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Optimal Payment Policy with Preservation Technology Investment and Shortages under Trade Credit

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Abstract

Deterioration rate is assumed as uncontrolled variable but through preservation techniques the deteriorating nature of the items can be controlled up to certain level. Hence to study the effect of preservation techniques on inventory control system. In this paper we have developed a mathematical model with preservation technology investment for deteriorating inventory. The whole study is carried out under the effect of inflation and trade credit. The demand rate is directly dependent on time and partially backlogged shortages are permitted in this model. Our main objective of this study is to find optimal payment time and optimal vale of total cost. Numerical illustration and sensitivity analysis is given at the end of this paper. All the calculations are done with the help of mathematical software Mathematica7.

Keywords: Demand Rate, Partially Backlogged Shortages, Preservation Technology Investment, Trade Credit

1. Introduction

The continuous degradation in quality of a product is known as deterioration. The concept of deterioration is first introduced by Ghare and Schrader[11]. They have studied the constant rate of deterioration. Further many researchers have focused on deteriorating inventory like Shah and Jaiswal[28], Padmanabhana and Vratb[22], and Yang and Wee[36] etc.

In literature deterioration rate is assumed as a uncontrolled variable. But in realistic conditions deteriorating nature can be controlled up to a certain level. It is only possible though the preservation techniques. Preservation can be done through procedural changes or through a specialized equipment acquisition. Due to its significance as well as social and environment changes Preservation techniques becomes very important in deteriorating inventory systems. In literature the effect of preservation techniques has been studied by Leea and Dye[20]. Mishra[21] is another who has incorporated the concept of preservation technology investment for deteriorating inventory.

Demand rate plays a crucial role in study of deteriorating inventory. The concept of constant demand rate has been studied by Chung and Lin[8]. Resh, Friedman, Barbosa[25] are the first who have incorporated the concept of linear trends in demand. Further many authors have focused on this concept like Donaldson[10], Giri, Chakrabarty, Chaudhuri[13], Chang, Hung, Dye[3], Chu and Chen[7], Balkhi[1], Teng and Yang[35], Jaggi and Verma[16], Singh and Singh[32] etc.

Due to excess demand stock level reaches at zero. In such conditions suppliers try to retain the customers for this they have considered the partially backlogged shortages. Shortage conditions have been discussed by several researchers like Singh and Singh[31], Singh, Kumari, Kumar[29], Hung[15], Pentico and Drake[24], Taleizadch and Pentico[33], Ghiami, Williams, Wu[12] etc.

After 1970's it is observed that many countries are suffered from inflation and time value of money. Effect of inflation has been introduced first by Buzacot[2]. After his work several researchers have implemented their work in different ways. We can find other interesting ideas

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in Dye, Mandal, Maiti[9], Hsieh and Dye[14], Sarkar, Sana, Chaudhuri[26,27], Patra and Ratha[23] etc.

To attract more customers supplier provides a period to settle the account i.e. permissible delay in payment (Trade Credit). After this period an interest is charged on unsold items. For literature review we can go through the work of researchers like Teng, Chang, Goyal[34], Kumari, Singh, Kumar[19], Kumar, Tripathy, Singh[18], Chang, Wu, Chen[4], Chen and Kang[5], Chen and Cheng[6], Jaggi, Goel, Mittal[17], Singh, Kumari, Kumar[30], Zhou, Zhong, Li[38] and Yadav, Singh, Kumari[37] etc.

In this paper we have formulated a mathematical model for deteriorating inventory with partially backlogged shortages and demand as well as holding cost both are treated as linear function of time. Further we have studied the effect of preservation technology on deterioration rate under the effect of inflation and trade credit. We have divided this article in six different sections. In the second assumptions and notation are given for mathematical model formulation which elaborated in the third section. Numerical illustration and sensitivity analysis is mentioned in fourth and fifth sections respectively. We have concluded our model in the sixth section.

2. Assumptions and Notations

We have used following assumptions and notations for the formulation of mathematical model.

2.1 Assumptions

(1) The demand is treated as linear function of time as follows
$D(t) = \alpha + \beta t;\ \alpha > 0,\ \beta > 0.$
(2) Partially backlogged shortages are permitted.
(3) Holding cost is time varying i.e. $h(t)= h_1+h_2 t$, where $h_1, h_2>0$
(4) Replenishment is instantaneous.
(5) Infinite time horizon.
(6) Lead time is zero.
(7) Preservation Technology is used to reduce deterioration rate.
(8) Deterioration rate is constant.
(9) Inflation rate is constant.
(10) Suppliers permit delay (M) in payment to a purchaser, during this period purchaser deposits their revenue in a interest bearing account. At the end of delay period they have two choices that is they can pay the amount at the end of delay period or after the delay period (payment time between M and T_1). Supplier charges high interest for unsold times when purchaser choose the payment time (K) between M and T_1.

