|\|_2$	$\|\|u\|\|_\infty$
CONSTANT RATE REACHNING LAW	112.912	3.1579	15.6513	3.3145
CONSTANT PLUS PROPORTIONAL RATE REACHING LAW	31.56	2.3671	25.87	2.5955
SUPER-TWISTING CONTROLLER	50.9674	23.5510	93.9545	23.6076

3.2 Considering the Transfer Function[12]

$$\frac{0.0506e^{-6s}}{s}$$

As in (1)(2)
Converting to state space

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & -0.16 \end{bmatrix} \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

3.2.1 Linear State Feedback

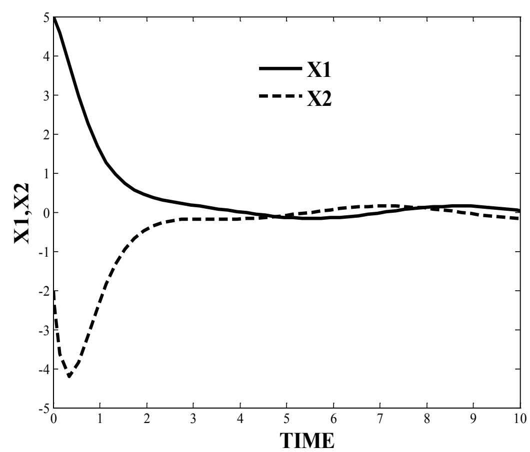

Figure 11. Evolution of state variables

Table 3. Control energy

$\frac{0.0506e^{-6s}}{s}$	WITH DISTURBANCE		WITHOUT DISTURBANCE	
	$\|\|u\|\|_2$	$\|\|u\|\|_\infty$	$\|\|u\|\|_2$	$\|\|u\|\|_\infty$
LINEAR STATE FEEDBACK CONTROLLER	16.3944	13	16.3944	13

3.2.2 *Constant Rate Reaching Law*

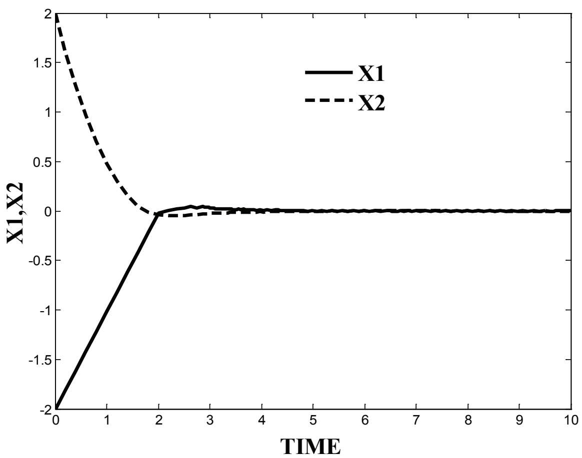

Figure 12. Evolution of state variables.

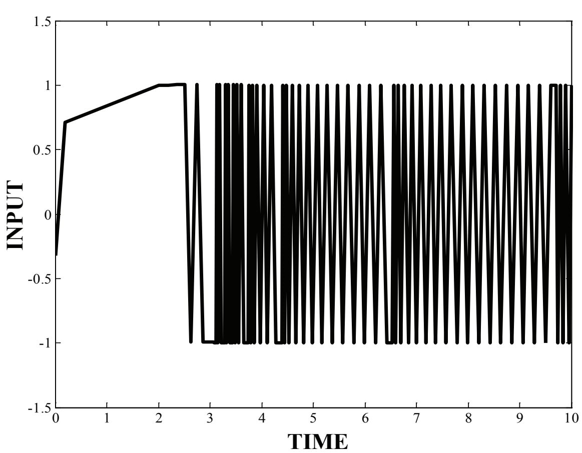

Figure 13. Control input u(t).

