

A Novel Cascaded Image Transform by Varying Energy Density to Convert an Image in to Sparse

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Abstract

Background: All natural signals are subjected to sparsity when they are properly represented by a basis function. Sparsity helps us to sample the signals less than Nyquist rate which clearly explained by the recent theory known as compressive sensing. **Methods:** This paper explains that DFT does a good job in converting the given image into sparse when the energy density of the image is varied and also a cascaded transform DFT and DWT is proposed. Qualitative measures for the cascaded transform were observed to be good. **Result:** It helps us to convert a given image signal into sparse without loss in information content present in that image. **Application:** While converting an analog signal into digital, sparsity will help to compress a given analog signal before conversion. So the number of samples obtained by sampling the compressed signal becomes less.

Keywords: Compressive Sensing, Energy Density, Image Transforms Information Preservation Capability, Sparsity

1. Introduction

Compressive sensing is a recent theory which proves that even if a given signal is under sampled, it can be reconstructed without much loss. Here the sampling is done in information rate rather than the Nyquist rate. Basically Discrete Fourier Transform is used to compress 1D signal whereas Discrete Cosine Transform and Discrete Wavelet Transform are good for compressing 2D signals¹⁻⁸. This paper explains that DFT does well with 2D signals say images. If the energy density ranges of an image after applying DFT is varied then the PSNR and MSE values of the reconstructed image improves. So that one can prove that the reconstructed image quality can be improved by varying the energy density of the image. In this paper DFT was applied to the reference images Lena and Cameraman, where the energy density of the images varied to improve the qualitative measures of the reconstructed images using inverse DFT. Also the second order wavelet transforms were cascaded with the DFT to find

better qualitative measures than the measures observed for only DWT applied over the same images. However HAAR transform stands best in compressing image, so leaving HAAR other second order wavelet transforms were cascaded with DFT to obtain better qualitative measures.

2. Transforms and Sparsity

Sparsity plays a key role in the modern known as compressive sensing which explains that a signal can be reconstructed with fewer samples measured. It is very helpful in some applications like analog to digital converter which is used to sample radio frequency signals where high rate ADC is required and also bandwidth required to transmit the samples is high. This makes the system costlier, sparsity helps in sampling the given signal in information rate rather than the Nyquist rate so that a low rate ADC is enough for sampling. So that the number of samples measured will be less and also the bandwidth

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required for transmitting the samples is less which makes the system cost effective. Sparsity always deals with the signals that are loosely bound. It is defined as the fractions of zero elements present in sparse matrix¹⁷. A sparse matrix is a matrix that has zero and nonzero components of a signal. We know that all natural signals are sparse in nature if they are represented by a proper basis function or transforms. Various transforms discussed in this paper are DFT and DWT. In DWT the following wavelets SYM2, COIF1, DB2, DB10 and DMDEY are discussed¹²⁻¹⁶.

Discrete Fourier transform DFT is a sampled transform which has only group of large samples that describes the spatial domain image. For a $N \times N$ image, the two dimensional DFT is given by the equation 1,

$$F(k,l) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} f(i,j) e^{-i2\pi(\frac{ki}{N} + \frac{lj}{N})} \quad (1)$$

where $f(i,j)$ is the image, exponential term is the basis function corresponding to each point $F(k,l)$ in Fourier space. The inverse Fourier transform required to reconstruct the image is shown in equation 2.

$$f(a,b) = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} F(k,l) e^{i2\pi(\frac{ka}{N} + \frac{lb}{N})} \quad (2)$$

Daubechies wavelet was proposed by a mathematician called Ingrid Daubechies. It is an orthogonal wavelet mostly chosen to get higher number of vanishing moments,

$$N=2A$$

Where N is the length of taps and A is the vanishing moments. It can be represented in two ways, in terms of length of the taps say DM or in terms of number of vanishing moments for e.g. dbA. In this paper we have taken db2 and db10 for analysis. Ingrid Daubechies, the mathematician made some modifications in his previously proposed wavelet Daubechies, to derive symlets. So symlets and daubechies wavelets have same properties. There are several symlet wavelets ranging from sym2 to sym20. In this paper sym2 are taken for analysis. This wavelet was invented by Ronald Coifman proposed the wavelet called COIF. This wavelet is symmetric and has vanishing moment of $A/3$ and a Tap length of $A/3-1$ ⁹⁻¹¹.

