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Survivable Network Design Problem in the Case of Arc Failure

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Abstract

This paper is concerned with the problem of designing a survivable capacitated fixed-charge network. This problem is about selecting a set of fixed-charge arcs inorder to build a survivable network with the cost being kept to a minimum and the flow requirements being satisfied. The system is subject to failure and capacity restriction. Failure is defined as simultaneous failure of arcs. To solve the problem, first a mixed integer linear program is introduced. Then, an appropriate Benders Decomposition (BD) method is developed. Finally, in order to reduce the number of iterations of using the BD approach, a new strategy is proposed on the basis of s-t cut theorem. Using this strategy, the length of time required to solve the problem was reduced by 35% on average.

Keywords: Benders Decomposition, Network Design, Survivable

1. Introduction

Network design problems arise in telecommunications, transportation, logistics, power systems, and production planning[1–3]. In many practical situations, a network should be designed in such a way that it is survivable and can satisfy the pertinent demands. The concept of survivability in network design can be defined as the continued ability of the network to remain operational in the face of component failures. Failure can occur as a result of errors in the routing software or facility of the network or in consequence of lost network facilities. When some components of the network fail, it is quite difficult to update the network so that it is cost-efficient and can do operational tasks. In the previous literature there are several network modeling approaches which take account of survivability requirements[4–6].

In designing a large number of practical networks, such as airlines, shipping companies, pipelines, and electrical lines, we should pay a price in the form of "fixed" costs for using arcs or installing facilities[7–9]. This problem is known as Fixed-Charge Network Design Problem (FNDP). In other words, we must incur fixed cost for each arc if the arc is part of the solution.

A large number of approaches have been proposed to solve FNDP. One of the most powerful approaches in this regard is the "Benders Decomposition" (BD) method. This method is based on the idea of decomposing the structure of the problem and dividing it up in to a integer Master Problem (MP) and several smaller linear problems, called sub-problems. Two advantages of the BD method over other approaches are comparative ease of solving sub-problems and more power in solving large-scale problems. The present paper considers a direct network as a set of nodes and potential capacitated arcs. The network has a known origin, destination, and demand, which is the amount of flow that should be sent from origin to destination. Associated with each arc is a construction cost and a failure cost, both of which are assumed to be non-negative. We consider a set of arcs that could simultaneously fail and lose a part of their capacity. We call each set of failing arcs a failure scenario. The Survivable Capacitated Fixed-Charge Network Design Problem (SCFNDP) that we consider in this paper attempts to design a network

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that minimizes the total cost of construction and failure such that a feasible flow exists in the event of any failure scenario. Much research has been performed in this area, but here we will mention only a few. Sridhar[10] introduced a Benders-and-cut algorithm for a capacitated fixed-charge network design problem which incorporated both Benders and polyhedral cuts into an implicit branch-and-bound algorithm. Georgios et al.[11] presented a new method to reinitialize the Benders master problem for a fixed-charge network problem using a series of valid inequalities. Gendron[12] focused on three methods used to solve large-scale instances of the multi-commodity capacitated fixed-charge network design problem. These approaches were cutting-plane, Benders decomposition, and Lagrangian relaxation.

Costa[13] presented a review of the BD approach applied to fixed-charge network design problems. Luss and Wong[14] proposed simple heuristic algorithms that ensure survivability under scenarios of a single failure of a link on a node. Atamturka and Rajanb[15] studied the design of capacitated survivable networks using directed p-cycles and employed a branch-and-cut algorithm to solve the problem.Bley et al.[16] considered a location problem with survivability constraints and proposed a branch-and-cut algorithm on the basis of the BD method in order to obtain an optimal solution. Ljubi etal.[17] introduced stochastic survivable network design problems with finite scenarios. They used a two-stage branch-and-cut algorithm for solving the decomposed model to obtain optimality. Garg and Cole[18] considered the design of a survivable multi commodity flow network with failure scenarios. They were mainly concerned with the development of a BD algorithm for solving this problem and explored the various models for the Benders master problem.

