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Performance Analysis and Comparison of Different Tuning Strategies of PI Controller in Conical Tank

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Abstract

Background/Objectives: An adaptive Proportional Integral (PI) controller for a conical tank liquid level process is simulated in MATLAB platform. **Method/Statistical Analysis:** The Scheduled PI and Internal model control (IMC) based PI performance are compared in terms of settling time rise time, overshoot, Integral of Squared Error (ISE), Integral of Absolute Error (IAE), Integral Time Absolute Error (ITAE) and disturbance rejection. For IMC based PI tuning, the exact model of the process obtained by on line estimation of process gain and time constant. **Findings:** The IMC based tuning procedure ensures good performance by Proportional and Integral (PI) controller. The Ziegler Nichols (ZN) tuning rules and estimation based tuning are used for scheduled Proportional and Integral (PI) and internal model control based PI controller. **Applications/Improvements:** Derivative controller can be added to this PI controller for distillation process.

Keywords: Disturbance Rejection, IMC based PI, Integral of Square Error, Scheduled PI

1. Introduction

A system performance description by mathematical equation is highly difficult in nonlinear systems. Most of the process in industries are non linear whose mathematical expression cannot provide general solutions. The nonlinear processes are very less predictable than linear process. Tanks requirements in the industries are more where transferring and storing the liquids and fluids plays lively role[1]. The primary problem in industries is taking a perfect control action in order to have desired tank level. The conical tank is well known nonlinear system which ensures the settle free at the bottom of the tank. The normal PI controllers are to take perfect control action only for linear process[2-4]. The adaptive controllers are perfect for nonlinear process to reach the set point without overshoot[1,5].

Gain scheduled PI controller provides better performance for nonlinear liquid process of level maintain in conical and spherical tank[6]. But gain scheduling is the time consuming process. Because of the approximation is always behind the process. In scheduling nonlinear system consider as many linear systems and design the PI controller for different region (linear region)[7]. Sometimes this scheduled PI controller less effective due to the approximation for linearization.

To overcome this problem on line tuning of parameters of the controller was done based on gain and time constant of the process[8]. So that online estimations of gain and time constant is needed to tune the controller parameters. Estimation of time constant and gain can be done by the present and past values of the input and output of the process. Model based[9] internal model control is one of the best tuning algorithm for tuning the parameters of the PI controller[10-12]. Comparison of estimation based PI with scheduled PI (Figure 5, Figure 7 and Figure 9) shows that the estimation based PI provides tight control action and ensures less time consuming during the tuning of PI controller (Section 2.2 and 2.4). Here ZN tuning rules and IMC based tuning were used to tune the scheduled and estimation based PI controller.

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2. Process Description

The process contains a conical tank whose height and radius are 10cm and 7cm and outlet valve coefficient is 0.2. The connections of Inlet valve (control valve, Figure 1) is from pump to tank and outlet valve is from tank to reservoir.

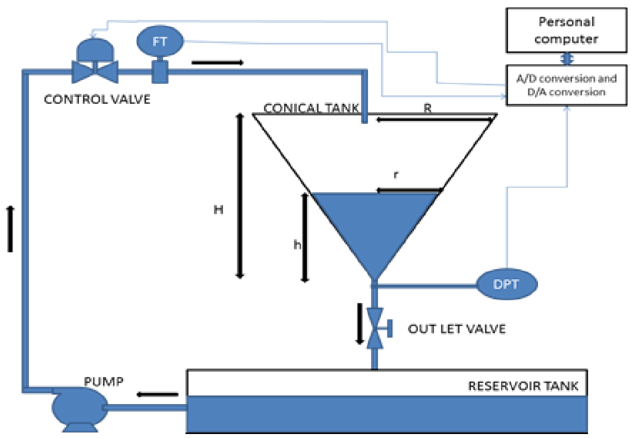

Figure 1. Experimental setup of liquid level process.

Pump can suck the liquid from reservoir and gives to tank via control valve. Control valve gets control signal at the rate of 4-20mA from controller and regulates the flow of liquid depending upon the level in the conical tank.

Level in the tank is measured by using a differential transducer and is transmitted to the interfacing unit in the form of current at the rate of 4-20mA. Controller compares the set point with transmitted out and produce control a signal based on the error.

