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On the Results of Impulsive Neutral Integrodifferential Control Systems with an InfiniteDelay

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Abstract

This paper deals with the sufficient conditions for the controllability of impulsive neutral functional integrodifferential systems with infinite delay in Banach space are established by means of the nonlinear alternative of Leray-Schauder type.

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1. Introduction

Many researchers have investigate the impulsive neutral functional differential equations in abstract space[3,4,8,9,12,13,14,15]. These type of equations occur in the study of heat conduction in material with memory and many other physical phenomena. So it is interesting to study the controllability of impulsive neutral integrodifferential systems in infinite delay[3]. Balachandran et al[5] discussed the controllability of impulsive neutral functional integrodifferential systems in abstract phase space with the help of Schauder's fixed point theorem. The purpose of this paper to study the controllability of impulsive neutral functional integrodifferential systems with infinite delay by using Nonlinear Alternative of Leray-Schauder Type.

In this paper, we study the controllability results for impulsive neutral functional integrodifferential evolution equation with infinite delay in a real Banach space E of the form

$$\frac{d}{dt}[u(t)-k_1(t,u_t)] = A(t)[u(t)-k_1(t,u_t)] + Gx(t) + k_2\left(t, u_t, \int_0^t e(t,s,u_s)ds\right), t\in J, \quad (1.1)$$

$$u_0 = \phi \in \mathcal{B}, \quad (1.2)$$

$$\Delta u(t_k) = I_k(u(t_k^-)), k=1,2,\ldots,m, t\neq t_k, 0<t_1<t_2<\ldots<t_m<a. \quad (1.3)$$

where $k_1 : J\times\mathcal{B}\to E$, $k_2 : J\times\mathcal{B}\times E\to E$, $e : J\times J\times\mathcal{B}\to E$, $I_k : E\to E$ and $\phi\in\mathcal{B}$ are given functions, the control $x(.)$ is given in $L^2(J;E)$, the Banach space of admissible control function with E is a real seperable Banach space with the norm $|.|$, G is bounded linear operator from E into E and $\{A(t)\}_{0\le t\le a}$ is a family of linear closed (not necessarily

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bounded) operators form E into E that generates an evolution system of operators $\{U(t,s)\}_{(t,s)\in J\times J}$ for $0 \le s \le t \le a$. The history $u_t : (-\infty, 0] \to X$, $u_t(\theta) = u(t+\theta)$, belongs to some abstract phase space B defined axiomatically; $0 = t_0 < t_1 < t_2 < \dots < t_m < t_{m+1} = a$, $\Delta u(t_k) = u(t_k^+) - u(t_k^-)$, $u(t_k^+)$ and $u(t_k^-)$ are respectively the right and left limits of u at $t = t_k$.

Sufficient conditions are establish here to get the controllability of mild solutions which are fixed points of appropriate corresponding operators using the nonlinear alternative of Leray-Schauder type Leray-Schauder type (see[7,19]).

2. Preliminaries

We introduce the notations, definitions and theorems which are used throughout this paper. Let $PC(J, E)$ be the Banach space of continuous functions with the norm $\|u\|_\infty = \sup\{|u(t)| : 0 \le t \le a\}$ and $B(E)$ be the space of all bounded linear operators from E into E, with the norm $\|N\|_{B(E)} = \sup\{|N(u)| : |u| = 1\}$. A measurable function $u : J \to E$ is Bochner integrable if and only if $|u|$ is Lebesgue integrable. (For the Bochner integral properties, see Yosida[20] for instance). Let $L^1(J, E)$ be the Banach space of measurable functions $u : J \to E$ which are Bochner integrble normed by $\|u\|_{L^1} = \int_0^a |u(t)|\,dt$.

Consider the following space $B_T = \{u : (-\infty, a] \to E : u|J \in PC(J; E),\ u_0 \in \mathcal{B}\}$, where $u|J$ is the restriction of u to J.

