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System Reliability Evaluation of a Stochastic - Flow Network using Spanning Trees

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Abstract

In this study, a new method is presented to evaluate the system reliability of a flow network using spanning trees with flow. The proposed method includes two main tasks: 1. Identifying the spanning trees without flow by representing each network link as a binary string of length N (N is the number of nodes) and performing all possible combinations between N – 1 links, and 2. Using the generated spanning trees without flow to find the spanning trees with flow by calculating the total flow carried by the links. The proposed method is tested on different examples from the literature to illustrate its efficiency in generating the spanning trees with flow and calculating the system reliability.

Keywords: Stochastic - flow Network, Spanning Tees, System Reliability

1. Introduction

A spanning tree of a graph (G) with N nodes is a sub graph of G with N - 1 links containing no circuits. Generating all spanning trees of G without flow has been much studied, for example[1–5]. These spanning trees without flow have been used to calculate the system reliability of a computer network[6–8]. A spanning tree with flow has been proposed[9] and uses an algorithm based on two major steps: The use of the Cartesian product of all paths to generate spanning trees without flow, followed by summing the flow carried by each arc (or link) to find the spanning trees with flow.

It is known that, each arc in the network connects two nodes and that the spanning tree contains N - 1 arcs if the network has N nodes. This is achieved by performing all possible combinations between N - 1 arcs. The number of all possible combinations (the number of candidate spanning trees) is equal to $\binom{m}{n-1}$, where m is the number of links.

In the case of flow network, the system reliability has traditionally been evaluated based on minimal paths or minimal cuts[10–28]. The paper presents a new method based on spanning trees with flow to evaluate the system reliability of a flow network.

2. Generating all Spanning Trees using Links Encoding

Let G (N, A) be an undirected network where N is the set of n nodes and A is the set of m arcs. A and N are defined as: A={a_i; 1≤ i ≤ m} and A={n_i; 1≤ i ≤ n} . Each link a_i can be represented as a binary string of length N elements. The position of the element in the string matches the label of the node: 1 if the node exists and 0 otherwise.

The spanning tree without flow is represented as a binary string of length m. The position of the element in the string matches the link number: 1 if the link exists and 0 otherwise.

Consider the following network with 4 (n=4) nodes and 5 (m=5)arcs, shown in Figure 1. The network can be represented by set of nodes, N = {n_1, n_2, n_3, n_4} and the set of arcs A = {a_1, a_2, a_3, a_4, a_5}.

The Table 1 shows the binary representation of each link.

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Table 1. Binary representation of links

Link a_i	node pair connected	Binary representation
a_1	(n_1, n_2)	(1 1 0 0)
a_2	(n_1, n_4)	(1 0 0 1)
a_3	(n_2, n_3)	(0 1 1 0)
a_4	(n_2, n_4)	(0 1 0 1)
a_5	(n_3, n_4)	(0 0 1)

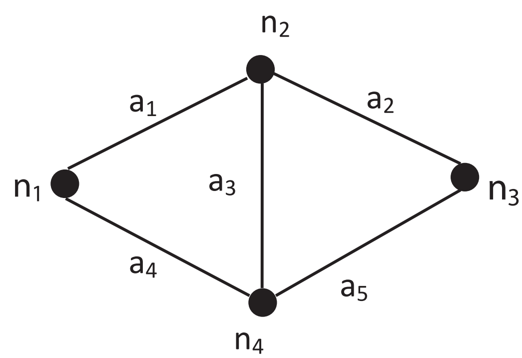

Figure 1. Network with 4 nodes and 5 links.

2.1 The Pseudocode of the Algorithm

Begin

Read all m arcs as a string of n elements

Calculate the number of candidate spanning trees ncsp $= \binom{m}{n-1}$

Set nsp = 0

For i = 1 to ncsp do

Combine n-1 arcs to generate a candidate spanning tree

Calculate the sum of bits of the candidate spanning tree sb(connected)

If sb = n and the candidate spanning tree has no circuit, then keep it and increase nsp.