2.1 Modeling for Conical Tank

The rate of change of volume is given by Mathematical equation for liquid level system is derived from mass balance equation.

$$\frac{\text{Total mass of accumulation}}{\text{time}} = \frac{\text{Total mass of input}}{\text{time}} - \frac{\text{Total mass of output}}{\text{time}} \tag{1}$$

The mass balance equation states that change in volume equal to difference between inlet and outlet flow rate of the conical tank[1].

$$\frac{dv}{dt} = f_{in} - f_{out} \tag{2}$$

$$\frac{dv}{dt} = A*(f_{in} - f_{out}) \tag{3}$$

$$\frac{dv}{dt} = \frac{1}{3}\pi r^2 h*(f_{in} - f_{out}) \tag{4}$$

2.1.1 Parameters for Modeling

The development of model for the process is needed the parameters are tabulated (Table 1) as analytical values.

Table 1. Process parameters

	Values
Maximum height of the tank	10 cm
Maximum radius of the tank	7 cm
Outlet valve coefficient	0.2
Maximum inlet flow rate	6.3cm^3/sec

2.1 Method of Gain Scheduling

The model of the system created by using MATLAB software to obtain non linear response of the process. The process response is shown in Figure 2.

The parameters of the process can be obtained by process reaction curve method for different operating regions. From the process parameters, Controller parameters are determined by ZN tuning rules for various regions to design the scheduled PI with the help of process parameters[1]. Controller parameters of process are proportional gain and Integral gain which are obtained from the below polynomial equations.

$$K_p = -0.0004013*SP^4 + 0.009184*SP^3 - 0.06767*SP^2 + 0.1513*SP + 0.6018 \tag{5}$$

$$K_i = -0.0001042*SP^4 + 0.002512*SP^3 - 0.02106*SP^2 + 0.07382*SP + 0.00829 \tag{6}$$

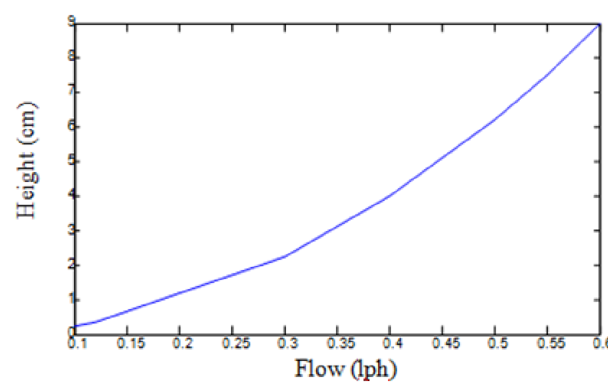

Figure 2. Open loop response of conical tank.

2.3 Linear Process Parameter Estimation

$$\frac{y(s)}{u(s)} - \frac{K}{\tau s+1} \tag{7}$$

$$y(s)\tau s+y(s)=Ku(s) \tag{8}$$

$$\frac{dy(t)}{ts}\tau + y(t) = Ku(t) \tag{9}$$

$$\frac{y(t)-y(t-1)}{ts}\tau + y(t) = ku(t) \tag{10}$$

$$y(t) = \frac{ku(t) + \frac{y(t-1)\tau}{ts}}{\left(\frac{\tau}{ts}+1\right)} \tag{11}$$

The digital form of the equation (11)

$$y(n) = \frac{ku(n) + \frac{y(n-1)\tau}{ts}}{\left(\frac{\tau}{ts}+1\right)} \tag{12}$$

Let us consider the past data samples n →(n-1) in above discrete model

$$y(n-1) = \frac{ku(n-1) + \frac{y(n-2)\tau}{ts}}{\left(\frac{\tau}{ts}+1\right)} \tag{13}$$

By solving the equations (12) and (13), the gain and time constant are estimated as shown in (14) and (15)

$$K(n) = \frac{u(n)}{y(n)}\frac{-T(n)(y(n-1)-y(n))}{u(n)} \tag{14}$$

$$T(n) = \frac{u(n)y(n-1)-y(n)u(n-1)}{u(n)(y(n-1)-y(n-2))-u(n-1)(y(n-1-y(n))} \tag{15}$$

Where

K(n)-gain of the model

T(n)-Normalized time constant

The Estimated values of gain K and time constant can be calculated shortly as much as possible to design perfect controller [13].

