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A Study on Algebraic Solutions of Trajectory Equations

Hwan IlKang[1*], Jae Jeong Pyun[2] and Hyun Soo Kim[3]

[1]Department of Information & Communication Engineering, Myongji University, Yongin, 449-718, South Korea; hwanilkang@naver.com
[2]Agency for Defense Development, Yuseong-Gu, Daejon, 305-600, South Korea; jjpyun@add.re.kr
[3]Department of Information & Communication Engineering, Myongji University, Yongin, 449-718, South Korea; saints7777@naver.com

Abstract

The exact trajectory of the direct fire gun results from the standard differential equations given in (1, 2, 3, 4). Under the condition that the elevation firing angle has a very small angle, the standard differential equations may be changed into the solvable differential equations. We compare two trajectories from the exact and the solvable differential equations. For a range distance of 570m, the error between the exact and approximate trajectory is 0.0055m if there is neither crosswind nor range wind. Under the condition of existence of the range and cross-range, their error is 0.2016m for a range distance of 570m. It turns out that the derived new trajectory has a very small discrepancy compared to the exact trajectory derived from the standard differential equations.

Keywords: Approximation, Standard Differential Equations, Solvable Differential Equations, Trajectory

1. Introduction

For the design of combat vehicle, we consider four requirements: mobility, fire power, the operability and the survivability. The first is that how fast it drives on the battlefield. The second is the fire power of the gun associated with the combat vehicle.

One of the third requirements is the human factor: so called the ride comfort. In addition, the survivability is related to the robustness against enemy's fire power. We need some requirement for the fire performances such as hit probability and rate of fire. Each subsystems are designed so that the systems satisfy the fire performances. Combat vehicle has five subsystems affecting hit probabilities: the first one is the bullet specifications such as the propellant mass and projectile mass. The second subsystem is the arm (gun) specification such as gun tube length, the muzzle velocity and vertical gun jump. The third subsystem is the sensor errors. Sensor errors include radar distance error, vertical and azimuth error, encoder error and resolver error. The fourth one has the meteorological factors such as the atmosphere pressure, density and temperature. The fifth one is related to the vehicle dynamics such as the platform vertical displacement, pitch variation and roll variation. For the vertical displacement, a quarter car model may be used. For the pitch variation, the half car model is used and for the roll variation, the full car model is used. In this paper, we focus on the fire power for the combat vehicle. To calculate the hit probability, trajectory analysis and exterior ballistics play important roles.

*Author for correspondence

Design factors such as muzzle velocity, pitch variation and roll variation are expressible as variance values. These variances carry over to the target through the trajectory. So the vertical and horizontal displacement on the target may be determined by the variances of design factors and the unit partials which mean the vertical or horizontal variations with respect to the unit variation of each design parameter at a given range. In this paper, the trajectory for direct fire gun is focused. The trajectory for the direct fire gun comes from the standard differential equations. Standard differential equations may be solved by the Runge-Kutta method[2] or an Adams-Bashforth-Moultons' method[5]. Runge-Kutta method is a single step numerical solutions while the Adams-Bashforth-Moultons' method is a predator-corrector method. Under the condition that the elevation firing angle is less than 5.71 degree, the differential equations may be changed into the solvable differential equations. That is, the new trajectory results from the algebraic equations. It turns out that the derived new trajectory has a very small error compared to the exact trajectory derived from the standard differential equations.

2. Differential Equations for Trajectories

2.1 The Standard Differential Equations

The standard differential equations (1, 2, 3) are described as

$$\dot{v}_x = -C_d^- v\,(v_x - w_x)$$
$$\dot{v}_z = -C_d^- v(v_z - w_z)$$
$$\dot{v}_y = -C_d^- v(v_y - w_y) - g$$

where v_x, v_z, v_y are range, cross range and upward velocity, respectively and w_x, w_z, w_y are range, cross range and upward velocity of winds, respectively. The gravitational acceleration is denoted by g. Figure 1 shows the trajectory and notations.