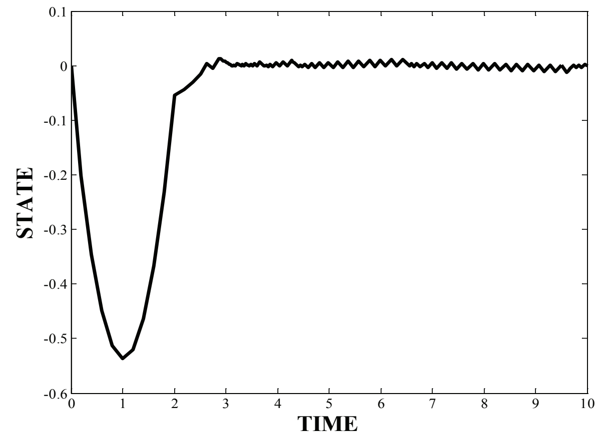

Figure 14. Evolution of sliding surface

3.2.3 *Constant Plus Proportinal Rate Reaching Law*

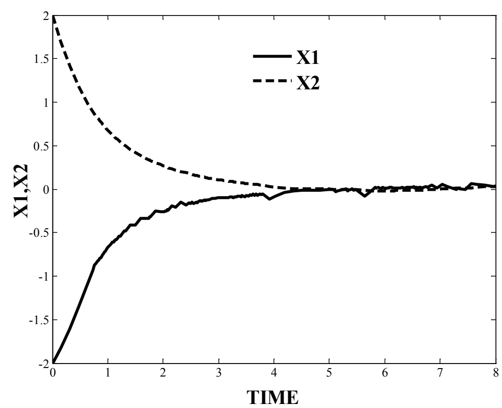

Figure 15. Evolution of state variables.

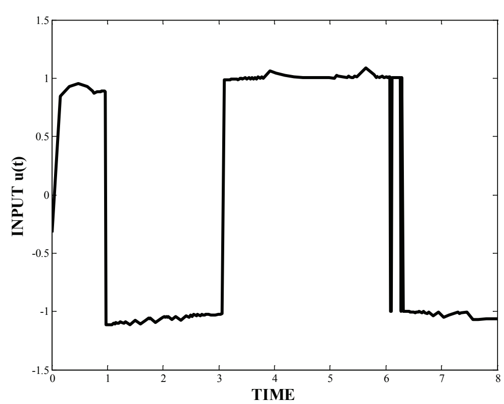

Figure 16. Control input u(t).

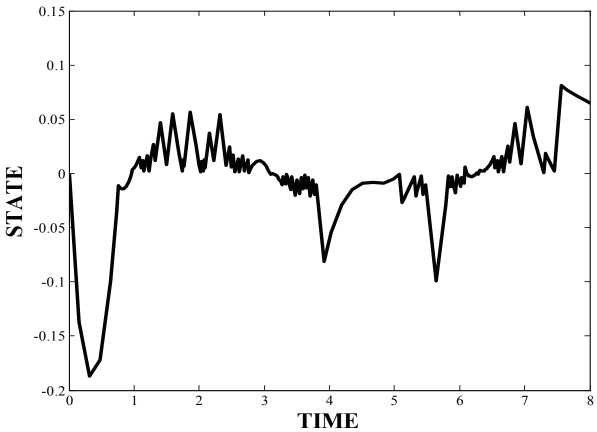

Figure 17. Evolution of sliding surface.

3.2.4 Super Twisting Controller

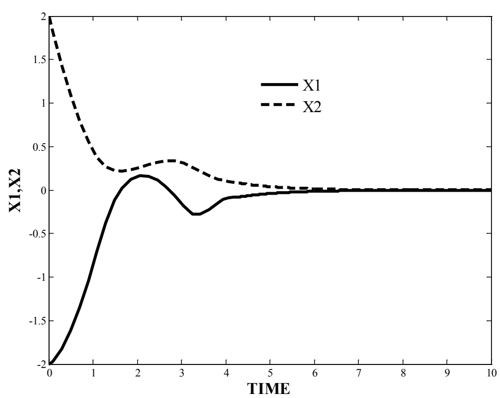

Figure 18. Evolution of state variables.

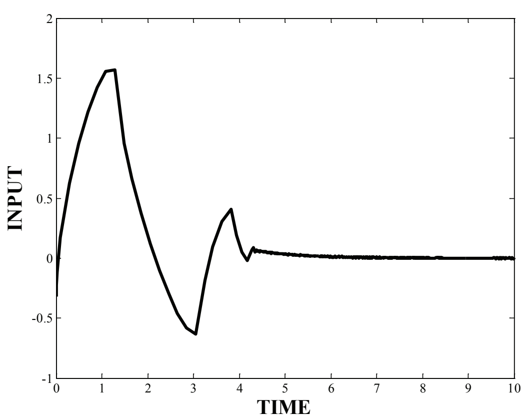

Figure 19. Control input u(t).