2.2 Notations

A: The unit ordering cost
p: The unit selling price
C: The unit purchasing cost
C_1: The unit backorder cost
C_2: The unit lost sale cost
h(t): The time varying holding cost excluding interest charges
θ: The deterioration rate
m(ξ): Reduced deterioration rate due to use of Preservation Technology and $m(\xi)= \theta\,(1- e^{-a\xi})$ where a>0.
ξ: Preservation Technology (PT) cost to preserve the product through which deterioration rate is also reduced ξ >0.
τ_p: Resultant Deterioration rate, $\tau_p= (\theta - m(\xi))$.
r: constant represents the difference between discount rate and inflation rate, where $0 \le r < 1$.
I_e: interest earned per \$ per year.
I_p: Interest paid by purchaser per \$ in stock per year, which is charged by supplier.
M: Permissible delay in payment (i.e., trade credit for purchaser to settle the account).
K: Payment time.
T: The length of cycle time.
T_1: The time at which shortages occurs.
$I_1(t)$: Inventory level during time period $[0, T_1]$.
$I_2(t)$: Inventory level during time period $[T_1, T]$.
Q: IM+IB, The order quantity during cycle length [0, T].
IM: Maximum inventory level during cycle time $[0, T_1]$.
IB: Maximum inventory level during shortage period $[T_1, T]$.
$TC_2(T, K, \xi)$: Present worth of Total relevant cost per time unit, when $M \le K \le T_1$ and $T_1=\gamma T$ where $0<\gamma<1$.
$TC_1(T, \xi)$: Present worth of Total relevant cost per time unit, when $T_1 \le M$.
Note: The total relevant cost includes following costs

(1) Cost of placing order(OC)
(2) Cost of purchasing(PC)
(3) Holding cost excluding interest charges(HC)
(4) Backordered cost (BC)

(5) Lost sales cost (LC)
(6) Interest paid for unsold times at initial time or after the permissible delay M (IP).
(7) Interest earned from sales revenue during permissible delay in payment (IE).

3. Mathematical Model Formulation and Solution

The inventory level gradually depleted mainly due to demand and partially due to reduced deterioration during time $[0, T_1]$ and during $[T_1, T]$ shortages occurs and partially backlogged. Inventory depletion during cycle length $[0, T]$ is represented by following differential equation:

$$\frac{dI_1(t)}{dt} + \tau p I_1(t) = -(\alpha + \beta t); \qquad 0 \le t \le T_1$$

$$\frac{dI_1(t)}{dt} + (\theta - m(\xi)) I_1(t) = -(\alpha + \beta t); \qquad 0 \le t \le T_1 \qquad (1)$$

and

$$\frac{dI_2(t)}{dt} = -(\alpha + \beta t) e^{-\delta t}; \qquad T_1 \le t \le T \qquad (2)$$

With boundary conditions I_1 (t=0)=IM, I_1(t=T_1)=0 and I_2(t=T_1)=0.

Now solving (1), (2) and using boundary conditions, we get

$$I_1(t) = \left[\left(\alpha(T_1 - t) + (\beta - \alpha(\theta - m(\xi))) \left(\frac{T_1^2 - t^2}{2} \right) - \beta(\theta - m(\xi)) \left(\frac{T_1^3 - t^3}{3} \right) \right) (1 - (\theta - m(\xi))t) \right] \qquad (3)$$

$$I_2(t) = \left[\left(\alpha(T_1 - t) + (\beta - \alpha\delta) \left(\frac{T_1^2 - t^2}{2} \right) - \beta\delta \left(\frac{T_1^3 - t^3}{3} \right) \right) \right] \qquad (4)$$

Now the order quantity Q= IM + IB

At t=0 the inventory level is maximum. Hence

IM= I_1 (t=0) therefore from (3), we have

$$IM = \left[\left(\alpha(T_1) + (\beta - \alpha(\theta - m(\xi))) \left(\frac{T_1^2}{2} \right) - \beta(\theta - m(\xi)) \left(\frac{T_1^3}{3} \right) \right) \right] \qquad (5)$$

The maximum backordered inventory level is IB= $-I_2$ (t=T), therefore from (4), we have

$$IB = -\left[\left(\alpha(T_1 - T) + (\beta - \alpha\delta) \left(\frac{T_1^2 - T^2}{2} \right) - \beta\delta \left(\frac{T_1^3 - T^3}{3} \right) \right) \right] \qquad (6)$$

Therefore the order size Q is as follows:

$$Q = \left[\left(\alpha(T_1) + (\beta - \alpha(\theta - m(\xi))) \left(\frac{T_1^2}{2} \right) - \beta(\theta - m(\xi)) \left(\frac{T_1^3}{3} \right) \right) - \left(\alpha(T_1 - T) + (\beta - \alpha\delta) \left(\frac{T_1^2 - T^2}{2} \right) - \beta\delta \left(\frac{T_1^3 - T^3}{3} \right) \right) \right] \qquad (7)$$