In this paper, we will employ an axiomatic definition of the phase space $\mathcal{B}$ introduced by Hale and Kato in[10] and follow the terminology used in[11]. Thus, $(\mathcal{B}, \|.\|_{\mathcal{B}})$ will be a seminorm linear space of functions mapping $(-\infty, 0]$ into E, and satisfying the following axioms:

(A) If $u : (-\infty, a] \to X$, is continuous on J and $u_0 \in \mathcal{B}$, then for every $t \in J$ the following conditions hold:
  (i) u_t is in $\mathcal{B}$;
  (ii) There exists a positive constants H such that $|u(t)| \le H\|u_t\|_{\mathcal{B}}$;
  (iii) There exists two functions $K(.)$, $M(.) : R_+ \to R_+$ independent of $u(t)$ with K continuous and M is locally bounded such that:

$$\|u_t\|_{\mathcal{B}} \le K(t)\sup\{|u(s)| : 0 \le s \le t\} + M(t)\|u_0\|_{\mathcal{B}}.$$

  Denote $K_a = \sup\{K(t) : t \in J\}$ and $M_a = \sup\{M(t) : t \in J\}$.

(B) For the function $u(.)$ in (A), u_t is a $\mathcal{B}$ valued function on $[0, a]$.

(C) The space $\mathcal{B}$ is complete.

Remark 2.1

1. Condition (ii) in (A) is equivalent to $|\phi(0)| \le H\|\phi\|_{\mathcal{B}}$ for every $\phi \in \mathcal{B}$.
2. Since $\|.\|_{\mathcal{B}}$ is a seminorm, two elements $\phi, \phi_1 \in \mathcal{B}$ can verify $\|\phi - \phi_1\|_{\mathcal{B}} = 0$ without necessarily $\phi(\theta) = \phi_1(\theta)$ for all $\theta \le 0$.
3. From the equivalence of (ii), we can see that for all $\phi, \phi_1 \in \mathcal{B}$ such that $\|\phi - \phi_1\|_{\mathcal{B}} = 0$. This implies necessarily that $\phi(0) = \phi_1(0)$.

Hereafter are some examples of phase spaces. For other details we refer, for instance to the book by Hino et al[11] and Selma Baghli et al[18].

Definition 2.1

A function $k_2 : J \times B \times E \to E$ is said to be an L^1-Caratheodory function it if satisfies:

(i) for each $t \in J$ the function $k_2(t, ., .) : \mathcal{B} \times E \to E$ ks continuous;
(ii) for every $\phi \in \mathcal{B}$, $x \in E$ the function $k_2(., \phi, u) : J \to E$ is measurable;
(iii) for every positive integer k there exists $h_k \in L^1(J, R^+)$ such that $|k_2(t, \phi, u)| \le h_k(t)$ for all $\|u\|_{\mathcal{B}} \le k$ and almost each $t \in J$.

In what follows, we assume that $\{A(t)\}_{t\ge 0}$ is a family of closed densely defined linear un- bounded operators on the Banach space E and with domain $D(A(t))$ independent of t.

Definition 2.2

A family of bounded linear operators $\{U(t, s)\}_{(t,s)\in\Delta}$: $U(t,s) : E \to E$ for $(t, s) \in \Delta := \{(t,s) \in J \times J : 0 \le s \le t \le a\}$ is called an evolution system if the following properties are satisfied:

1. $U(t, t) = I$ where I is the identity operator in E,
2. $U(t, s)U(s, r) = U(t, r)$ for $0 \le r \le s \le t \le a$,
3. $U(t, s) \in B(E)$ the space of bounded linear operators on E, where for every $(t, s) \in \Delta$ and for each $u \in E$, the mapping $(t, s) \to U(t, s)u$ is continuous.

More details on evolution systems and their properties could be found on the books of Ahmed[1], Engel and Nagel[6] and Pazy[16].

The proof of our result is based on the following fixed point theorem due to Nonlinear Alternative of Leray-Schauder Type.

Theorem 2.1

(Nonlinear Alternative of Leray-Schauder Type[7]). Let X be a Banach space, Y a closed, convex subset of E, v as an open subset of Y and $0 \in X$. Suppose that $N : \overline{v} \to Y$ is a continuous, compact map. Then either,

(C1) N has a fixed point in $\overline{v}$; or

(C2) There exists $\lambda \in (0, 1)$ and $u \in \partial v$ (the boundary u in Y) with $u = \lambda N(u)$.

3. Main Result

Before starting and proving the main result, we give first the definition of mild solution of problem (1.1)–(1.3).