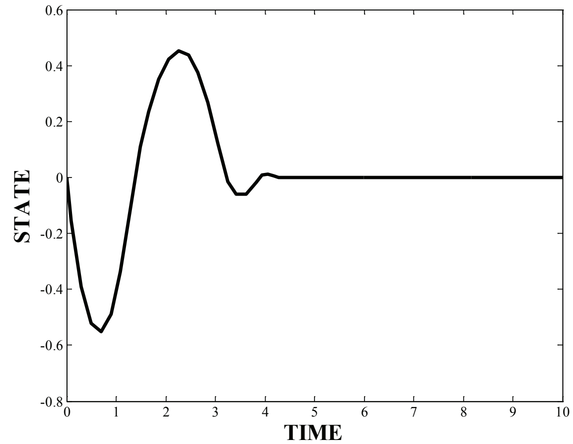

Figure 20. Evolution of sliding surface.

Table 4. Control energy

$\frac{0.0506e^{-6s}}{s}$	WITH DISTURBANCE		WITHOUT DISTURBANCE	
	$\|\|u\|\|_2$	$\|\|u\|\|_\infty$	$\|\|u\|\|_2$	$\|\|u\|\|_\infty$
CONSTANT RATE REACHNING LAW	12.6792	1.0981	11.7106	1.0054
CONSTANT PLUS PROPORTIONAL RATE REACHING LAW	15.6321	1.1497	13.5911	1.1054
SUPER-TWISTING CONTROLLER	3.2198	1.5856	9.5576	2.9095

3.3 Considering the Transfer Function[13-14]

$$\frac{e^{-4s}}{s(4s+1)}$$

As in (1)(2)
Converting to state space

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -0.0625 & -0.5 \end{bmatrix} B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

3.3.1 *Linear State Feedback*

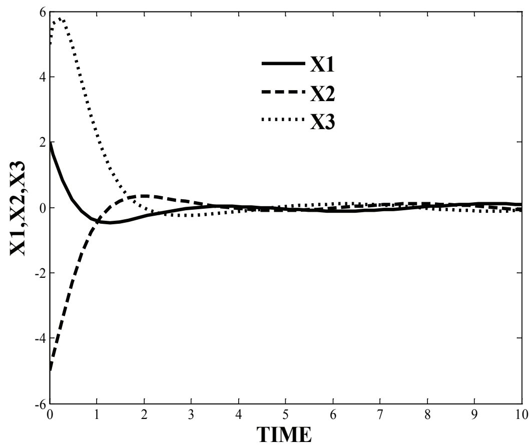

Figure 21. Evolutoin of state varaibles.

Table 5. Control energy

$\frac{e^{-4s}}{s(4s+1)}$	WITH DISTURBANCE		WITHOUT DISTURBANCE	
	$\lVert u \rVert_2$	$\lVert u \rVert_\infty$	$\lVert u \rVert_2$	$\lVert u \rVert_\infty$
LINEAR STATE FEEDBACK CONTROLLER	18.3601	13	16.3944	13

3.3.2 *Constant Rate Reaching Law*

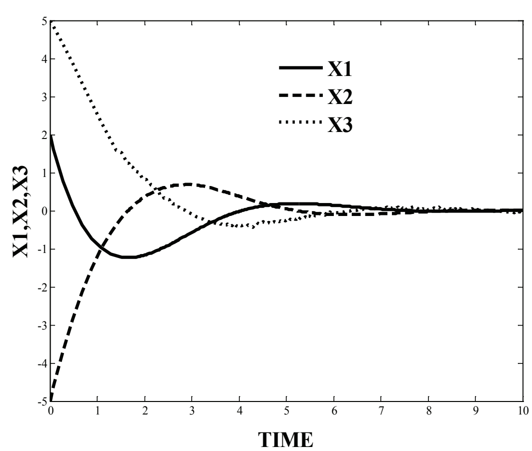

Figure 22. Evolution of state variables.