The constant value C_d^- is a modified drag coefficient value and the value is assumed to be $1.7701 \times 10^{-4} m^{-1}$. The value v is the scalar value of the velocity vector and is expressible as $\sqrt{v_x^2 + v_z^2 + v_y^2}$ m/sec. The above standard differential equations are solved by the numerical analysis. One of the numerical method is the Runge-Kutta method. The Runge-Kutta method is used in this paper.

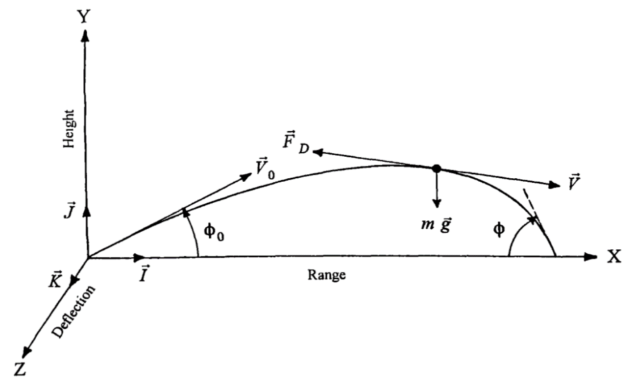

Figure 1. Bullet trajectory.

2.2 The Solvable Differential Equations

Consider the situation that the elevation angle is less than 5.71 degree[3]. Under this condition, $\left|\frac{v_y}{v_x}\right| \ll 1$ and $\left|\frac{v_y}{v_x}\right| \ll 1$. So, the value v can be described by

$$v = \sqrt{v_x^2 + v_z^2 + v_y^2} = v_x\sqrt{\frac{v_y^2}{v_x^2} + \frac{v_z^2}{v_x^2} + 1} \cong v_x$$

The family of the given differential equations are described by

$$\dot{v}_x = -C_d^- v_x(v_x - w_x)$$
$$\dot{v}_z = -C_d^- v_x(v_z - w_z)$$
$$\dot{v}_y = -C_d^- v_x(v_y - w_y) - g$$

The first equation in the above may be solved easily since the differential equation has a variable $v_x(t)$ only. Using the explicit expression for $v_x(t)$, the second and third equations may be solvable. Finally, the algebraic equations can be derived. The solutions are

$$x(t) = W_x t + \frac{f}{C_d^-}$$

$$_z(t) = W_y t - \frac{W_y f}{C_d^- V_w}$$

$$_y(t) = -0.25gt^2 + \frac{(g - 2V_w W_y C_d^-)f}{2C_d^- V_w^2} + \frac{(-0.5gt + V_w W_y C_d^- + V_y(0)f)}{V_w C_d^-}$$

where

$V_w = V_x(0) - W_x$

and

$f = [\ln(1 + t\, \bar{C_d}\, V]_w)$.

3. Results

3.1 Experiments with No Winds

Assume that the initial positions for the range, cross-range and upward displacement are (0,0,0) and the initial velocities for the range, cross-range and upward displacement are (300,0,10) m/sec, (300,0,7) m/sec and (300,0,5) m/sec respectively for the three experiments. The constant winds for the range, cross-range and upward displacement are (0,0,0) m/sec for three cases. The range and upward displacement are obtained for three trajectories from the Runge-Kutta method and the algebraic equations. We use the built-in function *ode45*, which implements versions of Runge-Kutta 4th/5th-order[6]. For the Runge-Kutta method derived from the standard differential equations, the range and upward displacement at the time of 1.0 [sec] are 292.0741m and 0.0538m, respectively. For the algebraic equations, the range and upward displacement are 292.0744m and 0.0538m, respectively at the time of 1.0 [sec]. The cross range displacements are zeroes for the three cases during all the trajectories. The errors are the Euclidean distance between two points from the Runge-Kutta method and the approximate method. The other two cases are the situations at the time of 1.4 [sec] and 2.0 [sec], respectively. We assume that the modified drag coefficient $\bar{C_d}$ to be $1.7701 \times 10^{-4} m^{-1}$. Figure 2 shows comparison of exact and approximate trajectories for three cases. Table 1 lists the comparisons between two trajectories for three different times of flight.