Definition 3.3

We say that the function $u(.) : R \to E$ is a mild solution of (1.1)–(1.3) if the following hold: $u(t) = \phi(t)$ for all $t \in (-\infty, 0]$, $\Delta u|_{t=t_k}$, $k = 1, 2, \ldots, .$; the restriction of $u(.)$ to the interval J is continuous and u satisfies the following integral equation

$$u(t) = U(t,0)[\phi(0) - k_1(0,\phi(0))] + k_1(t,u_t) + \int_0^t U(t,s)Gx(s)ds + \int_0^t U(t,s)k_2\left(s, u_s, \int_0^s e(s,\tau,u_\tau)d\tau\right)ds + \sum_{0<t_k<t} U(t,t_k)I_k(u(t_k^-)),\ t \in J. \tag{3.1}$$

Definition 3.4

The neutral evolution problem (1.1)–(1.3) is said to controllable on the interval J if for every initial function $\phi \in \mathcal{B}$ and $u_1 \in E$ there exists a control $x \in L^2(J, E)$ such that the mild solution $u(.)$ of (1.1)–(1.3) satisfies $u(a) = u_1$.

We will need to introduce the following hypotheses which are assumed hereafter:

(H1) $U(t, s)$ is a compact operator whenever $t - s > 0$ and there exists a constant $M \geq 1$ such that $\|U(t, s)\|_{B(E)} \leq M$ for every $(t, s) \in \Delta$.

(H2) The function $k_2 : J \times \mathcal{B} \times E \to E$ satisfies the following conditions

(1) For each $u : (-\infty, a] \to E$, $u_0 = \varphi \in \mathcal{B}$ and $u|J \in \mathcal{PC}$, the function $t \to k_2\left(t, u_t, \int_0^t e(t,s,u_s)ds\right)$ is strongly measurable and $k_2(t, ., .)$ is continuous for a.e. $t \in J$;

(2) There exists an integrable function $\alpha : J \to [0, +\infty)$ and a monotone continuous dondecreasing function $\Omega : [0, +\infty) \to (0, +\infty)$ such that

$$|k_2(t,\phi,u)| \leq \alpha(t)\Omega(\|\phi\|_{\mathcal{B}} + \|u\|), t \in J, (\phi,u) \in \mathcal{B} \times E$$

(3) There exists a positive constant L_f such that

$$\|k_2(t,\phi_1,x_1) - f(t,\phi_2,x_2)\| \leq L_f(\|\phi_1 - \phi_2\|_{\mathcal{B}} + \|x_1 - x_2\|)$$

where $0 < L_f < 1$, $(t, \phi_i, x_i) \in J \times \mathcal{B} \times E$, $i = 1, 2$.

(H3) The impulsive function I_k are continuous and there exist positive constants β_k such that $\|I_k(u)\| \leq \beta_k$, $k = 1, 2, \ldots, m$, for each $u \in E$

(H4) The linear operator $W : L^2(J, E) \to E$ is defined by $Wx = \int_0^a U(a,s)Gx(s)ds$, has an induced invertible operator W^{-1} which takes values $\dfrac{L^2(J,E)}{\ker W}$ and there exists positive constants M_1 and M_2 such that : $\|G\| \leq M_1$ and $\|W^{-1}\| \leq M_2$.

(H5) There exist a constant $M_0 > 0$ such that $\|A^{-1}(t)\|_{B(E)} \leq M_0$ for all $t \in J$.

(H6) The function $k_1 : J \times \mathcal{B} \to E$ satisfies the following conditions

(1) There exists a constant $0 < L < 1$ such that

$$|A(t)k_1(t,\phi)| \leq L(\|\phi\|_{\mathcal{B}} + 1) \text{ for all } t \in J \text{ and } \phi \in \mathcal{B}.$$

(2) There exists a constant $0 < L_1 < 1$ such that

$$|A(t)k_1(s_1,\phi_1) - A(t)k_1(s_2,\phi_2)| \leq L_1(|s_1 - s_2| + \|\phi_1 - \phi_2\|_{\mathcal{B}})$$

for all $t, s_1, s_2 \in J$ and $\phi_1, \phi_2 \in \mathcal{B}$.