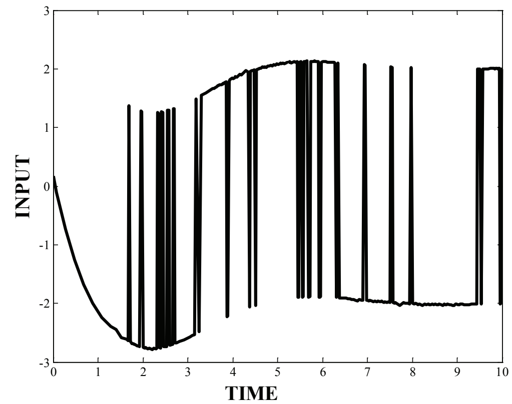

Figure 23. Control input u(t).

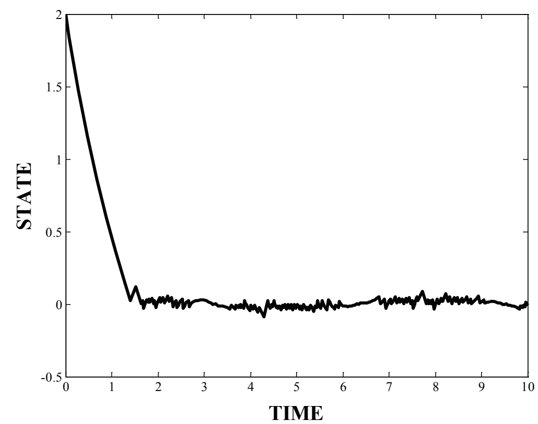

Figure 24. Evolution of sliding surface.

3.3.3 *Constant Plus Proportinal Rate Reaching Law*

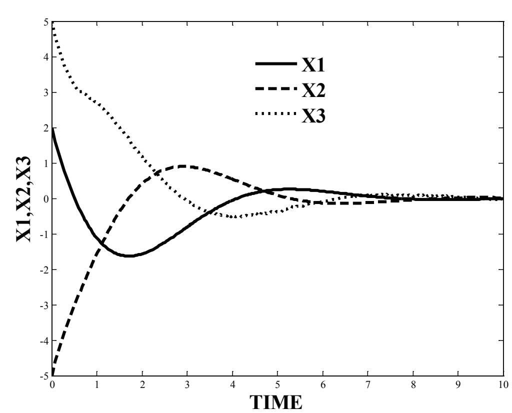

Figure 25. Evolution of state variables.

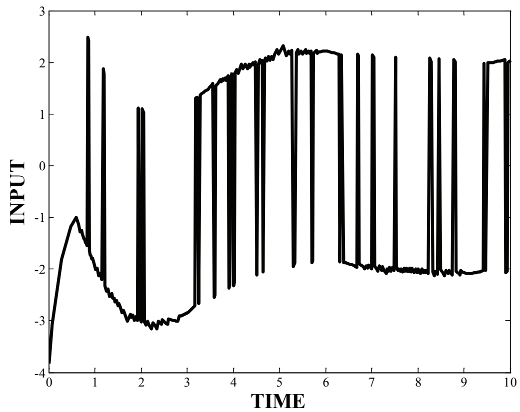

Figure 26. Control input u(t).

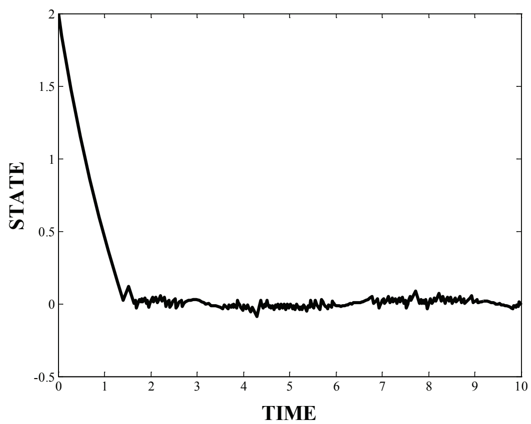

Figure 27. Evolution of sliding surface.

3.3.4 Super Twisting Controller

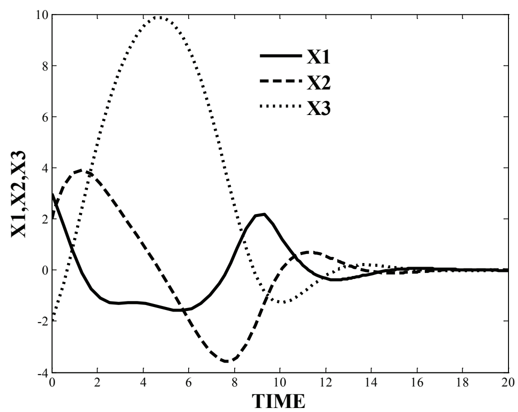

Figure 28. Evolution of state variables.