The rest of the paper is organized as follows. Section 2 introduces a mixed integer programming model for designing survivable capacitated fixed-charge networks and proposes an approach on the basis of the BD method in order to solve the model. In Section 3, two theorems are stated in order for the optimal solution to a certain scenario to be the optimal solution to the original problem. This section also presents a procedure for scenario selection which uses the BD method in order to faster obtain the optimal solution to the original problem. Computational results are given in Section 4. The paper ends with a summary of the main conclusions and suggestions future research in section 5.

2. SurvivableFixed-Charge Capacitated Network Design Model

We have a directed graph $G = (N, A)$ where N is the set of nodes and A is the set of arcs. Associated with each arc $(i, j) \in A$ is a construction cost c_{ij} and a failure cost c_{ij}^f, both of which are assumed to be non-negative. Each arc (i, j) has a capacity $u_{ij} > 0$ Let d denote system demand, which is the amount of flow that should be sent from origin to destination .A flow that satisfies origin-destination demands, conservative constraints, and arc capacity requirements is called a feasible flow.

We consider sets of arcs that could simultaneously fail and lose part of their capacity. We called each set of failing arcs a failure scenario.

The SCFNDP seeks to design the required flow network which minimizes the total cost of construction and failure such that a feasible flow exists in the event of any failure scenario.

To formulate this problem, p_k $k = 1, 2, \ldots, K$

be K fraction numbers in interval (0, 1) f_{p_k} and a failure scenario. That is, f_{p_k} is a set of arcs that could simultaneously fail and lose the $(1 - p_k)$ th of their capacity. Let $\bar{u}_{ij}$ be the capacity of arc (i, j) after failure. Therefore, $f_{p_k} = \left\{(i, j) \middle| (i, j) \in A \ \& \ \bar{u}_{ij} = p_k u_{ij}\right\}$ We define $F = \left\{f_{p_k} \middle| k = 1, 2, \ldots, K\right\}$ To mathematically represent the problem, we introduce the following notations:

$N^+(i) = \{(i, j) \in A | j \in N\}$: set of arcs that emanate from node i.

$N^-(i) = \{(j, i) \in A | j \in N\}$: set of arcs that terminate at node i.

$x_{ij}^{p_k}$: The amount of flow on arc (i, j) when the scenario f_{p_k} happens.

$$w_{ij}^{p_k} = \begin{cases} p_k & (i, j) \in f_{p_k} \\ 1 & (i, j) \notin f_{p_k} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & if\ arc\ (i,j)\ is\ constructed \\ 0 & o.w \end{cases}$$

The model can be stated as follows:

$$(MP) \quad \min . \sum_{(i,j)\in A} (c_{ij}^{f} + c_{ij}) y_{ij} \tag{1}$$

$s.t$

$$x_{ij}^{p_k} \le y_{ij}\, w_{ij}^{p_k}\, u_{ij} \qquad \forall f_{p_k} \in F, \forall (i,j) \in A \tag{2}$$

$$\sum_{(i,j)\in N^+(s)} x_{ij}^{p_k} - \sum_{(i,j)\in N^-(s)} x_{ij}^{p_k} = d \qquad \forall f_{p_k} \in F \tag{3}$$

$$\sum_{(i,j)\in N^+(t)} x_{ij}^{p_k} - \sum_{(i,j)\in N^-(t)} x_{ij}^{p_k} = -d \qquad \forall f_{p_k} \in F \tag{4}$$

$$\sum_{(i,j)\in N^+(v)} x_{ij}^{p_k} - \sum_{(i,j)\in N^-(v)} x_{ij}^{p_k} = 0 \qquad \forall v \in N - \{s,t\}, \forall f_{p_k} \in F \tag{5}$$

$$y_{ij} \in \{0,1\} \quad \forall (i,j) \in A \tag{6}$$

$$x_{ij}^{p_k} \ge 0 \quad \forall f_{p_k} \in F, \forall (i,j) \in A \tag{7}$$

The objective function (1) represents the total cost of arc construction and failure which has to be minimized. Constraints (2) imply that flow exists on an arc if the arc has been constructed, and limit the flow on the arc to be no more than its capacity in each failure scenario. Constraints (3) and (4) guarantee that the demand between each origin-destination pair is satisfied in a failure scenario. Constraints (5) show the flow balance for the other nodes (except the origin-destination node).