(3) The function k_1 is completely continuous and for any bounded set $Q \subseteq B_a$ the set $\{t \to k_1(t, u_t) : u \in Q\}$ is equicontinuous in $PC(J, E)$

Remark 3.2

For the construction of W and W^{-1} see the paper by Carmichael and Quinn[17].

Theorem 3.1

Suppose that hypotheses (H1)–(H6) are satisfied and moreover there exists a constant $M_3 > 0$ such that

$$\frac{\|z\|_a}{\beta + MK_a \frac{1+MM_1M_2a}{1-M_0LK_a} \times \Omega(\|z\|_a + L_0\mu(\|z\|_a))\|\xi\|_{L^1}} > 1, \tag{3.2}$$

with

$$\beta = \beta(\phi,\mu_1) = (K_aMH + M_a)\|\phi\|_{\mathcal{B}}$$
$$+\frac{K_a}{1-M_0LK_a}\Big\{M_0L(M+1)(1+MM_1M_2a)$$
$$+MM_1M_2a(1+M_0LK_a)|x_1| + M[M_0L(1+MM_1M_2a)$$
$$+M_1M_2a(MH+M_0LM_a)]\|\phi\|_B + M_0La_1$$
$$+M(1+MM_1M_2a)\sum_{k=1}^{m} d_k\Big\}$$

then the neutral evolution problem (1.1)–(1.3) is controllable on $(\infty, a]$.

Proof.

Transform the problem (1.1)–(1.3) into a fixed point problem. Consider the operator $N : B_a \to B_a$ definedby:

$$h(t) = \begin{cases} \phi(t), \quad if\ t \in (-\infty, 0) \\ U(t,o)[\phi(0) - k_1(0,\phi(0))] + k_1(t,u_t) \\ \quad + \int_0^t U(t,s)Gx(s)ds \\ + \int_0^t U(t,s)k_2(s,u_s,\int_0^s e(s,\tau,u_\tau)d\tau)ds \\ \quad + \sum_{0<t_k<t} U(t,t_k)I_k(u(t_k^-)), t \in J. \end{cases} \tag{3.3}$$

Using the assumption (H3), for arbitrary function $x(.)$, we define the control

$$x_u(t) = W^{-1}\Bigg[u_1 - U(a,0)[\phi(0) - k_1(0,\phi(0))] + k_1(a, x_a)$$
$$+\int_0^a U(a,s)k_2\left(s,u_s,\int_0^s e(s,\tau,u_\tau)d\tau\right)ds$$
$$+\sum_{0<t_k<a} U(a,t_k)I_k(u(t_k^-))\Bigg](t).$$

From the hypotheses, we get

$$|x_u(t)| = M_2\Big[|u_1| + M(H+M_0L)\|\phi\|_{\mathcal{B}}$$
$$+M_0L(M+1) + M_0L\|u_a\|_{\mathcal{B}}\Big]$$
$$+M_2M\int_0^a \alpha(s)\Omega\big(\|u_s\|_{\mathcal{B}} + L_0\mu(\|u_\tau\|_{\mathcal{B}})\big)ds$$
$$+M_2M\sum_{k=0}^{m} d_k. \tag{3.4}$$

We shall show that using this control the operator N has a fixed point $u(.)$, which is a mild solution of the neutral evolution system (1.1)–(1.3).

For $\phi \in \mathcal{B}$, we will define the function $u(.) : R \to E$ by

$$u(t) = \begin{cases} \phi(t), & if\ t \in (-\infty, 0] \\ U(t,0)\phi(0) & if\ t \in J, \end{cases}$$

Then $x_0 = \phi$. For each function $z \in B_T$, set

$$u(t) = z(t) + y(t). \tag{3.5}$$

It is obvious that u satisfies (3.1) if and only if z satisfies $z_0 = 0$ and for $t \in J$, we get

$$z(t) = g(t, z_t + y_t) - U(t,0)k_1(0,\phi(0)) + \int_0^t U(t,s)Gx_z(s)ds$$
$$+\int_0^t U(t,s)\times k_2\left(s, z_s + y_s, \int_0^s e(s,\tau,z_\tau + y_\tau)d\tau\right)ds$$
$$+\sum_{0<t_k<t} U(t,t_k)I_k\left(z(t_k^-) + y(t_k^-)\right), t \in J.$$