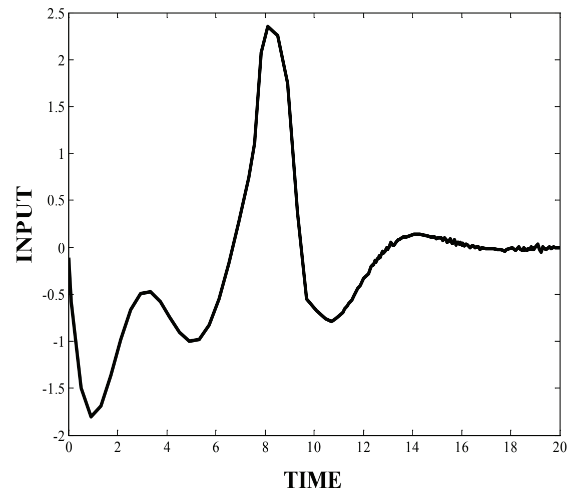

Figure 29. Control input u(t).

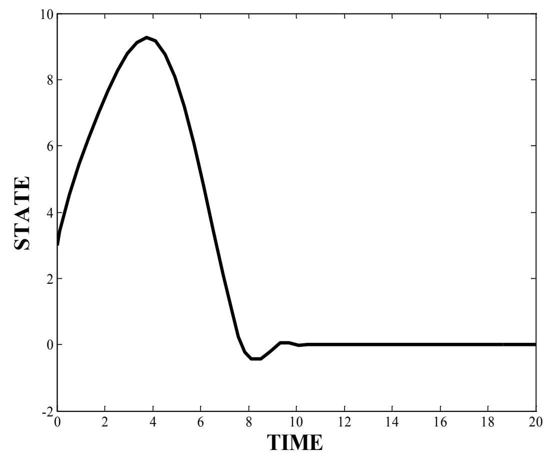

Figure 30. Evolution of sliding surface.

Table 6. Control energy

$\frac{e^{-4s}}{s(4s+1)}$	WITH DISTURBANCE		WITHOUT DISTURBANCE	
	$\|\|u\|\|_2$	$\|\|u\|\|_\infty$	$\|\|u\|\|_2$	$\|\|u\|\|_\infty$
CONSTANT RATE REACHNING LAW	43.211	2.7826	20.7847	2.8335
CONSTANT PLUS PROPORTIONAL RATE REACHING LAW	39.624	3.8215	25.5803	3.8215
SUPER-TWISTING CONTROLLER	7.9376	3.0824	10.8222	3.3665

3.4 Considering the Transfer Function[15]

$$\frac{e^{-1.4s}}{(s-1)}$$

As in (1)(2)
Converting to state space

$$A = \begin{bmatrix} 0 & 1 \\ 0.7142 & 0.2858 \end{bmatrix} B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

3.4.1 Linear State Feedback

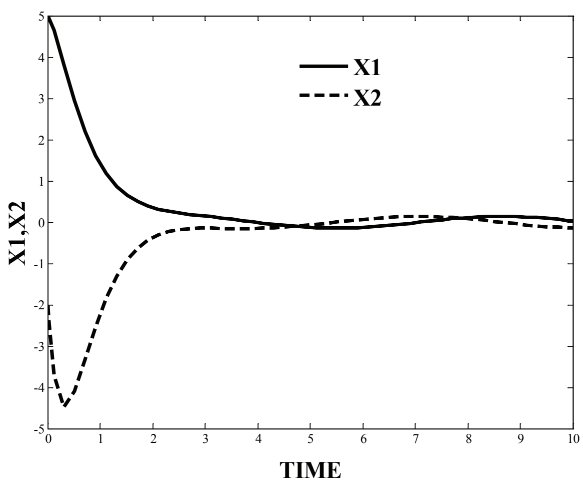

Figure 31. Evolution of state variables.