Else discard it.

End for

Printout the number of spanning trees (nsp) and list the generated spanning trees.

End

2.2 Illustrative Example

In the following steps, we demonstrate how the algorithm given in section 2.1 generates the spanning trees without flow for the network example shown in Figure 1.

1) The arcs are :
   a_1 = (1 1 0 0), a_2 = (0 1 1 0), a_3 = (0 1 0 1), a_4 = (1 0 0 1), and a_5 = (0 0 1 1).
2) The number of candidate spanning trees ncsp = $\binom{m}{n-1} = \binom{5}{3} = 10.$
3) i = 1
4) Combine a_1, a_2, and a_3 will result the string (1 1 1 1) with the sum = 4 and has no circuit. Then nsp =1 and the spanning tree is a_1 a_2 a_3. In binary string representation (1 1 1 0 0).
5) i = 2
6) Combine a_1, a_2, and a_4 will result the string (1 1 1 1) with the sum = 4 and has no circuit. Then nsp =2 and the spanning tree is a_1 a_2 a_4
7) i = 3
8) Combine a_1, a_2, and a_5 will result the string (1 1 1 1) with the sum = 4 and has no circuit. Then nsp = 3 and the spanning tree is a_1 a_2 a_5
9) i = 4
10) Combine a_1, a_3, and a_4 will result the string (1 1 1 1) with the sum = 4 and has no circuit. Then nsp = 4 and the spanning tree is a_1 a_3 a_4
11) i = 5
12) Combine a_1, a_3, and a_5 will result the string (1 1 1 1) with the sum = 4 and has a circuit. Then the spanning tree is discarded
13) i = 6
14) Combine a_1, a_4, and a_5 will result the string (1 1 1 1) with the sum = 4 and has no circuit. Then nsp = 5 and the spanning tree is a_1 a_4 a_5
15) i = 7
16) Combine a_2, a_3, and a_4 will result the string (1 1 1 1) with the sum = 4 and has a circuit. Then the spanning tree is discarded
17) i = 8
18) Combine a_2, a_3, and a_5 will result the string (1 1 1 1) with the sum = 4 and has a circuit. Then the spanning tree is discarded
19) i = 9
20) Combine a_2, a_4, and a_5 will result the string (1 1 1 1) with the sum = 4 and has no circuit. Then nsp = 7 and the spanning tree is a_2 a_4 a_5
21) i = 10, this is the last combination because ncsp = 10.
22) Combine a_3, a_4, and a_5 will result the string (1 1 1 1) with the sum = 4 and has no circuit. Then nsp = 8 and the spanning tree is a_3 a_4 a_5.

23) The number of spanning trees is nsp = 8 and the panning trees are:
$a_1 a_2 a_3$, $a_1 a_2 a_4$, $a_1 a_2 a_5$, $a_1 a_3 a_4$, $a_1 a_4 a_5$, $a_2 a_3 a_5$, $a_2 a_4 a_5$, and $a_3 a_4 a_5$

In Binary representations:

(1 1 1 0 0), (1 1 0 1 0), (1 1 0 0 1), (1 0 1 1 0), (1 0 0 1 1), (0 1 1 0 1), (0 1 011) and (00 1 11)

The number of candidate spanning trees (ncsp) is shown in Table 2 and the number of generated spanning trees (nsp) using the proposed algorithm given in 2.1, applied on the networks given in Figures 2 – 5 taken from[1, 6, 7, 9]. Table 3 shows the spanning trees generated for each network by the presented algorithm.

Note: The number of spanning trees in the case of fully connected network is equal to $n^{(n-2)}$ where n the number of nodes[3].

3. Generating Spanning Trees with Flow

The spanning tree with flow can be obtained by generating all paths to that spanning tree without flow and calculating the total flow carried by each link.