Let $B_a^0 = \{z \in B_a : z_0 = 0\}$. For any $z \in B_a^0$ we have

$$\|z\|_a = \sup\{|z(t)| : t \in J\} + \|z_0\|_{\mathcal{B}} = \sup\{|z(t)| : t \in J\}.$$

Thus $\left(B_a^0, \|.\|_a\right)$ is a Banach space. Define the operator $F : B_a^0 \to B_a^0$ by :

$$F(z)(t) = k_1(t, z_t + y_t) - U(t,0)k_1(0,\phi(0)) + \int_0^t U(t,s)Gx_z(s)ds$$
$$+\int_0^t U(t,s)\times k_2\left(s, z_s + y_s, \int_0^s e(s,\tau, z_\tau + y_\tau)d\tau\right)ds$$
$$+\sum_{0<t_k<t} U(t,t_k)I_k\left(z(t_k^-) + y(t_k^-)\right). \tag{3.6}$$

Obviously the operator N has a fixed point is equivalent to F has one, so it turns to prove that F has a fixed point. The proof will be given in several steps.

Let us first show that the operator F is continuous and compact.

Step 1. F is Continuous.
Let (z_n) be a sequence in B_a^0 such that $z_n \to z$ in B_a^0. Then using (3.4) and k_2 is L^1-Caratheodory, we get , we obtain by Lebesgue dominated convergence theorem

$$|F(z_n)(t) - F(z)(t)| \to 0 \text{ as } n \to +\infty.$$

Thus F is continuous.

Step 2. F maps bounded sets of B_a^0 into bounded sets. For any $d > 0$, there exists a positive constant l such that for each $z \in B_d = \{z \in B_a^0 : ||z||_a \le d\}$ one has $||F(z)||_a \le$ l.

Let $z \in B_d$. By (3.4) and using the assumption (A1), we get

$$\begin{aligned} ||z_s + y_s||_{\mathcal{B}} &\le ||z_s||_{\mathcal{B}} + ||y_s||_{\mathcal{B}} \le K(s)|z(s)| + M(s)||z_0||_{\mathcal{B}} \\ &\quad + K(s)|x(s)| + M(s)||x_0||_{\mathcal{B}} \\ &\le K_a|z(s)| + (K_a MH + M_a)||\phi||_{\mathcal{B}}. \end{aligned}$$

Set $a_1 = (K_a MH + M_a)||\phi||_{\mathcal{B}}$ and $b = K_a d + a_1$. Then

$$||z_s + y_s||_{\mathcal{B}} \le K_a|z(s)| + a_1 \le b. \tag{3.7}$$

Using the nondecreasing character of Ω, we get for each $t \in J$

$$\begin{aligned} |F(z)(t)| &\le [MM_0 + M_0 LH + MM_1M_2aM(H + M_0L)]||\phi||_{\mathcal{B}} \\ &+ MM_1M_2a|u_1| + M_0L[M + 1 + b + MM_1M_2a(M + 1 + b)] \\ &+ M(1 + MM_1M_2a)\int_0^a \alpha(s)\Omega(b + L_0\mu(b))ds \\ &+ M(1 + MM_1M_2a)\sum_{k=0}^m d_k := l. \end{aligned}$$

Thus there exists a positive number l such that $|F(z)(t)| \le l$.

Hence $F(B_d) \subset B_d$.

Step 3. F maps a bounded sets into equicontinuous sets of B_a^0. We consider B_d in Step 2 and we show that $F(B_d)$ is equicontinuous.

Let $\tau_1, \tau_s \in J$ with $\tau_2 > \tau_1$ and $z \in B_d$.

By the inequalities (3.4) and (3.7) and using the nondecreasing character Ω, we get

$$\begin{aligned} |x_u(t)| &\le M_2\Big[|u_1| + M(H + M_0L)\|\phi\|_{\mathcal{B}} + M_0L(M + 1) \\ &\quad + M_0L\|x_\alpha\|_{\mathcal{B}}\Big] + M_2M\|\alpha\|_{L^1}\,\Omega(b + L_0\mu(b))ds \\ &\quad + M_2M\sum_{k=0}^m d_k := w \end{aligned} \tag{3.8}$$

Noting that $|F(z)(\tau_2) - F(z)(\tau_1)|$ tends to zero as $\tau_2 - \tau_1 \to 0$ independently of $z \in B_d$. The right-hand side of the above inequality tends to zero as $\tau_2 - \tau_1 \to 0$.