2.4 IMC based Tuning Algorithm

The normal PI controller is the most transparent and simple controller. IMC describes internal model control (Figure 3) which plays crucial role for tuning the parameter of PI controller in order to have a set point tracking and disturbance rejection[14]. The frequent tuning and desired response tracking of controller are more convenient by using IMC based PI[15].

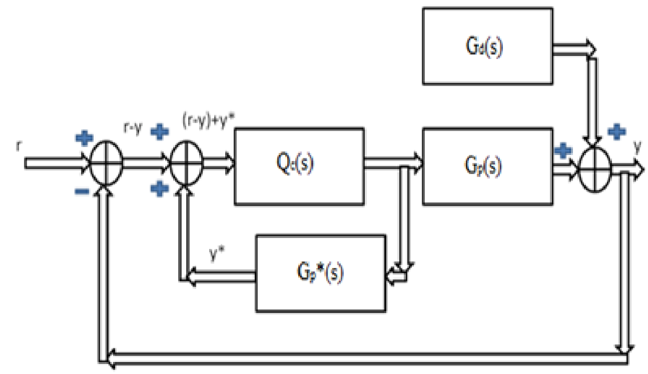

Figure 3. Block diagram for IMC based PI design.

The Process G_P can be controlled by the controller Q_c. The process deviates from the model due to disturbance occurring of the process. Controller Q_c will be equal to the Inverse of the model if process is exactly same as the model[16].

Q_c=Inverse of G_P* if $G_P = G_P$*

The models of the transfer function represented by two parts, they are non invertible and invertible part. Invertible and non invertible parts are good stuff and bad stuff portions.

$$yGP^*(s)= GP^*(+)(s)\ GP^*(-)(s) \tag{16}$$
$$GP^*(s)=1.K^*/[\tau^*(s)+1] \tag{17}$$

Here non invertible part will be cancelled and get

$$Q_c^*(s)=inv[G_P^*(-)(s)]= [\tau^*(s)+1]/K^* \tag{18}$$

$$Q_c(s)=Q_c^*(s).f(s) = [\tau^*(s)+1]/[K^*.(lem(s)+1] \tag{19}$$

Equivalent feedback controller

$$G_c(s)=Q_c(s)/(1-Q_c(s)G_P^*(s)) =(\tau s+1)/(K_p.lem.s \tag{20}$$

$$G_c(s)=[Kc.(\tau i.s+1)]/(\tau i.s) \tag{21}$$

The controller parameters are $Kc= \tau/ K$ lem and $\tau_i= \tau$[15]
The equation for PI is

$$K_C\left[e(t)+\frac{1}{\tau i}\int e(t)\right] \tag{22}$$

3. Result and Discussion

A Conical tank whose process parameters are varying with respect to time is controlled by gain scheduled PI and IMC based PI controller. The Gain scheduled PI provides less efficient control action than IMC based PI controller. Here the control performance is obtained for various desired response and compared both controllers for the set points of 4cm, 6cm and 8cm.

Performance of IMC based PI and parameter scheduled PI is compared for conical tank. The servo and regulatory response were stimulated using MATALB.

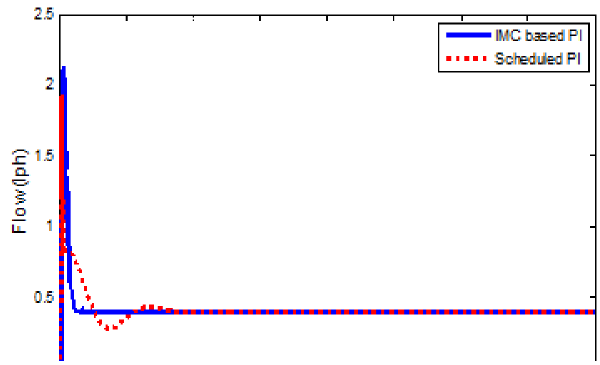

Figure 4. Controller output of the Scheduled PI and IMC based PI for the set point of 4 cm.