Given a failure scenario $f_{p_k} \in F$, Problem (1)-(7) may be written as:

$$\min \sum_{(i,j)\in A} (c_{ij}^{f} + c_{ij}) y_{ij} + 0 x_{ij}^{p_k}$$

$$x_{ij}^{p_k} \le y_{ij}\, w_{ij}^{p_k}\, u_{ij} \qquad \forall (i,j) \in A$$

$$\sum_{(i,j)\in N^+(s)} x_{ij}^{p_k} - \sum_{(i,j)\in N^-(s)} x_{ij}^{p_k} = d$$

$$\sum_{(i,j)\in N^+(t)} x_{ij}^{p_k} - \sum_{(i,j)\in N^-(t)} x_{ij}^{p_k} = -d$$

$$\sum_{(i,j)\in N^+(v)} x_{ij}^{p_k} - \sum_{(i,j)\in N^-(v)} x_{ij}^{p_k} = 0 \qquad \forall v \in N - \{s,t\}$$

$$y_{ij} \in \{0,1\} \qquad \forall (i,j) \in A$$
$$x_{ij}^{p_k} \geq 0 \qquad \forall (i,j) \in A$$

Now let us define $Y = \left\{ y_{ij} \mid \left(i,j\right) \in A \;\&\; y_{ij} \in \{0,1\} \right\}$ and suppose that $\overline{y}_{ij}$, $\left(i,j\right) \in A$ are fixed. The following problem can be solved:

$$\begin{aligned}
&\min \quad 0x_{ij}^{p_k} \\
&x_{ij}^{p_k} \leq \overline{y}_{ij}\, w_{ij}^{p_k}\, u_{ij} \\
&\sum_{(i,j)\in N^+(v)} x_{ij}^{p_k} - \sum_{(i,j)\in N^+(v)} x_{ij}^{p_k} = \begin{cases} d & v = s \\ 0 & v = N - \{s,t\} \\ -d & v = t \end{cases} \\
&x_{ij}^{p_k} \geq 0 \\
&\overline{y}_{ij} \in Y
\end{aligned}$$

Thus, we can express the Problem (1)-(7)

$$\min_{\overline{y}_{ij}\in Y}\left\{ \sum_{(i,j)\in A} (c_{ij}^f + c_{ij})\overline{y}_{ij} + \min_{x_{ij}^{p_k}\geq 0}\left\{ 0.x_{ij}^{p_k} \,\middle|\, x_{ij}^{p_k} \leq \overline{y}_{ij}\, w_{ij}^{p_k}\, u_{ij} \,,\, \sum_{(i,j)\in N^+(v)} x_{ij}^{p_k} - \sum_{(i,j)\in N^+(v)} x_{ij}^{p_k} = \begin{cases} d & v = s \\ 0 & v = N - \{s,t\} \\ -d & v = t \end{cases} ,\, x_{ij}^{p_k} \geq 0 \right\}\right\} \quad (8)$$

We introduce the dual variables β_{ij} and π_i for the inner minimization problem and write its dual as:

$$(SP)\ \min. \sum_{(i,j)\in f_{p_k}} \beta_{ij} u_{ij} p_k y_{ij} + d(\pi_s - \pi_t) \tag{9}$$

$$\begin{aligned}
&s.t. \\
&\beta_{ij} + \pi_i - \pi_j \geq 0 \\
&\beta_{ij} \geq 0
\end{aligned}$$