The total relevant cost consists following cost parameters

1. The ordering cost (OC)= A (8)
2. The Purchase Cost (PC)= C ∗Q

$$PC = C \left[\left(\alpha(T_1) + (\beta - \alpha(\theta - m(\xi))) \left(\frac{T_1^2}{2} \right) - \beta(\theta - m(\xi)) \left(\frac{T_1^3}{3} \right) \right) - \left(\alpha(T_1 - T) + (\beta - \alpha\delta) \left(\frac{T_1^2 - T^2}{2} \right) - \beta\delta \left(\frac{T_1^3 - T^3}{3} \right) \right) \right] \qquad (9)$$

3. The Present worth of Holding cost(HC)

$$HC = \int_0^{T_1} (h_1 + t h_2) I_1(t) e^{-rt} dt$$

$$\begin{aligned} HC = \Bigg[& h_1 \left\{ \alpha \left(\frac{T_1^2}{2} \right) + \left(\beta - \alpha \left(\theta - m(\xi) \right) \right) \frac{T_1^3}{3} - \beta \left(\theta - m(\xi) \right) \frac{T_1^4}{4} \right\} \\ & + \left(h_2 - h_1 \left(\theta - m(\xi) + r \right) \right) \left\{ \alpha \left(\frac{T_1^3}{6} \right) + \left(\beta - \alpha \left(\theta - m(\xi) \right) \right) \frac{T_1^4}{8} \right. \\ & \left. - \beta \left(\theta - m(\xi) \right) \frac{T_1^5}{10} \right\} - h_2 \left(\theta - m(\xi) + r \right) \left\{ \alpha \left(\frac{T_1^4}{12} \right) \right. \\ & \left. + \left(\beta - \alpha \left(\theta - m(\xi) \right) \right) \frac{T_1^5}{15} - \beta \left(\theta - m(\xi) \right) \frac{T_1^6}{18} \right\} \Bigg] \end{aligned} \qquad (10)$$

4. Present worth of Backordered Cost (BC)

$$BC = -C_1 \int_{T_1}^{T} I_2(t) e^{-rt} dt$$

$$BC=-C_1\left[\left\{\alpha\left(T_1T-\frac{T_1^2}{2}-\frac{T^2}{2}\right)+(\beta-\alpha\delta)\left(\frac{T_1^2T}{2}-\frac{T_1^3}{3}-\frac{T^3}{6}\right)-\beta\delta\left(\frac{T_1^3T}{3}-\frac{T_1^4}{4}-\frac{T^4}{12}\right)\right\}-r\left\{\alpha\left(\frac{T_1T^2}{2}-\frac{T_1^3}{6}-\frac{T^3}{3}\right)+(\beta-\alpha\delta)\left(\frac{T_1^2T^2}{4}-\frac{T_1^4}{8}-\frac{T^4}{8}\right)-\beta\delta\left(\frac{T_1^3T^2}{6}-\frac{T_1^5}{10}-\frac{T^5}{15}\right)\right\}\right] \quad (11)$$

5. Lost sales cost (LC)

$$LC=C_2\int_{T_1}^{T}\left(1-e^{-\delta t}\right)(\alpha+\beta t)e^{-rt}dt$$

$$LC=C_2\left[\delta\left\{\alpha\left(\frac{T^2-T_1^2}{2}\right)+(\beta-r\alpha)\left(\frac{T^3-T_1^3}{3}\right)-r\beta\left(\frac{T^4-T_1^4}{4}\right)\right\}\right] \quad (12)$$

Now we will find Interest paid and earned by purchaser, for this there are two cases

(i) $T_1 < M$ (ii) $T_1 \geq M$. These two cases are graphically represented in Figure 1.

Case 1: $T_1 < M$ (M=K)

The permissible delay period M is greater than the total inventory depletion period i.e., T_1. Therefore there is no interest paid by purchaser to the supplier for the items. However purchaser will uses the sales revenue to earn interest at the rate of I_e during time period $[0, T_1]$ and interest from cash invested during period $[T_1, M]$. Therefore the total value of interest earned under effect of inflation is

$$IE_1=I_e p\left[\int_0^{T_1} e^{-rt}(\alpha+\beta t)\,t\,dt+T_1\int_{T_1}^{M} e^{-rt}(\alpha+\beta t)dt\right]$$

$$IE_1=I_e p\left[\left\{\alpha\left(\frac{T_1^2}{2}\right)+(\beta-r\alpha)\left(\frac{T_1^3}{3}\right)-r\beta\left(\frac{T_1^4}{4}\right)\right\}+T_1\left\{\alpha(M-T_1)+(\beta-r\alpha)\left(\frac{M^2-T_1^2}{2}\right)-r\beta\left(\frac{M^3-T_1^3}{3}\right)\right\}\right] \quad (13)$$