Since $U(t, s)$ is a strongly continuous operator and the compactness of $U(t, s)$ for $t > s$ implies the continuity in the uniform operator topology (see[2,16]). As a consequence of Steps 1 to 3 together with the Arzela-Ascoli theorem it suffices to show that the operator F maps $z \in B_d$ into a precompact set in E.

Let $t \in J$ be fixed and let $\in$ be the real number satisfying $0 < \in < t$. For $z \in B_d$ we define

$$\begin{aligned} F_\in(z)(t) &= k_1(t, z_t + y_t) - U(t,0)k_1(0, \phi(0)) \\ &\quad + U(t, t-\in)\int_0^{t-\in} U(t-\in, s)\, G\, x_z(s)ds \\ &\quad + U(t, t-\in)\int_0^{t-\in} U(t-\in, s)\, k_2(s, z_s + y_s, \\ &\quad \int_0^s e(s, \tau, z_\tau + y_\tau)d\tau)ds \\ &\quad + U(t, t-\in)\sum_{0<t_k<t-\in} U(t-\in, s)|I_k(z(t_k^-) + y(t_k^-))|. \end{aligned}$$

Since $U(t, s)$ is a compact operator, the set $z_\in(t) = \{F_\in(z)(t) : z \in B_d\}$ is precompact in E for every $\in$ sufficiently small, $0 < \in < t$. Moreover using (3.8), we have

$$\begin{aligned} |F(z)(t) - F_\in(z)(t)| &\le M_1 w\int_{t-\in}^t \|U(t, s)\|_{B(E)}\, ds \\ &\quad + \Omega(b + L_0\mu(b))\int_{t-\in}^t \|U(t, s)\|_{B(E)}\, \alpha(s)\, ds \\ &\quad + \sum_{k=0}^m \|U(t, s)\|_{B(E)}\, dk. \end{aligned}$$

Therefore there are precompact sets arbitrary close to the set $\{F(z)(t) : z \in B_d\}$. Hence the set $\{F(z)(t) : z \in B_d\}$ is precompact in E. So we deduce from Steps 1, 2 and 3 that F is a compact operator.

Step 4. For applying Theorem 2.1, we must chect (C2): i.e. it remains to show that the set

$$\zeta = \{z \in B^a : z = \lambda F(z) \text{ for some } 0 \le \lambda < 1\}$$

is bounded.

Let $z \in \zeta$. By (3.4), we have for each $t \in J$

$$\begin{aligned} |z(t)| &\le M_0L(M + 1)(1 + MM_1M_2a) + MM_1M_2a|u_1| \\ &\quad + M\Big[M_0L(1 + MM_1M_2a) + MM_1M_2aH\Big]\|\phi\|_{\mathcal{B}} \\ &\quad + MM_1M_2aM_0L||z_a + y_a||_{\mathcal{B}} + M_0\mathrm{L}||z_t + y_t||_{\mathcal{B}} \\ &\quad + M(1 + MM_1M_2a)\int_0^a \alpha(s)\,\Omega\Big(||z_s + y_s||_{\mathcal{B}} \\ &\quad + L_0\mu(||z_\tau + y_\tau||_{\mathcal{B}})\Big)ds. \\ &\quad + M(1 + MM_1M_2a)\sum_{k=1}^m d_k. \end{aligned}$$

Noting that we have $||z_a + y_a||_{\mathcal{B}} \le K_a|u_1|+M_a||\phi||_{\mathcal{B}}$ and using the first inequality $||z_s + y_s||_{\mathcal{B}} \le K_a|z(s)| + a_1$ in (3.7), then by nondecreasing character Ω, we obtain

$$\begin{aligned}|z(t)| &\le M_0L(M+1)(1+MM_1M_2a)+MM_1M_2a(1+M_0LK_a)|u_1| \\ &+M\left[M_0L(1+MM_1M_2a)+M_1M_2a(MH+M_0LM_a)\right]\|\phi\|_{\mathcal{B}} \\ &+M_0L(K_a|z(t)|+a_1)+M(1+MM_1M_2a)\sum_{k=1}^{m}d_k \\ &+M(1+MM_1M_2a)\int_0^a \alpha(s)\,\Omega\left((K_a|z(s)|+a_1)\right. \\ &\left.+L_0\mu(K_a|z(\tau)|+a+1)\right)ds.\end{aligned}$$