Because the objective function is zero, and with respect to duality theory, we use "min" instead of "max" in the objective function of the sub-problem. Problem (9) is called a sub-problem (SP) of BD. Drawing upon the duality theory; we can rewrite Problem (8) as:

$$\min_{Y}\left\{ \sum_{(i,j)\in A} (c_{ij}^f + c_{ij})y_{ij} + \min_{\substack{\beta_{ij}\geq 0 \\ \pi_s}}\left\{ \sum_{(i,j)\in f_{p_k}} \beta_{ij} u_{ij} p_k y_{ij} + d(\pi_s - \pi_t) \mid \beta_{ij} + \pi_i - \pi_j \geq 0 \right\}\right\} \quad (10)$$

The feasible space of SP should not be empty; otherwise, the primal problem is either infeasible or unbounded.

Suppose that $(z_\beta^p, z_\pi^p)^t \quad p = 1,2,\ldots,P$ are extreme points and $(v_\beta^q, v_\pi^q)^t \quad q = 1,2,\ldots,Q$

are the extreme directions of the feasible space of SP. The SP can take either bounded or unbounded value. In the unbounded case, there is a direction $(v_\beta^q, v_\pi^q)^T$ such that . $(u_{ij} p_k y_{ij}, d)(v_\beta^q, v_\pi^q)^t < 0$ The unboundedness of SP makes the overall problem infeasible. To prevent the unbounded case, we add the following restrictions:

$$(u_{ij} p_k y_{ij}, d)(v_\beta^q, v_\pi^q)^t \geq 0 \qquad q = 1,2,\ldots,Q$$

When SP is bounded, every solution is a linear combination of extreme points only. Thus, we can rewrite Problem (10) as:

$$\min_{Y} . \left\{ \sum_{(i,j)\in A} (c_{ij}^f + c_{ij}) y_{ij} + \min_{1\leq p\leq P} \left\{ (u_{ij} p_k y_{ij}, d)(z_\beta^p, z_\pi^p)^t \mid p = 1,2,\ldots,P \right\} \right\} \tag{11}$$
$$s.t.$$
$$(u_{ij} p_k y_{ij}, d)(v_\beta^q, v_\pi^q)^t \geq 0 \qquad q = 1,2,\ldots,Q$$

By introducing the auxiliary variable T to the inner optimization problem, Problem (11) can be rewritten as below:

$$\min . \sum_{(i,j)\in A} (c_{ij}^f + c_{ij}) y_{ij} + T \tag{12}$$
$$s.t.$$
$$T \leq (u_{ij} p_k y_{ij}, d)(z_\beta^p, z_\pi^p)^t \qquad p = 1,2,\ldots,P \tag{13}$$
$$(u_{ij} p_k y_{ij}, d)(v_\beta^q, v_\pi^q)^t \geq 0 \qquad q = 1,2,\ldots,Q \tag{14}$$
$$y_{ij} \in Y \tag{15}$$

The problem above, (12)-(15), is a BD. In practical situations, there are typically too many extreme points and rays. This is a disadvantage of the above-said formulation. To overcome this weakness, we delay generating Constraints (13) and (14). Initially, only the last Constraint (15), is considered. Thus, the first MP is as follows:

$$\min \sum_{(i,j)\in A} (c_{ij}^f + c_{ij}) y_{ij} + T \tag{16}$$
$$s.t$$
$$y_{ij} \in Y$$

Once Problem (16) is solved, SP would be solved using a trial configuration of . If SP is unbounded, we add a constraint of type (14) to the MP. However, if it is bounded, a constraint of type (13) is added. We solve these problems iteratively until the optimality condition is obtained.

When we obtain an optimal solution for a certain scenario, we use it in the SP for another scenario. Ifthe optimal value for the objective function of SP is equal to zero for each failure scenario, we obtain an optimal solution to the original problem; otherwise, we apply this solution to the SP of another scenario and obtain an optimal solution. We iteratively do this process until we obtain an optimal solution to the original problem.