Table 2. The ncsp and nsp for each network

Network Serial Number	Number of nodes	Number of edges	Number of ncsp	Number of nsp	Note
1	4	5	10	8	Non fully connected
2	4	6	20	16	Fully connected
3	5	7	35	21	Non fully connected
4	5	10	210	125	Fully connected
5	6	8	56	30	Non fully connected

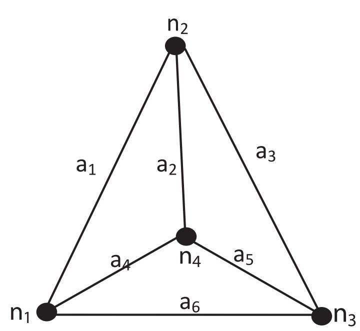

Figure 2. Network with 4 nodes and 6 links.

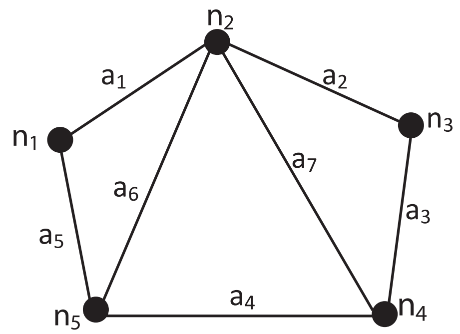

Figure 3. Network with 5 nodes and 7 links.

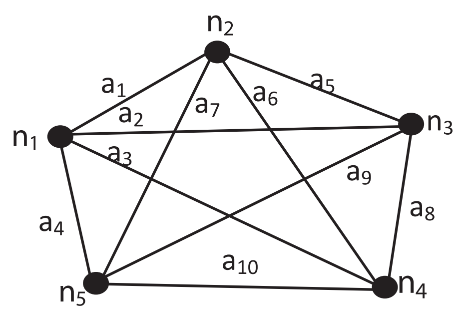

Figure 4. Network with 5 nodes and 10 links.

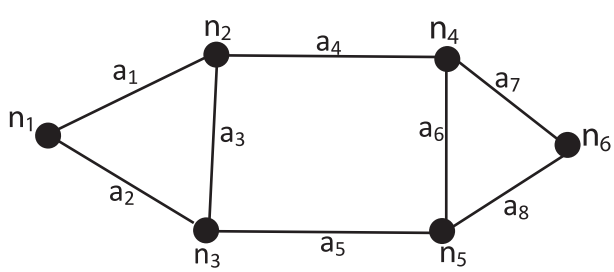

Figure 5. Network with 6 nodes and 8 links

3.1 The Pseudocode of the Algorithm

Begin
For i = 1 to nsp do
Generate all paths to the spanning tree
Calculate the total flow carried by each link.
Printout the obtained spanning tree after replaced the value 1 by the flow value.
End for End

3.2 Example

Consider the spanning tree without flow (1 1 1 0 0). Its links are a_1 a_2 a_3, and it can be represented as shown in Figures 6.