Therefore total relevant cost under the effect of inflation in first case is

$$TC_1(T, T_1, \xi) = [OC + PC + HC + BC + LC - IE_1]$$

$$TC_1(T_1,\xi)=\left[A+C\left[\left(\alpha(T_1)+\left(\beta-\alpha(\theta-m(\xi))\right)\left(\frac{T_1^2}{2}\right)-\beta(\theta-m(\xi))\left(\frac{T_1^3}{3}\right)\right)-\left(\alpha(T_1-T)+(\beta-\alpha\delta)\left(\frac{T_1^2-T^2}{2}\right)-\beta\delta\left(\frac{T_1^3-T^3}{3}\right)\right)\right]\right]$$

$$+\left[h_1\left\{\alpha\left(\frac{T_1^2}{2}\right)+\left(\beta-\alpha(\theta-m(\xi))\right)\frac{T_1^3}{3}-\beta(\theta-m(\xi))\frac{T_1^4}{4}\right\}+\left(h_2-h_1(\theta-m(\xi)+r)\right)\left\{\alpha\left(\frac{T_1^3}{6}\right)+\left(\beta-\alpha(\theta-m(\xi))\right)\frac{T_1^4}{8}-\beta(\theta-m(\xi))\frac{T_1^5}{10}\right\}-h_2(\theta-m(\xi)+r)\left\{\alpha\left(\frac{T_1^4}{12}\right)+\left(\beta-\alpha(\theta-m(\xi))\right)\frac{T_1^5}{15}-\beta(\theta-m(\xi))\frac{T_1^6}{18}\right\}\right]$$

$$-C_1\left[\left\{\alpha\left(T_1T-\frac{T_1^2}{2}-\frac{T^2}{2}\right)+(\beta-\alpha\delta)\left(\frac{T_1^2T}{2}-\frac{T_1^3}{3}-\frac{T^3}{6}\right)-\beta\delta\left(\frac{T_1^3T}{3}-\frac{T_1^4}{4}-\frac{T^4}{12}\right)\right\}-r\left\{\alpha\left(\frac{T_1T^2}{2}-\frac{T_1^3}{6}-\frac{T^3}{3}\right)+(\beta-\alpha\delta)\left(\frac{T_1^2T^2}{4}-\frac{T_1^4}{8}-\frac{T^4}{8}\right)-\beta\delta\left(\frac{T_1^3T^2}{6}-\frac{T_1^5}{10}-\frac{T^5}{15}\right)\right\}\right]$$

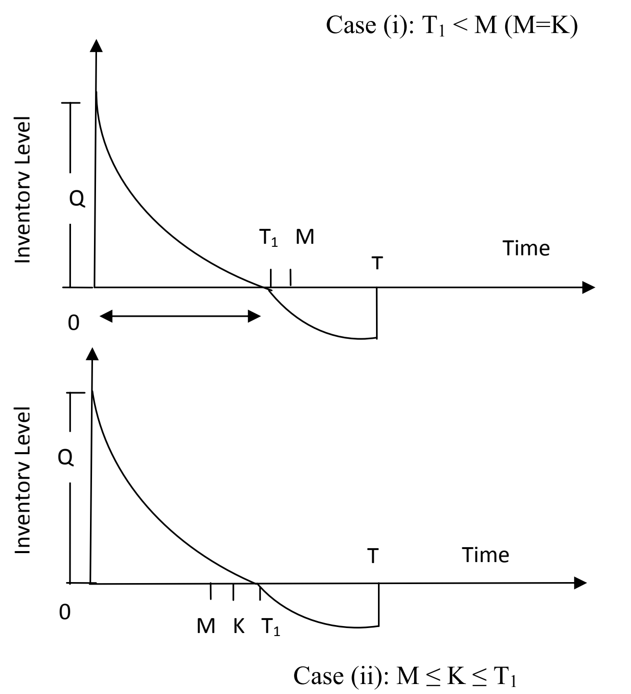

Figure 1. Case (i): $T_1 < M$ (M=K) and Case (i): $T_1 < M$ (M=K).

$$+C_2\left[\delta\left\{\alpha\left(\frac{T^2-T_1^2}{2}\right)+(\beta-r\alpha)\left(\frac{T^3-T_1^3}{3}\right)-r\beta\left(\frac{T^4-T_1^4}{4}\right)\right\}\right]$$

$$I_e p\left[\left[\left\{\alpha\left(\frac{T_1^2}{2}\right)+(\beta-r\alpha)\left(\frac{T_1^3}{3}\right)-r\beta\left(\frac{T_1^4}{4}\right)\right\}\right.\right.$$
$$+T_1\left\{\alpha(M-T_1)+(\beta-r\alpha)\left(\frac{M^2-T_1^2}{2}\right)\right.$$
$$\left.\left.\left.-r\beta\left(\frac{M^3-T_1^3}{3}\right)\right\}\right]\right] \quad (14)$$