3. Optimality Conditions for a Certain Scenario and Modified BD

First, we regard a certain scenario as fixed. At the first iteration of BD, the MP is

$$\min \sum_{(i,j)\in A} (c_{ij}^{f} + c_{ij}) y_{ij} + T \quad (17)$$

$$s.t$$

$$y_{ij} \in \{0,1\}$$

and we obtain $y_{ij} = 0 \quad \forall (i,j) \in A$.

As constraints of Forms (13) and (14) are added to the model, we would eventually build a set of paths from the origin to the destination. However, this initial process will result in several iterations. In order to find a better initial solution, we consider one of the scenarios and find a path from the origin to the destination. Finding this path helps us reduce the number of MP iterations. Of course, the choice of scenario also affects this reduction. We define the capacity of a path Q under scenario f_k as follows:

$$u(Q, f_k) = \min\left\{ w_{ij}^{p_k} u_{ij} \,|\, (i,j) \in Q \right\} \quad (18)$$

Suppose that $Q^* = \left(Q_1^*, Q_2^*, \ldots, Q_t^*\right)$ is an optimal solution to the problem. This solution consists of several paths from the origin to the destination with several arcs in common. Now, the following theorem is stated in order to decide on a scenario the optimal solution to which is the optimal solution to the original problem.

Theorem 1 :If the optimal solution under scenario f_k is Q^* with the capacity of paths not decreasing in the other scenario, then Q^* is the optimal solution to the original problem.

Proof: The optimal paths do not include any cycle. Otherwise, we can find a better solution without passing through the cycles. Without loss of generality, suppose that the optimal solution under scenario f_{p_1} is Q_1^* , which has the aforementioned property and suppose that Q^* ($Q_1^* \neq Q^*$) is the optimal solution to the original problem. Since the capacity of Q_1^* does not decrease under any other scenario, satisfies the network demand under other scenarios. The solution is an optimal solution if it satisfies the destination demand and is with minimum cost. Since Q^* is an optimal solution, the following condition should obtain in the scenarios except f_{p_1} :

$$\sum_{(i,j)\in Q^*} (c_{ij} + c_{ij}^{f}) < \sum_{(i,j)\in Q_1^*} (c_{ij} + c_{ij}^{f}) \quad (19)$$

Now, we consider Q^* under scenario f_{p_1} . Q^* satisfies the network demand under f_{p_1} and it is optimal. Thus,

$$\sum_{(i,j)\in Q^*} (c_{ij} + c_{ij}^f) < \sum_{(i,j)\in Q_1^*} (c_{ij} + c_{ij}^f) \tag{20}$$

Now, let us state a definition which can be used to find a failure scenario which helps us find the optimal solution to the original problem.

Before stating Theorem 2, we define the capacity of an s-t cut $(s, \overline{s})$ under as f_{p_k}

$$U_{p_k}(s,\overline{s}) = \sum_{(i,j)\in(S,\overline{S})} p_k u_{ij} \tag{21}$$

And also define $Z_{f_{p_k}}$ as the capacity of the minimum cut:

$$Z_{f_{p_k}} = \min_{(s,\overline{s})} \left\{ \sum_{(i,j)\in(S,\overline{S})} p_k u_{ij} \right\} \tag{22}$$

Theorem 2: The optimal solution under f_{p_k}, which is $Q_{p_k}^*$, is optimal to the original problem if and only if the capacity of every s-t cut of $Q_{p_k}^*$ under other scenarios is greater than or equal to the system demand.

