

A New Approach to Measuring Congestion in DEA with Common Weights

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Abstract

Congestion is an important topic in economics and data envelopment analysis. It occurs when the output increases by reducing one or more inputs without improving any other inputs or outputs. Conversely, congestion occurs when the output reduces by increasing one or more inputs without improving any other inputs or outputs. In this article, we have focused on a work that proposes less computation for measuring congestion. In addition, this article also propose a new method for measuring congestion in decision making units by using common weights based on comparison of inputs.

Keywords: Common Weights, Congestion, Data Envelopment Analysis

1. Introduction

In economics, congestion is said to occur when an increase in one or more inputs can be associated with a decrease in one or more outputs, without improving any other inputs or outputs¹. First, the research on congestion began by Fare and Svensson in 1980². Then, it was completed in 1983 and 1985 by the Fare and Grosskopf^{3,4}. They presented a model according to the concept of Data Envelopment Analysis (DEA). Another approach was presented by Cooper in 1996⁵. Brockett et al in 1996⁶ and Cooper et al. in 2001⁷ developed a new DEA-based approach to measure input congestion. While a significant literature exists on this subject^{8,9}; but, the two latter methodologies are considered to be fundamental congestion consideration. Another notable method for measuring congestion is the method of Noura et al¹⁰, which is based on a comparison of inputs. But recently it is shown that this method does not identify congestion in some examples. So in this paper we revise the method of Noura et al. and then we focus on proposing a new methodology for congestion by common weights.

To clarify the concept of congestion, we first analysis Cooper et al's method as extended in¹¹. Suppose we have n Decision Making Units (DMUs) with m inputs and s outputs, and the vectors $x_j = (x_{1j}, x_{2j}, \dots, x_{mj})^T$

and $y_j = (y_{1j}, y_{2j}, \dots, y_{sj})^T$ denote the input and output values of DMU_j , $j = 1, \dots, n$, respectively. First, they solve the output-oriented BCC (Banker, Charnes, Cooper)^{12–14} model (model 1), in order to obtain the efficiency of each DMU.

$$\begin{aligned} \phi_o^* &= \text{Max } \phi + \varepsilon \left[\sum_{i=1}^m s_i^- + \sum_{r=1}^s s_r^+ \right] \\ \text{s.t. } \sum_{j=1}^n \lambda_j x_{ij} + s_i^- &= x_{io} \quad i=1, \dots, m \\ \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ &= \phi y_{ro} \quad r=1, \dots, s \\ \sum_{j=1}^n \lambda_j &= 1 \\ \lambda_j &\geq 0 \quad j=1, \dots, n \\ s_i^- &\geq 0 \quad i=1, \dots, m \\ s_r^+ &\geq 0 \quad r=1, \dots, s \end{aligned} \quad (1)$$

In above model, $\varepsilon > 0$ is a Non-Archimedean element smaller than any positive real number.

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For an optimal solution $(\varphi^*, \lambda^*, S^{+*}, S^{-*})$ of (1), we can rewrite its constraints in the following form:

$$\sum_{j=1}^n \lambda_j^* x_{ij} = x_{io} - s_i^{-*} \quad i = 1, \dots, m \quad (2)$$

$$\sum_{j=1}^n \lambda_j^* y_{rj} = \varphi^* y_{ro} + s_r^{+*} \quad r = 1, \dots, s \quad (3)$$

Now, we can use the values on the right-hand side (RHS) in (2) and (3) to define new outputs and inputs, $\hat{y}_{ro}; r = 1, \dots, s$ and $\hat{x}_{io}; i = 1, \dots, m$, as in the following:

$$\hat{y}_{ro} = \varphi^* y_{ro} + s_r^{+*} \geq y_{ro} \quad r = 1, \dots, s \quad (4)$$

$$\hat{x}_{io} = x_{io} - s_i^{-*} \leq x_{io} \quad i = 1, \dots, m \quad (5)$$

Note that $\hat{y}_{ro}, \hat{x}_{io}$ are coordinates of points on the efficiency frontier. In this method, inefficiency is a necessary condition for the presence of congestion. Therefore, they first use (1) to identify whether DMU_o is inefficient. If it is found to be so, they utilize (2) and (3) to formulate (6):

$$\begin{aligned} & \text{Max} \sum_{i=1}^m \delta_i^- \\ \text{s.t.} \quad & \hat{y}_{ro} = \varphi^* y_{ro} + s_r^{+*} = \sum_{j=1}^n \lambda_j y_{rj} \quad r = 1, \dots, s \\ & \hat{x}_{io} = x_{io} - s_i^{-*} = \sum_{j=1}^n \lambda_j x_{ij} \quad i = 1, \dots, m \\ & \sum_{j=1}^n \lambda_j = 1 \\ & s_i^{-*} \geq \delta_i^- \quad i = 1, \dots, m \\ & \delta_i^- \geq 0 \quad i = 1, \dots, m \\ & \lambda_j \geq 0 \quad j = 1, \dots, n \end{aligned} \quad (6)$$