Our objective is to minimize total cost TC_1 (T, ξ) where $T_1=\gamma T$; $0 < \gamma < 1$. The necessary conditions for minimization are

$$\frac{\partial TC_1(T,\xi)}{\partial T}=0\ \&\ \frac{\partial TC_1(T,\xi)}{\partial \xi}=0$$

Solving above equations we can find optimal values, T^*, $T_1^*=\gamma T^*$ and ξ^*. The optimal value of Total relevant cost is determined provided T^*, ξ^* must satisfy the sufficient conditions

$$\frac{\partial^2 TC_1(T^*,\xi^*)}{\partial T^2}>0\ \&\ \frac{\partial^2 TC_1(T^*,\xi^*)}{\partial \xi^2}>0$$

$$\left[\left(\frac{\partial^2 TC_1(T^*,\xi^*,)}{\partial T^2}\right)\left(\frac{\partial^2 TC_1(T^*,\xi^*,)}{\partial \xi^2}\right)\right]-\left[\frac{\partial^2 TC_1(T^*,\xi^*,)}{\partial T\partial \xi}\right]^2>0$$

Case 2: $M \le K \le T_1$

In this case the permissible delay period M expires before the total inventory depletion period T_1; hence purchaser will have to pay interest charged on unsold items during (M, K). Therefore Present worth of interest paid by purchaser is

$$IC_2=I_pC\int_M^K\left[I_1(t)e^{-rt}\right]dt$$

$$IC_2=I_pC\left[\left(\alpha\left(T_1(K-M)-\frac{(K^2-M^2)}{2}\right)\right.\right.$$
$$+\left(\beta-\alpha(\theta-m(\xi))\right)$$
$$\left(\frac{T_1^2(K-M)}{2}-\frac{(K^3-M^3)}{6}\right)$$
$$\left.\left.-\beta(\theta-m(\xi))\left(\frac{T_1^3(K-M)}{3}-\frac{(K^4-M^4)}{12}\right)\right)\right.$$
$$-(\theta-m(\xi)+r)\left\{\alpha\left(T_1\frac{(K^2-M^2)}{2}-\frac{(K^3-M^3)}{3}\right)\right.$$
$$+\left(\beta-\alpha(\theta-m(\xi))\right)\left(\frac{T_1^2(K^2-M^2)}{4}-\frac{(K^4-M^4)}{8}\right)$$
$$\left.\left.-\beta(\theta-m(\xi))\left(\frac{T_1^3(K^2-M^2)}{6}-\frac{(K^5-M^5)}{15}\right)\right\}\right]$$

(15)

Now the Present worth of interest earned during positive inventory and interest from invested cost is (This expression has been used by Chang et. Al[4]).

$$IE_2=I_e\left[p\int_0^{T_1}(\alpha+\beta t)te^{-rt}dt-CT_1\int_K^{T_1}(\alpha+\beta t)e^{-rt}dt+pK_1\int_K^{T_1}(\alpha+\beta t)e^{-rt}dt\right]$$

$$IE_2=I_e\left[p\left\{\alpha\left(\frac{T_1^2}{2}\right)+(\beta-r\alpha)\left(\frac{T_1^3}{3}\right)-r\beta\left(\frac{T_1^4}{4}\right)\right\}\right.$$
$$+(pK-CT_1)\left\{\alpha(K-T_1)+(\beta-r\alpha)\left(\frac{K^2-T_1^2}{2}\right)\right.$$
$$\left.\left.-r\beta\left(\frac{K^3-T_1^3}{3}\right)\right\}\right]$$

(16)

Hence the Present worth of total relevant cost is

$TC_2(T, T_1, K, \xi) = [OC + PC + HC + BC + LC + IC_2 - IE_2]$

$$TC_2(T,T_1,K,\xi)=\left[A+C\left[\left(\left(\alpha(T_1)+\left(\beta-\alpha(\theta-m(\xi))\right)\left(\frac{T_1^2}{2}\right)\right.\right.\right.\right.$$
$$\left.\left.-\beta(\theta-m(\xi))\left(\frac{T_1^3}{3}\right)\right)\right)-\left(\alpha(T_1-T)+(\beta-\alpha\delta)\left(\frac{T_1^2-T^2}{2}\right)\right.$$
$$\left.\left.\left.-\beta\delta\left(\frac{T_1^3-T^3}{3}\right)\right)\right]\right]$$

$$+\left[h_1\left\{\alpha\left(\frac{T_1^2}{2}\right)+\left(\beta-\alpha\left(\theta-m(\xi)\right)\right)\frac{T_1^3}{3}-\beta\left(\theta-m(\xi)\right)\frac{T_1^4}{4}\right\}\right.$$