Table 3. The spanning trees generated by the presented algorithm

Network Serial Number	Spanning trees
1	11100, 11010, 11001, 10101, 10011, 01110, 01011, 00111.
2	111000, 110010, 110001, 101100, 101010, 100110, 100101, 100011, 011100, 011001, 010110, 010101, 010011, 001110, 001101,001011.
3	1111000, 11010100, 1110010, 1101100, 1101010, 1101001,1100101, 1100011, 1011100, 1011010, 1011001, 1010101,1010011, 0111100, 0110110, 0101110, 0101101, 0100111,0011110, 0011101, 0010111.
4	,1111000000,1110001000,1110000010,1110000001, 1101010000,1101000100,1101000001,1100011000, 1100010010,1100010001,1100001100,1100001001, 1100000110,1100000101,1100000011,1011100000, 1011000100,1011000010,1010101000,1010100010, 1010100001,1010001100,1010001010,1010000110, 1010000101,1010000011,1001110000,1001100100, 1001100001,1001010100,1001010010,1001000110, 1001000101,1001000011,1000111000,1000110010, 1000110001,1000101100,1000101001,1000100110, 1000100101,1000100011,1000011100,1000011010, 1000010110,1000010101,1000010011,1000001110, 1000001101,1000001011,0111100000,0111010000, 0111001000,0110101000,0110100010,0110100001, 0110011000,0110010010,0110010001,0110001010, 0110001001,0101110000,0101100100,0101100001, 0101011000,0101010100,0101010001,0101001100, 0101001001,0100111000,0100110010,0100110001, 0100101100,0100101001,0100100110,0100100101, 0100100011,0100011100,0100011010,0100010110, 0100010101,0100010011,0100001110,0100001101, 0100001011,0011110000,0011101000,0011100100, 0011100010,0011010100,0011010010,0011001100, 0011001010,0010111000,0010110010,0010110001, 0010101100,0010101001,0010100110,0010100101, 0010100011,0010011100,0010011010,0010010110, 0010010101,0010010011,0010001110,0010001101, 0010001011,0001111000,0001110010,0001110001, 0001101100,0001101001,0001100110,0001100101, 0001100011,0001011100,0001011010,0001010110, 0001010101,0001010011,0001001110,0001001101, 0001001011
5	11011010, 11011001, 11010110, 11010101, 11010011, 11001110, 11001101, 11001011, 10111010, 10111001, 10110110, 10110101, 10110011, 10101110, 10101101, 10101011, 10011110, 10011101, 10011011, 01111010, 01111001, 01110110, 01110101, 01110011, 01101110, 01101101, 01101011, 01011110, 01011101, 01011011.

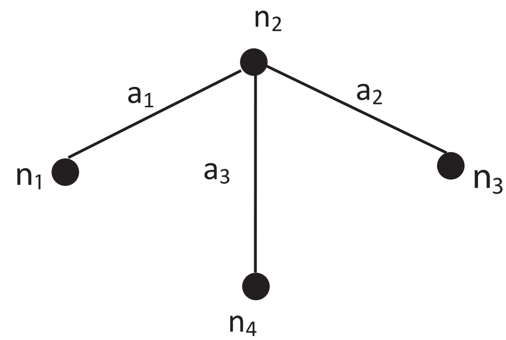

Figure 6. Spanning tree (11100).

The paths and the flow requirement of each path and the total flow carried by each link in the spanning tree are shown in Tables 3 and 4 respectively.

The spanning tree with flow is (6360 0); achieved by replacing 1 with the flow carried by the link in the spanning tree without flow (1 1 1 0 0). The obtained spanning trees with flow to the network given in Figure 1 are shown in Table 5.

4. Using Spanning Trees with Flow for Evaluating Reliability

The reliability can be evaluated using spanning trees with flow by considering each span as a capacity vector and applying the Inclusion - exclusion rule[29].

If $X^1, X^2, \ldots, X^{nsp}$ are the capacity vectors, then the system reliability R is defined by

$$R = \Pr\{\bigcup_{i=1}^{nsp}\{Y \mid Y \geq X^i\}\} \tag{1}$$

Where $\Pr\{Y\} = \Pr\{y_1\}\cdot\Pr\{y_2\}\cdot\ldots\cdot\Pr\{y_n\}$

The inclusion-exclusion rule is as follows:

If $A_1 = \{Y \mid Y \geq X^1\}$, $A_2 = \{Y \mid Y \geq X^2\}$, ..., $A_{nsp} = \{Y \mid Y \geq X^{nsp}\}$, then apply the inclusion - exclusion rule to calculate **R** using the following relationship**:**

$$R = \sum_i \Pr\{A_i\} - \sum_{i \neq j} \Pr\{A_i \cap A_j\} + \sum_{i \neq j \neq k} \Pr\{A_i \cap A_j \cap A_k\} - \ldots + (-1)^{nsp-1}\Pr\{A_1 \cap A_2 \cap \ldots \cap A_{nsp}\} \tag{2}$$

Note: It is not necessary to check that each capacity vector is minimal because the network is acyclic[30]. Otherwise, use[12] to remove non - minimal vectors.