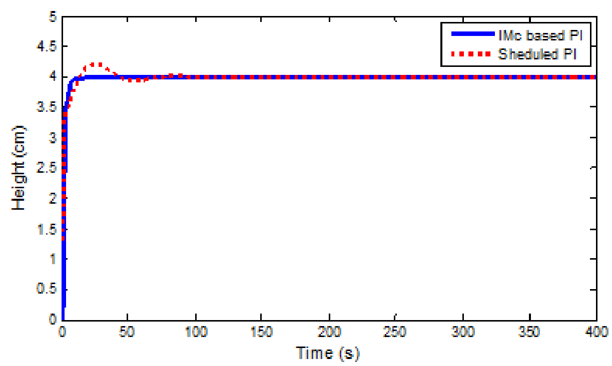

Figure 5. Output response using Scheduled PI and IMC based PI for the set point of 4 cm.

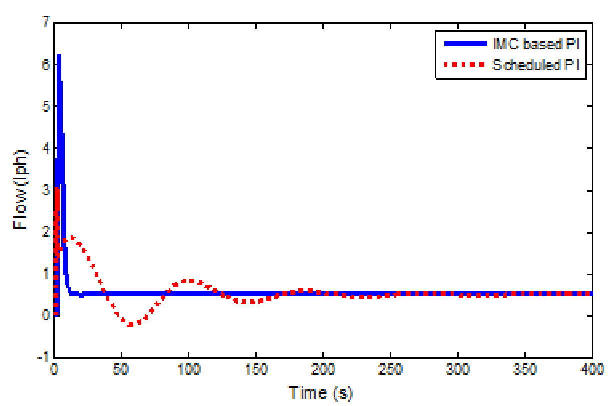

Figure 6. Controller output of the Scheduled PI and IMC based PI for the set point of 6 cm.

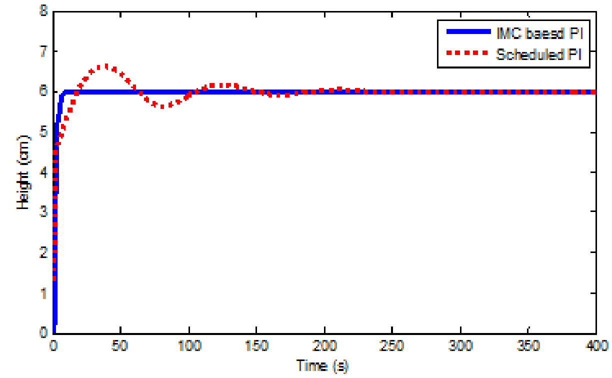

Figure 7. Output response using Scheduled PI and IMC based PI for the set point of 6 cm.

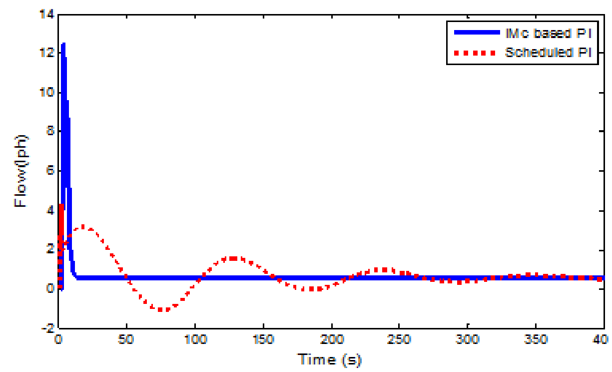

Figure 8. Controller output of the Scheduled PI and IMC based PI for the set point of 8 cm.

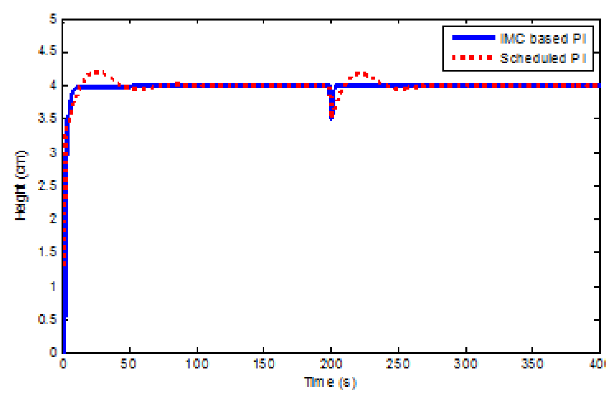

Figure 9. Output response using Scheduled PI and IMC based PI for the set point of 8 cm.