$$+\left(h_2-h_1\left(\theta-m(\xi)+r\right)\right)\left\{\alpha\left(\frac{T_1^3}{6}\right)+\left(\beta-\alpha\left(\theta-m(\xi)\right)\right)\frac{T_1^4}{8}\right.$$

$$\left.-\beta\left(\theta-m(\xi)\right)\frac{T_1^5}{10}\right\}-h_2\left(\theta-m(\xi)+r\right)$$

$$\left.\left\{\alpha\left(\frac{T_1^4}{12}\right)+\left(\beta-\alpha\left(\theta-m(\xi)\right)\right)\frac{T_1^5}{15}-\beta\left(\theta-m(\xi)\right)\frac{T_1^6}{18}\right\}\right]$$

$$-C_1\left[\left[\left\{\alpha\left(T_1T-\frac{T_1^2}{2}-\frac{T^2}{2}\right)+(\beta-\alpha\delta)\left(\frac{T_1^2T}{2}-\frac{T_1^3}{3}-\frac{T^3}{6}\right)\right.\right.\right.$$

$$\left.-\beta\delta\left(\frac{T_1^3T}{3}-\frac{T_1^4}{4}-\frac{T^4}{12}\right)\right\}-r\left\{\alpha\left(\frac{T_1T^2}{2}-\frac{T_1^3}{6}-\frac{T^3}{3}\right)\right.$$

$$\left.\left.\left.+(\beta-\alpha\delta)\left(\frac{T_1^2T^2}{4}-\frac{T_1^4}{8}-\frac{T^4}{8}\right)-\beta\delta\left(\frac{T_1^3T^2}{6}-\frac{T_1^5}{10}-\frac{T^5}{15}\right)\right\}\right]\right]$$

$$+C_2\left[\delta\left\{\alpha\left(\frac{T^2-T_1^2}{2}\right)+(\beta-r\alpha)\left(\frac{T^3-T_1^3}{3}\right)-r\beta\left(\frac{T^4-T_1^4}{4}\right)\right\}\right]\Bigg]$$

$$I_pC\left[\left[\left(\alpha\left(T_1(K-M)-\frac{(K^2-M^2)}{2}\right)+\left(\beta-\alpha\left(\theta-m(\xi)\right)\right)\right.\right.\right.$$

$$\left(\frac{T_1^2(K-M)}{2}-\frac{(K^3-M^3)}{6}\right)-\beta\left(\theta-m(\xi)\right)$$

$$\left.\left(\frac{T_1^3(K-M)}{3}-\frac{(K^4-M^4)}{12}\right)\right)$$

$$-\left(\theta-m(\xi)+r\right)\left\{\alpha\left(T_1\frac{(K^2-M^2)}{2}-\frac{(K^3-M^3)}{3}\right)\right.$$

$$+\left(\beta-\alpha\left(\theta-m(\xi)\right)\right)\left(\frac{T_1^2(K^2-M^2)}{4}-\frac{(K^4-M^4)}{8}\right)$$

$$\left.\left.\left.-\beta\left(\theta-m(\xi)\right)\left(\frac{T_1^3(K^2-M^2)}{6}-\frac{(K^5-M^5)}{15}\right)\right\}\right]\right]$$

$$I_e\left[p\left\{\alpha\left(\frac{T_1^2}{2}\right)+(\beta-r\alpha)\left(\frac{T_1^3}{3}\right)-r\beta\left(\frac{T_1^4}{4}\right)\right\}\right.$$

$$+(pK-CT_1)\left\{\alpha(K-T_1)+(\beta-r\alpha)\left(\frac{K^2-T_1^2}{2}\right)\right.$$

$$\left.\left.-r\beta\left(\frac{K^3-T_1^3}{3}\right)\right\}\right]\Bigg] \tag{17}$$

To minimize total relevant cost, we differentiate $TC_2(T, K, \xi)$ w. r. t to T, K and ξ, where $T_1=\gamma T$; $(0<\gamma<1)$ and for optimal value necessary conditions are

$$\frac{\partial TC_2(T,\xi)}{\partial T}=0,\frac{\partial TC_2(T,\xi)}{\partial K}=0,\frac{\partial TC_2(T,\xi)}{\partial \xi}=0$$

Provided the determinant of principal minor of hessian matrix are positive definite, i.e. det(H1)>0, det(H2)>0,det(H3)>0 where H1, H2, H3 is the principal minor 0f the Hessian-matrix.