3. Proposed Algorithm

A cascaded transform is proposed just by fusing DFT and DWT other than HAAR wavelet because HAAR wavelet

is incomparably best for image compression. But other second order wavelets don't produce good performance when compared to HAAR. So in order to increase the quality of the retrieved image, instead of using the DWT transforms like coif1, sym2, db2, etc separately on images they were cascaded with DFT. Before creating the cascaded transform, the apt energy density range of the image after applying the DFT is found to obtain good qualitative measure values for the reconstructed image. Figure 1 shows the block diagram for the proposed algorithm.

Initially DFT is applied to the input image so that the image gets decomposed into number of energy coefficients. Keeping the larger coefficients throw away all other coefficients and apply inverse DFT to reconstruct the input image which makes the image compressed. Now we say that the input image has been converted to sparse. But the performance measures like PSNR and MSE values are satisfactory. So the energy density range of the image was varied, initially while applying DFT the default energy density range was 16. This range was reduced to 10 first and then 6, it was found that PSNR and MSE values improved which means the quality of the reconstructed image was good. When the energy density range was reduced below 6 the values remained the same so keeping this 6 as the threshold value for the energy density range further analysis were made. When the qualitative measures of the reconstructed image by applying DFT keeping 0 to 6 as the energy density range were compared with the values obtained by applying DWT like sym2, coif1, db2, and db10 the later was found to less. So the compressed image using DFT was applied with second order DWT to once again decompose the image in to number of energy coefficients. All the smaller coefficients were discarded and inverse DWT was applied to the larger coefficients to reconstruct the original image. It was found that qualitative analysis of the reconstructed image was extremely good.

4. Simulation Result and Discussion

Reference images Lena and Cameraman which are of different in size and format were taken and applied with DFT to decompose them into number of energy coefficients. The energy density range was from 0 to 16, the energy coefficients less than 16 were discarded and only the larger coefficients were applied with inverse DFT to recover the image. The PSNR and MSE values were not

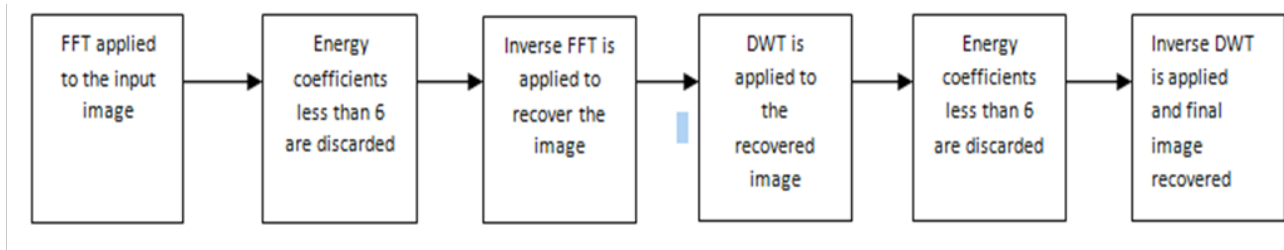


Figure 1. Block diagram of Cascaded Transform to convert an image into sparse.