Without loss of generality, suppose that $Q_{p_1}^*$ is the optimal solution under scenario f_{p_1} such that the capacity of every s-t cut of $Q_{p_1}^*$ under other scenarios is greater than or equal to the demand. Suppose that Q^* is the optimal solution to the original problem and that $Q_{p_1}^* \neq Q^*$ Since is the optimal solution under f_{p_1} ,it follows that

$$\sum_{(i,j)\in Q_{p_1}^*} (c_{ij} + c_{ij}^f) < \sum_{(i,j)\in Q^*} (c_{ij} + c_{ij}^f) \tag{23}$$

Now, we consider $Q_{p_1}^*$ under another scenario, say f_{p_2} . $Q_{p_1}^*$ is not the optimal solution under f_{p_2} .Since the capacity of every s-t cut of $Q_{p_1}^*$ under f_{p_2} is greater than or equal to the demand, the destination demand is satisfied. Therefore,

It can be seen that (23) is in contradiction with (24).

$$\sum_{(i,j)\in Q^*} (c_{ij} + c_{ij}^f) < \sum_{(i,j)\in Q_{p_1}^*} (c_{ij} + c_{ij}^f) \tag{24}$$

It can be seen that (23) is in contradiction with (24).

Now, suppose that the optimal solution under scenario f_{p_1} (which is $Q_{p_1}^*$) is the optimal solution to the the original problem. Without loss of generality, we consider $Q^*_{p_1}$ under scenario f_{p_2} and suppose that there is an s-t cut under f_{p_2} such that the capacity of $Q^*_{p_1}$ on this cut is less than the demand. Thus, $Q^*_{p_1}$ under f_{p_2} cannot satisfy the destination demand, indicating that $Q^*_{p_1}$ is not the optimal solution to the original problem.

Finally, note that this scenario may be difficult or even impossible to identify. We define $f_{\min}$ as a failure scenario that has the least s-t cut of all scenarios. In order to find a scenario which would allow us to obtain the optimal solution to the original problem at fewer iterations, we suggest to start the BD approach with $f_{\min}$ .We chose this scenario because it has the least s-t cut of all scenarios and the capacity of every s-t cut of its optimal solution is least likely to decrease under other scenarios.

We find the optimal solution under $f_{\min}$. This solution consists of several paths from the origin to the destination which can have several arcs in common. If the optimal solution to this scenario is optimal to the original problem (i.e., it satisfies the conditions of Theorem 2), the original problem is solved. Otherwise, we put the optimal solution obtained under $f_{\min}$ into others scenarios.

4. Computational Results

In this section we compare two approaches in terms of the length of time it takes them to obtain the optimal solution. The first approach only uses the BD method. However, the second approach, as suggested above, starts the BD method with $f_{\min}$.

For comparison purposes, four networks are created with 10, 15, 20, and 25 nodes. Four failure scenarios are defined for each network. We select Node 1 in each network as the origin and the last node as the destination.

The size of each problem and the demand pertinent to each problem are so chosen that it is possible to show the impact of the number of nodes on computational time of the two approaches above. We set the target arc density to 0.5, which is the ratio of the total number of network arcs to those arcs that can exist in the network. The capacity, construction cost, and failure cost of each arc are randomly set between 1 and 100 according to a uniform probability distribution. Demands are considered to be in the range of 80-90% of $f_{\min}$. Finally, four failure scenarios are defined depending on the percentage of lost capacity: 80%, 50%, 60%, and 20%. The results are summarized in Table 1.

Table 1. Comparison of computational time (in seconds) required by each approach

	10 nodes	15 nodes	20 nodes	25 nodes
BD	38	58	344	918
$f_{\min}$ and BD	21	31	135	327

5. Conclusion

This research aims to develop an effective algorithm for solving survivable network design problems. First, a mixed integer linear program which is NP-hard is formulated. To solve this problem, we employ the BD approach, which is significantly better than mixed integer stated in order to obtain the optimality condition for a certain scenario. To reduce the number of iterations of using the BD approach, a scenario with minimum capacity of s-t cut is used. Further research in this area could develop a branch-and-price-and-cut algorithm for survivable network design problems. We believe that the obtained results can be useful for those actual instances of network arc failure.

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