Table 1. Performance comparisons between two trajectories

	Range [m]	Height [m]	Initial velocities [m/sec]	Time of Flight [sec]	Error [m]
Runge-Kutta	569.3575	0.0401	(300,0,10)	2.0	0.0055
Algebraic Equation	569.3650	0.0402			
Runge-Kutta	404.6781	0.0706	(300,0,7)	1.4	0.0014
Algebraic Equation	404.6795	0.0706			
Runge-Kutta	292.0741	0.0538	(300,0,5)	1.0	3.6053e-4
Algebraic Equation	292.0744	0.0538			

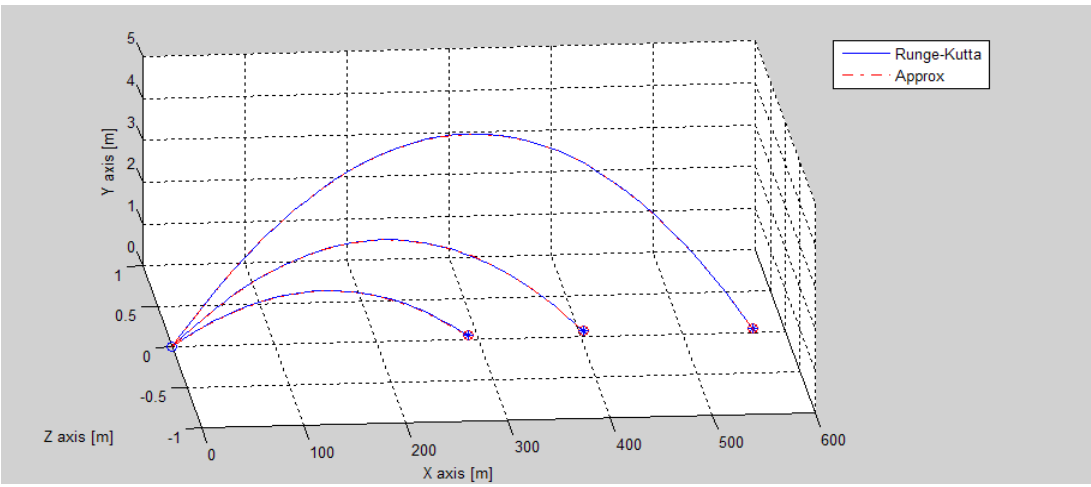

Figure 2. Comparison of exact and approximate of trajectories.

Table 2. Performance comparisons between two trajectories

	Range [m]	Height [m]	Deflection [m]	Initial Velocities [m/sec]	Time of Flight [sec]	Error [m]
Runge-Kutta	569.5550	0.0401	0.0511	(300,0,10)	2.0	0.2016
Algebraic Equation	569.7566	0.0425	0.0507			
Runge-Kutta	404.7778	0.0706	0.0255	(300,0,7)	1.4	0.1004
Algebraic Equation	404.8782	0.0714	0.0254			
Runge-Kutta	292.1260	0.0538	0.0132	(300,0,5)	1.0	0.0519
Algebraic Equation	292.1779	0.0541	0.0131			

3.2 Experiments with Existence of Winds

Now, we consider three cases for the existence of range and crosswind. All data are the same as in the case in Table 1 except the data of winds. Assume that the constant winds for the range, cross-range and upward displacement are (2,0.5,0) m/sec, respectively. The range, upward displacement and the deflection are obtained for three trajectories from the Runge-Kutta method and the algebraic equations. The measured time settings are 1, 1.4, and 2 [sec]. For the Runge-Kutta method from the standard differential equations, the range, upward displacement and deflection at the time of 1[sec] are 292.1260m, 0.0538m and 0.0132m, respectively. For the algebraic equations, the range, upward displacement and deflection are 292.1779m, 0.0541m and 0.0131m, respectively at the time of 1 sec. The error between two points at time 1.0 sec is 0.0519m. Table 2 lists the comparisons between two trajectories for three different times of flight with existence of winds.

4. Discussion

The exact trajectory of the direct fire gun results from the standard differential equations. Under the condition that the elevation firing angle is less than 5.71 degree, the standard differential equations may be changed into the solvable differential equations. It turns out that the derived new trajectory has a very small error compared to the exact trajectory from the trajectory derived from the standard differential equations. The exact trajectory may be derived from the standard differential equations through the Runge-Kutta method.

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