satisfactory so the energy density range was reduced to 10 and then 6. The PSNR and MSE values were good for the recovered image of the reduce energy density. The PSNR and MSE values of the reconstructed Lena and Cameraman image for various energy density is shown in Table 1 and Table 2 respectively. The various energy density diagrams for Lena and Cameraman is shown in Figure 2 and Figure 3 respectively. After this various DWT were applied for the Lena and Cameraman image, keeping this energy density range 6 as the threshold value lower coefficients less than 6 were omitted and inverse DWT were applied for the larger coefficients to reconstruct the original image and their performance measures were analyzed. Finally DFT and DWT were cascaded to find a cascaded image transform and also their qualitative measures were analyzed to find these measures were greater when compared to that of former. The performance measures of various transforms applied on Lena and Cameraman image is shown in Table 3 and 4 respectively. The various performance measures analyzed for the reconstructed Lena and Cameraman image are as follows. The MSE value should be as low as possible then only the

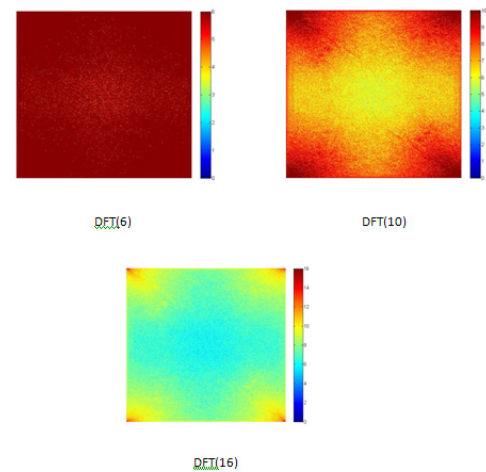


Figure 2. Various ranges of energy densities with their colour bar when DFT is applied on Lena image

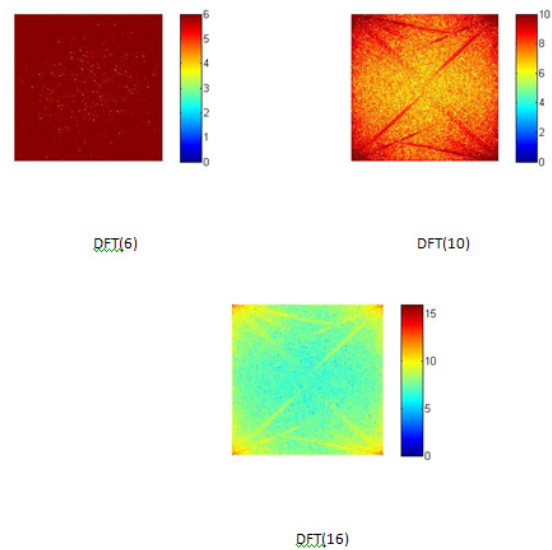


Figure 3. Various ranges of energy densities with their colour bar when DFT is applied on Lena image.

Table 1. MSE and PSNR values of the reconstructed Lena image shown for various energy densities

DFT	MSE	PSNR
DFT(16)	8.7730e-008	118.6993
DFT(10)	9.5626e-009	128.3591
DFT(6)	3.0270e-028	323.3547

Table 2. MSE and PSNR values of the reconstructed Cameraman image shown for various energy densities

DFT	MSE	PSNR
DFT(16)	1.1568e-008	127.5322
DFT(10)	1.1568e-008	127.5322
DFT(6)	1.0215e-028	328.0725

Table 3. Performance measures of various transforms applied on Lena image

TRANSFORM	MSE	PSNR	SSIM	NCC	FOM	ENTROPY	ENTROPY	CR
						ORIGINAL	RESTORED	
Db2	1.0046	48.14	0.9930	0.9999	0.9674	7.4460	1.7247	76.4268
Db10	0.9334	48.46	0.9937	0.9999	0.9731	7.4660	1.4227	76.6503
Coif1	0.9847	48.23	0.9931	0.9999	0.9667	7.4660	1.7117	77.0106
Sym2	1.0446	48.14	0.9930	0.9999	0.9674	7.4660	1.7247	76.4268
dmey	0.8771	48.73	0.9942	0.9999	0.9791	7.4660	1.2164	76.4268
dft+dmey	1.8271e-004	85.54	1	1	1	7.4460	4.5155	76.4268
dft+sym2	1.0443e-004	87.97	1	1	1	7.4460	4.6197	76.4268
dft+coif1	2.7568e-022	263.76	1	1	1	7.4460	4.5813	76.4268
dft+db10	0.0019	75.38	1	1	0.9999	7.4460	4.6473	76.4268
dft+db2	1.0443e-004	87.97	1	1	1	7.4460	4.6197	76.4268