Table 7. Control energy

$\frac{e^{-1.4s}}{(s-1)}$	WITH DISTURBANCE		WITHOUT DISTURBANCE	
	$\lVert u \rVert_2$	$\lVert u \rVert_\infty$	$\lVert u \rVert_2$	$\lVert u \rVert_\infty$
LINEAR STATE FEEDBACK CONTROLLER	33.8057	22.9994	33.7640	22.9994

3.4.2 Constant Rate Reaching Law

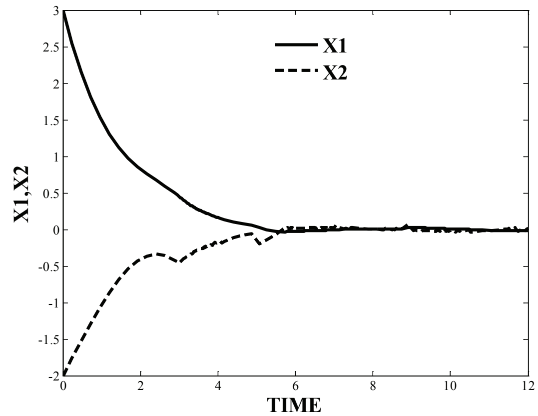

Figure 32. Evolution of state variables.

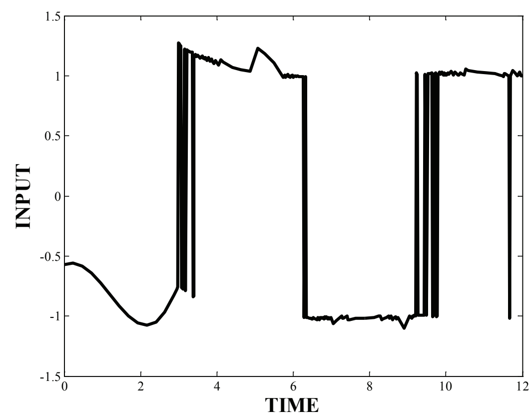

Figure 33. Control input u(t).

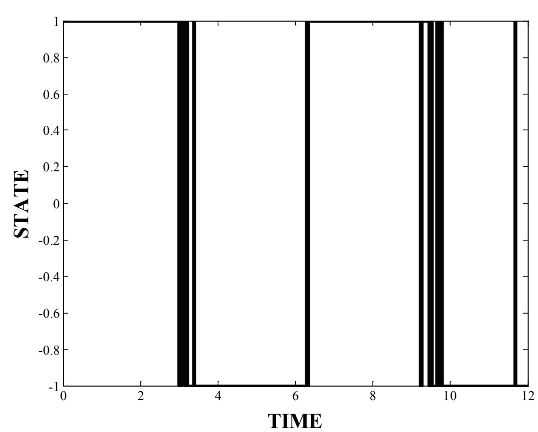

Figure 34. Evolution of sliding surface.

3.4.3 Constant Plus Proportional Rate Reaching Law

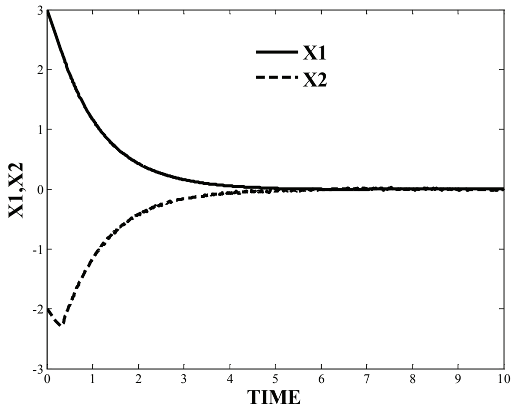

Figure 35. Evolution of state variables.

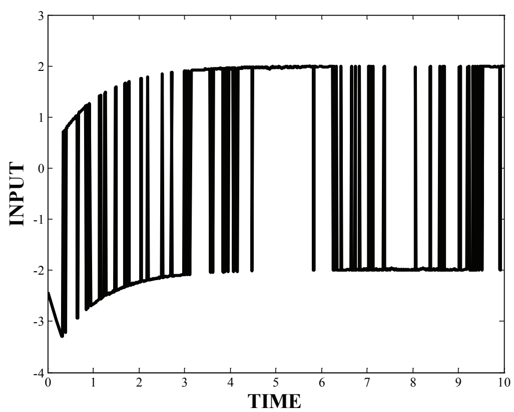

Figure 36. Control input u(t).