Table 2. Comparison Table of Scheduled PI and IMC PI for the Set point 4cm

Controller	Set point	Settling time	Rise time	
Scheduled PI	4cm	78s	17s	
IMC PI	4cm	20s	10s	
Controller	overshoot	ISE	IAE	ITAE
Scheduled PI	0.25cm	0.956	2.1	10.5
IMC PI	Nil	0.25	0.5	2.5

The IMC based PI provides no overshoot and small oscillation in output than Parameter scheduled PI controller. The Scheduled PI controller is designed for different nominal operating point (from top to bottom of the tank). The scheduled PI provides oscillation in the range between the nominal values (Figure 5, 7 and 9). This is mainly due to the cross sectional variations in entire region of the tank. But the IMC based tuning overcomes this drawback effectively, which is not able to attain by scheduled PI controller ((Figure 5, Figure 7 and Figure 9). Figure 4, 6 and 8 show the controller outputs for both controlling techniques. The scheduled PI controller has large settling time, Rise time, Overshoot, ISE (Integral of square error), IAE (Integral of absolute error) and ITAE (Integral of time absolute error) [Table 2].

$$ISE = \int_0^\infty |e(t)|^2 dt \tag{23}$$

$$ITAE = \int_0^\infty t\,|e(t)|^2 dt \tag{24}$$

$$IAE = \int_0^\infty |e(t)| dt \tag{25}$$

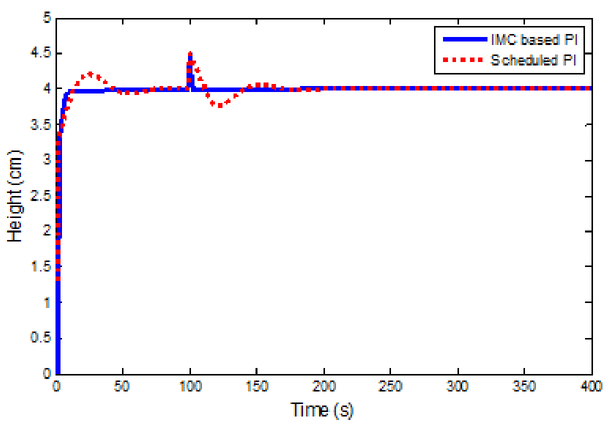

Figure 10. Positive Disturbance rejection performance using Scheduled PI and IMC based PI.

Table 3. Process parameters

Controller	Positive disturbance Occurring Time	Positive disturbance Rejection Time
Scheduled PI	100^{th} s	175^{th} s
IMC PI	100^{th} s	106^{th} s
Controller	Negative disturbance Occurring Time	Negative disturbance Rejection Time
Scheduled PI	200^{th} s	275^{th} s
IMC PI	200^{th} s	206^{th} s

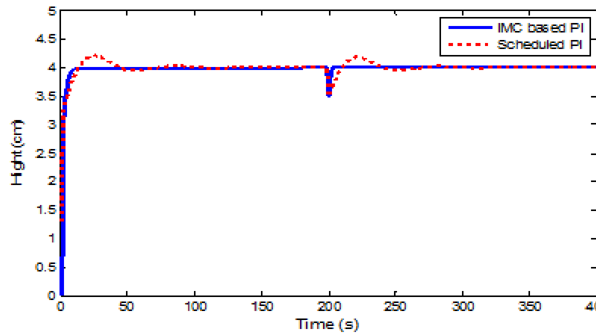

Figure 11. Negative Disturbance rejection performance using Scheduled PI and IMC based PI.

The Scheduled PI is less effective controller than IMC based PI controller for rejection of disturbance in the conical tank. The positive and negative disturbances are added to the process after the settling time at the 100^{th} second and 200^{th} second [Table 3]. The scheduled controller takes more time to retain the set point of 4cm. But IMC based PI retain the Set point as soon as possible by reducing the settling time.

4. Conclusion

In this work, the effectuality of the IMC based PI and ineffectual of scheduled PI for nonlinear system are illustrated. The regulatory and servo performance for both tuning methodology are simulated using MATLAB. The online estimation of gain and time constant based tuning of PI controller provides better control action and disturbance rejection for nonlinear process than Scheduled PI controller. The ISE, IAE and ITAE are calculated in order to analysis the regulatory response of the controller (settling time, rise time, overshoot and oscillation). The small value of ISE and IAE shows that, there is a small overshoot and settling time using IMC based tuning than other. The servo mechanism is analysed by adding the disturbance to the process and how quickly output is retained to the set point of the process. The proposed control algorithm does not require any difficult mathematical calculation. This IMC based PI controller tuning ensures low cost of computation and optimal control action even the disturbance is added to the process.