Table 4. Performance measures of various transforms applied on Cameraman image

TRANSFORM	MSE	PSNR	SSIM	NCC	FOM	ENTROPY	ENTROPY	CR
						ORIGINAL	RESTORED	
Db2	0.2084	54.9748	0.9999	1	0.9823	6.9719	2.2361	112.2192
Db10	0.2230	54.6820	0.9932	0.9999	0.9904	6.9719	2.4706	112.2192
Coif1	0.2091	54.9620	0.9939	0.9999	0.9851	6.9719	2.311.3	112.2192
Sym2	0.2084	54.9748	0.9939	1	0.9823	6.9719	2.3361	112.2192
dmey	0.2356	54.4423	0.9927	0.9999	0.9928	6.9719	2.2993	112.2192
dft+dmey	0.0043	71.7801	1	1	0.9996	6.9719	4.0204	112.2192
dft+sym2	0.0021	74.8502	1	1	0.9999	6.9719	4.1476	112.2192
dft+coif1	0.0016	76.0711	1	1	0.9999	6.9719	4.0397	112.2192
dft+db10	0.0030	73.4176	1	1	0.9998	6.9719	4.3971	112.2192
dft+db2	0.0021	74.8502	1	1	0.9999	6.9719	4.1476	112.2192

restored image will be of good quality. DFT applied with the wavelet COIF1 has the lowest MSE value when compared with other transforms. PSNR value of the restored image should be high enough to say that image is of good quality, DFT with COIF1 wavelet has the highest PSNR value when compared with other transforms. The value of SSIM should fall between 0 and 1 for a reconstructed image. If the value is 1 then the reconstructed image is said to have a good structural similarity when compared to the original image. NCC illustrates the similarity between reconstructed image and the original image, if the value of NCC is 1 then reconstructed image is said to be similar to the original image. FOM defines preservation of edges, the edges of reconstructed image is said to be well preserved if the FOM value is closer to 1. The qualitative analysis of lena and cameraman images by applying various cascaded image transforms are shown in Figure 4 and Figure 5 respectively.

5. Conclusion

Sparsity is the key role in compressive sensing which helps us to sample the given signal in information rate rather than the Nyquist rate. It is very useful in saving the bandwidth and also reduces the cost of transmitting high frequency sampled signal. To compress the given signal we need to represent it in a proper basis function or transform. DFT is better for compressing 1D signal whereas DCT and DWT are suitable for compressing 2D signals. In this it is proved from the simulation results that DFT does better for 2D signals by varying the energy density range of the image. Also a cascaded image transform is proposed in this paper which cascades DFT and DWT. Performance measures of various cascaded image transforms are analyzed to find which transform is best. DFT applied with the wavelet COIF1 has the lowest MSE value when compared with other transforms. It has the highest



Figure 4. Original and the reconstructed Lena image by applying various cascaded image transform.



Figure 5. Original and the reconstructed Cameraman image by applying various cascaded image transform.

PSNR value compared to other transforms which illustrates that the quality of the reconstructed image is good. The value of SSIM should fall between 0 and 1 for a reconstructed image. If the value is 1 then the reconstructed image is said to have a good structural similarity when compared to the original image. DFT applied with the wavelet COIF1 applied on Lena and Cameraman image shows good structural similarity between original and reconstructed image. If the value of NCC is 1 then reconstructed image is said to be similar to the original image. From the table it is understood that when DFT applied with COIF1 the reconstructed image is similar to the original image and also its edges are well preserved. DFT and COIF1 wavelet cascaded together is best to represent the given image in sparse.

6. References

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