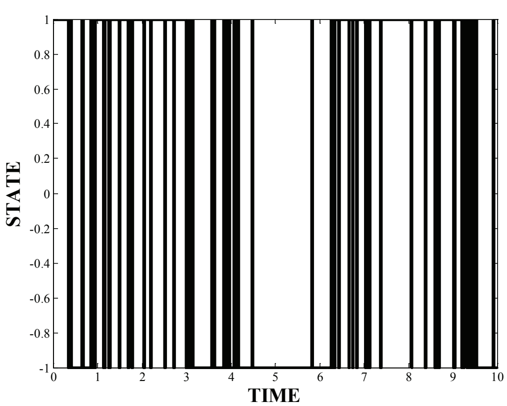

Figure 37. Evolution of sliding surface.

3.4.4 Super Twisting Controller

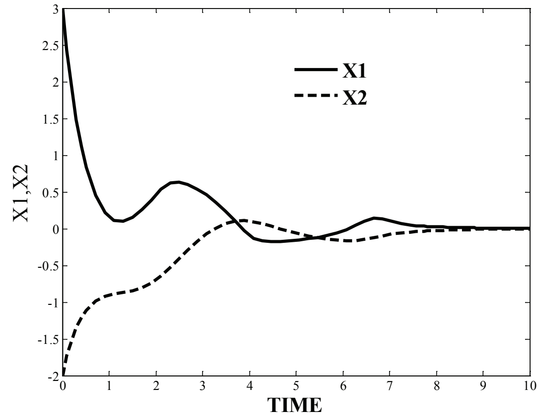

Figure 38. Evolution of state variables.

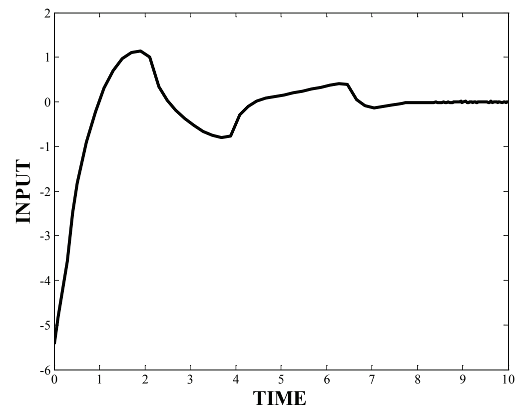

Figure 39. Control input u(t).

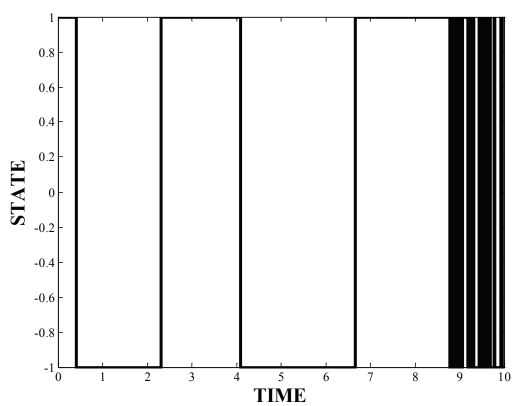

Figure 40. Evolution of sliding surface.

Table 8. Control energy

$\frac{e^{-1.4s}}{(s-1)}$	WITH DISTURBANCE		WITHOUT DISTURBANCE	
	$\|\|u\|\|_2$	$\|\|u\|\|_\infty$	$\|\|u\|\|_2$	$\|\|u\|\|_\infty$
CONSTANT RATE REACHNING LAW	49.9010	2.9994	26.7204	3.0513
CONSTANT PLUS PROPORTIONAL RATE REACHING LAW	50.4459	3.4334	32.5794	3.3727
SUPER-TWISTING CONTROLLER	16.0312	8.7327	16.8548	8.7327

4. Conclusion

Finite time asymptotic convergence is obtained using the Sliding Mode Control. When the linear state feedback is used, convergence of the state variables may not be achieved; this disadvantage is overcome using the Sliding Mode Control. The control energy for driving the state variables to zero is studied. The control energy required for linear state feedback is more as shown in Tables 1,3,5,7 compared to the sliding mode control as in Tables 2,4,6,8. The control input for this law is discontinuous as in Figures 3,13,23,33. The control input for this law is discontinuous as in Figures 6,16,26,36. The control input for the super-twisting control is continuous as in Figures 9,19,29,39.

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