Hessian Matrix of the total cost function is as follows:

$$\begin{bmatrix} \frac{\partial^2 TC_2(T^*,K^*,\xi^*,)}{\partial T^2} & \frac{\partial^2 TC_2(T^*,K^*,\xi^*,)}{\partial T\partial K} & \frac{\partial^2 TC_2(T^*,K^*,\xi^*,)}{\partial T\partial \xi} \\ \frac{\partial^2 TC_2(T^*,K^*,\xi^*,)}{\partial K\partial T} & \frac{\partial^2 TC_2(T^*,K^*,\xi^*,)}{\partial K^2} & \frac{\partial^2 TC_2(T^*,K^*,\xi^*,)}{\partial K\partial \xi} \\ \frac{\partial^2 TC_2(T^*,K^*,\xi^*,)}{\partial \xi\partial T} & \frac{\partial^2 TC_2(T^*,K^*,\xi^*,)}{\partial \xi\partial K} & \frac{\partial^2 TC_2(T^*,K^*,\xi^*,)}{\partial \xi^2} \end{bmatrix}$$

4. Numerical Illustration

For the Illustration of proposed model we consider following inventory system in which values of different parameters in proper units are $A=250, \alpha=50, \beta=80, C=20, C_1=3, C_2=5, h_1=1.2,$ $h_2=0.005, \theta=0.35, \delta=0.1, a=2, \gamma=0.1, r=0.01,$ $I_e=0.18, I_p=0.2, p=30.$ there are two cases according to the permissible delay period as follows:

Case: 1 for T < M, M= 0.342466, Using mathematical software Mathematica7 the output results are as follows

$T^*=0.32761096$, $\xi^*=0.0100623$, $TC_1{}^*(T^*, \xi^*)=337.591$, $Q^*(T^*, \xi^*)=4.08813$

Case: 2 for M≤ K ≤ T, M= 0.2191781, Using mathematical software Mathematica7 the output results are as follows

$T^*=0.32522$, $K^*=0.22006$, $\xi^*=0.473581$, $TC_2{}^*(T^*, K^*, \xi^*)=327.961$, $Q^*(T^*, K^*, \xi^*)=4.03941$

5. Sensitivity Analysis

The study of effect of change of different parameters on total relevant cost, a sensitivity analysis is performed by varying some parameters like permissible period M, inflation rate r, demand factors (α, β), partial backlogging parameter δ and deterioration rate θ. We have changed value of one parameter at a time and taking other parameters with their original values given in above numerical example.

A keen observation of Table 1 and Table 2 reveal the following

i. Variations in permissible delay period M: It is very often seen that Decision of purchaser directly depends on permissible delay period M. Hence delay period M is an important parameter in proposed model. In the both case we observe (from Table 1, 2) that a slight increase in M results in decrement in optimal values of Cycle length T^*, order quantity Q^* and optimal total relevant cost respectively. On the other in second case a slight increase in M results in increment in optimal values preservation cost ξ^*, optimal payment time K^*. These observations are very obvious regarding practical situations.

ii. Variations in inflation and time discounting rate r: In the both the case we observe (from Table 1, 2) that a slight increase in r results in decrement in optimal values of Cycle length T^*, order quantity Q^* and optimal total relevant cost, optimal payment time K^*, optimal preservation cost ξ^* respectively.

iii. Variations in time independent demand rate α: In both cases we observe (from Table 1, 2) that a slight increase in α results in decrement and increment in optimal values of Cycle length T^*, order quantity Q^*, optimal payment time K^* respectively in first and second case. Whereas optimal preservation cost ξ^* decrease in both the cases and optimal total relevant cost increases and decreases respectively in first and second case.

iv. Variations in time dependent demand factor β and deterioration rate θ: These factors are also play an important role in Decisions of purchaser. Hence we will discuss influence of variations in β and θ. From Table 1 and 2 one can easily say that, in both the cases a slight increase in β results in increment in optimal values of Cycle length T^*, order quantity Q^* and optimal total relevant cost. Whereas preservation cost ξ^* increase and decrease respectively in first and second case.

In case of deterioration rate which is controllable by using preservation techniques. A slight increase in θ results in increase in ξ^* i.e., increase in deterioration rate means it required more preservations which definitely increase the

Table 1. (When T < M)