Then

$$\begin{aligned}(1-M_0LK_a)|z(t)| &\le M_0L(M+1)(1+MM_1M_2a) \\ &+MM_1M_2a(1+M_0LK_a)|u_1| \\ &+M\left[M_0L(1+MM_1M_2a)+M_1M_2a(MH+M_0LM_a)\right]\|\phi\|_{\mathcal{B}} \\ &+M_0L\,a_1+M(1+MM_1M_2a)\sum_{k=1}^{m}d_k \\ &+M(1+MM_1M_2a)\int_0^a \alpha(s)\,\Omega\left((K_a|z(s)|+a_1)\right. \\ &\left.+L_0\mu(K_a|z(\tau)|+a_1)\right)ds.\end{aligned}$$

Set

$$\begin{aligned}\beta &:= a_1+\frac{K_a}{1-M_0LK_a}\{M_0L(M+1)(1+MM_1M_2a) \\ &+MM_1M_2a(1+M_0\,LK_a)|u_1| \\ &+M\left[M_0L(1+MM_1M_2a)+M_1M_2a(MH+M_0LM_a)\right]\|\phi\|_{\mathcal{B}} \\ &+M_0L\,a_1+M(1+MM_1M_2a)\sum_{k=1}^{m}d_k\}.\end{aligned}$$

Thus

$$\begin{aligned}K_a|z(t)|+a_1 &\le \beta+\frac{MK_a}{1-M_0LK_a} \\ &\times(1+MM_1M_2a)\int_0^a \alpha(s)\,\Omega\left((K_a|z(s)|+a_1)\right. \\ &\left.+L_0\mu(K_a|z(\tau)|+a_1)\right)ds.\end{aligned}$$

We consider the function r defined by

$$r(t):=\sup\{K_a|z(s)|+a_1\ :\ 0\le s\le t\},\ 0\le t\le a.$$

Let $t^* \in [0, t]$ be such that $r(t) = K_a|z(t^*)| + a_1$. If $t \in J$, by the previous inequality, we have $t \in J$

$$\begin{aligned}r(t) &\le \beta+\frac{MK_a}{1-M_0LK_a} \\ &\times(1+MM_1M_2a)\int_0^a \alpha(s)\Omega\left((r(s))+L_0\mu(r(\tau))\right)ds.\end{aligned}$$

Set $\xi(t) := \max(\alpha(s))$ for $t \in J$

$$r(t)\le \beta+MK_a\frac{1+MM_1M_2a}{1-M_0LK_a}\times\int_0^T \xi(s)\Omega\left((r(s))+L_0\mu(r(\tau))\right)ds.$$

consequently,

$$\frac{\|z\|_a}{\beta+MK_a\dfrac{1+MM_1M_2a}{1-M_0LK_a}\times\Omega\left(||z||_a+L_0\mu(||z||_a)\right)||\xi||_{L^1}}\le 1.$$

Then by (3.2), there exists a constant M_3 such that $||z||_a \ne M_3$. Set $v = \{z \in B_a^0 : ||z||_a \le M_3 + 1\}$. Clearly v is a closed subset of B_a^0. From the choice of u there is no $v \in \partial v$ such that $z = \lambda F(z)$ for some $\lambda \in (0, 1)$. Then the statement (C2) is theorem 2.1 does no hold. As a consequence of the nonlinear alternative of Leray-Schauder type([7]), we deduce that (C1) holds: i.e. the operator F has a fixed point z^*. Then $u*(t) = z*(t) + y(t)$, $t \in (-\infty, a]$ is a fixed point of the operator N, which is a mild solution of the problem (1.1)–(1.3). Thus the evolution system (1.1)–(1.3) is controllable on $(-\infty, a]$.

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