Table 4. Paths and flow requirement to the spanning (11100)

Nodes pair	Path	Flow required
(1, 2)	$\{a_1\}$	2
(1, 3)	$\{a_1, a_2\}$	1
(1, 4)	$\{a_1, a_3\}$	3
(2, 3)	$\{a_2\}$	1
(2, 4)	$\{a_3\}$	2
(3, 4)	$\{a_2, a_3\}$	1

Table 5. Total flow carried by each link in the spanning tree (11100)

Link	Total Flow
a_1	2+1+3=6
a_2	1+1+1 = 3
a_3	3+2+1=6
a_4	0
a_5	0

4.1 Example

The spanning trees with flow have been obtained for the network example shown in Figure 1. Each span in Table 6 serves as a capacity vector. Both the capacity and the corresponding probability for each link are shown in Table 7. We now possess all of the information needed to calculate R using the expressions mentioned in Section 4.

According to Table 6, there are eight capacity vectors:

X1 = (6 3 6 0 0), X2=(6 3 0 6 0), X3 =(6 7 0 0 6), X4 =(6 0 6 0 3), X5 = (5 0 0 7 3),X6 =(0 3 6 6 0), X7=(0 5 0 6 6), and X8 = (0 0 5 6 3)

Set $A_1 = \{X \mid X \geq X^1\}$, $A_2 = \{X \mid X \geq X^2\}$ and $A_8 = \{X \mid X \geq X^8\}$

Then, $\Pr\{A_1\} = \{X \mid X \geq (6\,3\,6\,0\,0)\}$, $\Pr\{A_2\} = \Pr\{X \mid X \geq (6\,3\,0\,6\,0)\}$ and $\Pr\{A_8\} = \{X \mid X \geq (0\,0\,5\,6\,3)\}$. The values of some different terms in equation 2:

$$\sum_{i=1}^{8} \Pr\{A_i\} = 3.580892$$

$$\sum_{i \neq j}^{8} \Pr\{A_i \cap A_j\} = 6.142880$$

Table 6. The spanning trees with flow.

Spanning trees without flow	Spanning trees with flow
(11100)	(6 3 6 0 0)
(11010)	(6 3 0 6 0)
(11001)	(6 7 0 0 6)
(10101)	(6 0 6 0 3)
(10011)	(5 0 0 7 3)
(01110)	(0 3 6 6 0)
(01011)	(0 5 0 6 6)
(00111)	(0 0 5 6 3)

Table 7. Capacities and corresponding probabilities of each link

a_n	0	1	2	3	4	5	6
a_1	0.001	0.001	0.003	0.004	0.005	0.005	0.006
a_2	0.001	0.003	0.003	0.004	0.005	0.007	0.007
a_3	0.002	0.002	0.003	0.006	0.007	0.007	0.010
a_4	0.001	0.001	0.002	0.003	0.005	0.008	0.010
a_5	0.001	0.002	0.009	0.012	0.020	0.040	0.050
7	8	9	10	11	12	13	14
0.007	0.010	0.015	0.060	0.150	0.733	0.000	0.000
0.008	0.009	0.010	0.943	0.000	0.000	0.000	0.000
0.012	0.015	0.017	0.919	0.000	0.000	0.000	0.000
0.011	0.012	0.015	0.015	0.016	0.020	0.025	0.856
0.060	0.806	0.000	0.000	0.000	0.000	0.000	0.000

$$\sum_{i \neq j \neq k}^{8} \Pr\{A_i \cap A_j \cap A_k\} = 7.721368$$

$$\vdots$$

$$\Pr\{A_1 \cap A_2 \cap \ldots \cap A_8\} = 0.097776$$

Thus, the system reliability (R) = 0.992997

5. Conclusions

We present a successful new method for generating spanning trees without flow using links. By generating paths to the spanning tree without flow, a spanning tree with flow can be obtained for a given flow network. Finally, system reliability has been evaluated in terms of spanning trees with flow. Network examples have been used to illustrate efficiency of the presented methods.

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