Variation in M				
Change in M	ξ^*	T^*	Q^*	TC_1
0.3424658	0.0100623	0.32761096	4.08813	337.591
0.34795	0.009925	0.3275534	4.0874	337.554
0.350685	0.009852	0.3275233	4.08673	337.501
Variation in inflation rate r				
Change in r	ξ^*	T^*	Q^*	TC_1
0.0101	0.003397	0.324825	4.02093	336.118
0.01011	0.002729	0.324548	4.01428	335.971
0.01012	0.002061	0.324274	4.00771	335.825
Variation in time independent demand factor α				
Change in α	ξ^*	T^*	Q^*	TC_1
40	0.0109701	0.3276521	4.11135	337.093
45	0.0105138	0.3276329	4.09977	337.351
55	0.0096156	0.327589	4.07649	337.863
Variation in time dependent demand factor β				
Change in β	ξ^*	T^*	Q^*	TC_1
70	0.009426	0.32758	3.56253	326.976
75	0.009764	0.327595	3.82531	332.291
85	0.0103273	0.32762192	4.35086	342.921
Variation in partial backlogging parameter δ				
Change in δ	ξ^*	T^*	Q^*	TC_1
0.2	0.00405423	0.3251233	3.67563	330.886
0.3	0.00216843	0.324351	3.30699	325.115
0.4	0.00124628	0.3239781	2.9495	319.584
Variation in deterioration rate θ				
Change in θ	ξ^*	T^*	Q^*	TC_1
0.36	0.0236717	0.327611	4.08813	337.607
0.37	0.0365454	0.327611	4.08813	337.607
0.38	0.0487416	0.327611	4.08813	337.607

Table 2. (When $M \le K \le T$)

Variation in M					
Change in M	ξ^*	K^*	T^*	Q^*	TC_1
0.13699	0.458119	0.1371178	0.32573973	4.05163	336.618
0.21918	0.473581	0.22006	0.32522	4.03941	327.961
0.24658	0.476583	0.246576	0.324781	4.02892	323.921
Variation in inflation rate r					
Change in r	ξ^*	K^*	T^*	Q^*	TC_1
0.009	0.473611	0.2196351	0.3559644	3.48856	345.388
0.01	0.473581	0.22006	0.32522	3.48856	327.926
0.011	0.473342	0.2239	0.301507	3.48856	314.692
Variation in time independent demand factor α					
Change in α	ξ^*	K^*	T^*	Q^*	TC_1
40	0.473589	0.2191792	0.3250137	4.03434	328.866
50	0.473581	0.22006	0.32522	4.03941	327.961
50.5	0.473402	0.222407	0.325888	4.05449	327.908
Variation in time dependent demand factor β					
Change in β	ξ^*	K^*	T^*	Q^*	TC_1
79.5	0.473609	0.2197263	0.3251206	4.01117	327.441
80	0.473581	0.220059	0.3252164	4.03941	327.961
81	0.47352	0.220804	0.3254274	4.09621	328.988
Variation in partial backlogging parameter δ					
Change in δ	ξ^*	K^*	T^*	Q^*	TC_1
0.2	0.473644	0.2191792	0.323858	3.65663	321.99
0.4	0.473643	0.2191792	0.323359	2.94818	311.058
0.6	0.473642	0.2191792	0.3232	2.24859	300.325
Variation in deterioration rate θ					
Change in θ	ξ^*	K^*	T^*	Q^*	TC_1
0.31	0.470213	0.219589	0.3250822	4.0361	327.957
0.32	0.471125	0.219816	0.325148	4.03768	327.958
0.34	0.472736	0.2209252	0.3254658	4.04533	327.959

optimal preservation cost ξ^*. Similarly other changes in other optimal values due to change in θ, are also feasible according to the practical situations.

v. Variation in partial backlogging parameter δ: A slight increase in δ results in decrement in optimal values of all parameters.

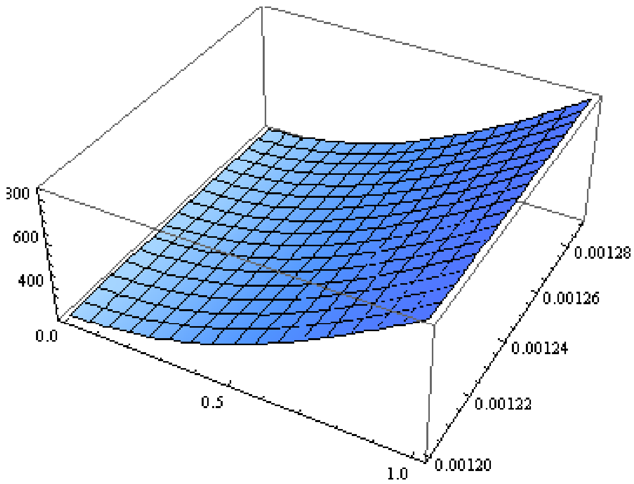

Figure 2. Convexity of Total Cost w. r. t. T^* and ξ^*.

6. Conclusion

In this model we have considered the deteriorating item with controllable deterioration rate by using preservation techniques and time dependent demand. Effect of inflation and permissible delay is also considered with partially backlogged shortages. Optimal value of total relevant cost, preservation cost, economic ordered quantity, payment time optimal cycle length are obtained by using Mathematical software Mathematica7. For sensitivity of proposed model a sensitivity analysis is performed by varying some parameters. This model is applicable in the market in which demand is varying with time (like fruits, vegetables, etc.). Proposed model can be extended by considering other factors of inventory control like shortages, price dependent demand etc.

7. References

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