5. References

1. Bhuvaneshwari NS, Uma G, Rangaswamy TR. Adaptive

and optimal control of an nonlinear process using intelligent controllers. Applied Soft Computing. 2009; 9(1):182-90.
2. Anandanatarajan R, Chidambaram M, Jayasingh T. Limitation of a PI controller for a first order nonlinear process with dead time. ISA Transactions. 2006; 45(2):185-99.
3. Babu SRR, Deepa S, Jothivel S. A closed loop control of quadratic boost converter using PID controlller. IJE Transaction B: Applications. 2014 Nov; 27(11):1653-62.
4. Astrom KJ, Hagglund T. The future of PID controller. Control Engineering Practice. 2001; 9(11):1163-75.
5. Tan KK, Haung S, Ferdous S. Robust self-tuning PID controller for nonlinear systems. Journal of Process Control. 2002; 12(7):753-61.
6. Kuamr DD, Meenakshipriya M. Design and implementation of nonlinear system using gain scheduled PI controller. Procedia Engineering. 2002; 38:3105-12.
7. Hizal NA. Gain scheduling adaptive model control. Journal of Engineering and Environmental Science. 1999; 23:209.
8. Schei TS. Automatic tuning of PID controllers based on transfer function estimation. Automatica. 1994; 30(12):1983-89.
9. Morari M, Zafiriou E. Prentice-Hall: Englewood Cliffs, NJ: Robust Process Control. 1989.
10. Volanova R. IMC based robust PID design tuning guidelines and automatic tuning. Journal of Process Control. 2008; 18(1):61-70.
11. Tan W, Liu J, Chen T, Marquez HJ. Comparison some well known PID tuning formulas. Computer and Chemical Engineering. 2006; 30(9):1416-23.
12. Arputha Vijayaselvi J, Rathakirushnan TK, Sundaram S. Performance assessment of PID and IMC tuning methods for a mixing process with time delay. ISA Transaction. 2007; 47(3):391-97.
13. Kesavan E, Rakesh Kumar S. PLC based adaptive PID control of NON linear liquid tank system using online estimation of linear parameters by difference equations. International Journal of Engineering and Technology. 2013; 5(2):910-16.
14. Arbogast JE, Cooper DJ. Extension of IMC tuning correlation for non-self regulating (integrating) process. ISA Transactions. 2007; 46(3):303-11.
15. Li D, Zeng F, Jin Q, Pan L. Application of an IMC based PID controller tuning strategy in atmospheric and vacuum distillation units. Non Linear Analysis Real World Application. 2009; 10(5):2729-39.
16. Porwal A, Vyas V. Rourkela: National Institute of Technology: Internal model control and IMC based PID controller. 2010.

NOMENCLATURE

$e(t)$	Error signal	Kc	Controller gain
$y(s)$	Process output	dv/dt	Change in volume
$u(s)$	Process input	dh/dt	Change in height
K	Process gain	A	Area of the tank
*	Model parameters	SP	Setpoint
f_{in}	Inlet flow, cm^3/s (lph)	u(n)	Present input
f_{out}	Outlet flow, cm^3/s (lph)	y(n)	Preset output
$G_p(s)$	Process	T(n)	Normalized time constant
$G_p{*}(s)$	Model of the process	Greek Symbols	
$G_d(s)$	Disturbance	τ	Time constant
H	Maximum height, cm	Abbreviations	
h	Height at liquid level of the conical tank, cm	P	Proportional
K_v	Outlet valve coefficient	I	Integral
K_p	Proportional gain	IMC	Internal model control
K_i	Integral gain	ZN	Ziegler Nichols
K(n)	Gain of the model	ISE	Integral squared error
$G_c(s)$	Controller of the process	IAE	Integral absolute error
R	Maximum radius, cm	ITAE	Integral of time absolute error
r	Radius at liquid level of the conical tank, cm	DPT	Differential pressure transmitter