Then, to obtain the level of congestion, they use the input constraints at the bottom of (6):

$$\sum_{j=1}^n x_{ij} \lambda_j - \hat{x}_{ij} = \delta_i^- \quad i = 1, \dots, m \quad (7)$$

$$s_i^{-c} = s_i^- - \delta_i^- \quad i = 1, \dots, m \quad (8)$$

Substituting (8) into (6), they can rewrite the latter as (9):

$$\begin{aligned} & \text{Min} \sum_{i=1}^m s_i^{-c} \\ \text{s.t.} \quad & \varphi^* y_{ro} + s_r^{+*} = \sum_{j=1}^n \lambda_j y_{rj} \quad r = 1, \dots, s \\ & x_{io} - s_i^{-c} = \sum_{j=1}^n \lambda_j x_{ij} \quad i = 1, \dots, m \\ & \sum_{j=1}^n \lambda_j = 1 \\ & s_i^{-c} \geq 0 \quad i = 1, \dots, m \\ & \lambda_j \geq 0 \quad j = 1, \dots, n \end{aligned} \quad (9)$$

By using ε , Cooper et al^{11,15,16} combined these steps into the single model of (10):

$$\begin{aligned} & \text{Max} \quad \varphi + \varepsilon \left[\sum_{r=1}^s s_r^+ - \sum_{i=1}^m s_i^{-c} \right] \\ \text{s.t.} \quad & \sum_{j=1}^n \lambda_j x_{ij} + s_i^{-c} = x_{io} \quad i = 1, \dots, m \\ & \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ = \varphi y_{ro} \quad r = 1, \dots, s \\ & \sum_{j=1}^n \lambda_j = 1 \\ & \lambda_j \geq 0 \quad j = 1, \dots, n \\ & s_i^{-c} \geq 0 \quad i = 1, \dots, m \\ & s_r^+ \geq 0 \quad r = 1, \dots, s \end{aligned} \quad (10)$$

In order to detect the presence of congestion in a DMU, Cooper et al¹¹ introduced the following theorem:

THEOREM 1.

Congestion exists if and only if in an optimal solution ($\varphi^*, \lambda^*, s^{+*}, s^{-c*}$) of (10), at least one of the following two conditions is satisfied:

- (i) $\varphi^* > 1$ and there is at least one $s_i^{-c*} > 0 (1 \leq i \leq m)$.
- (ii) There exists at least one $s_r^{+*} > 0 (1 \leq r \leq s)$ and at least one $s_i^{-c*} > 0 (1 \leq i \leq m)$.

2. Noura's Method for Measuring Congestion by Comparing Inputs

Noura et al¹⁰ have proposed a method for measuring congestion by comparing inputs. In this section, we review this method.

Suppose we have n DMUs with m inputs and s outputs, and the vectors $x_j = (x_{1j}, x_{2j}, \dots, x_{mj})^T$ and $y_j = (y_{1j}, y_{2j}, \dots, y_{sj})^T$ denote input and output values of $DMU_j, j = 1, \dots, n$, respectively. First in order to obtaining the efficiency of each DMU, they solve model 1 (BCC model with output-oriented). Then, they define set of E as follows:

$$E = \{j : \varphi_j^* = 1\} \quad (11)$$

Among the DMUs in set E , there exists at least one DMU; that has the highest consumption in its first input component compared with the first input component of the remaining DMUs of set E . This means that,

$$\exists (l \in E) \text{ s.t. } \forall j (j \in E) \Rightarrow x_{1l} \geq x_{1j} \quad (12)$$

They denote x_{1l} by x_1^* . Then they find, again, among the DMUs in E , a DMU, say DMU_t , that has the highest consumption in its second input component compared to the remaining DMUs in E . In other words,

$$\exists (t \in E) \text{ s.t. } \forall j (j \in E) \Rightarrow x_{2t} \geq x_{2j} \quad (13)$$

They denote x_{2t} by x_2^* . In a similar manner, for all input components $i = 1, \dots, m$; They can identify a DMU in E whose i th input consumption is higher than that of all other DMUs in the set. They denote such an input by $x_i^*, i = 1, \dots, m$. Note that $x_1^*, x_2^*, \dots, x_m^*$ need not necessarily be selected from a single DMU. Then, they define congestion as follows:

DEFINITION 1.

Congestion is present if and only if, in an optimal solution ($\varphi^*, \lambda^*, S^{+*}, S^{-*}$) of (1) for DMU_o , at least one of the following two conditions is satisfied:

- (i) $\varphi^* > 1$, and there is at least one $x_{io} > x_i^*, i = 1, \dots, m$.
- (ii) There exists at least one $s_r^* > 0 (r = 1, \dots, s)$, and at least one $x_{io} > x_i^*, i = 1, \dots, m$.

They denote the amount of congestion in the i th input of DMU_o by $s_i^{c'}$ where $x_{io} > x_i^*$ and define it as:

$$s_i^{c'} = x_{io} - x_i^* \quad (14)$$

Congestion is considered not present when $x_{io} > x_i^*$ and $s_i^{c'} = 0$. The sum of all $s_i^{c'}$ is the amount of congestion in DMU_o .

It should be noted that this definition only specifies the congestion of some points; particularly in dimension more than one. So revised this method is presented in the next section.

2.1 Modification of Noura et al.'s Method in Measuring Congestion

After reviewing the above method, it was determined that the methodology can't identify congestion in some cases. To clarify this important, the following example is introduced.

EXAMPLE 1.

Table 1 shows five DMUs with two inputs and two outputs. Moreover, It is included the results of model (1).

By applying the method of Noura et al. for above example and Due to the efficiency of DMUs, we have

$$E = \{DMU_1, DMU_2, DMU_3\}$$

So $x^* = (4, 6)$, therefore DMU_4 have congestion in the first input. But according to the definition of congestion

Table 1. Five DMUs with Two Inputs and Two Outputs

	I_1	I_2	O_1	O_2	I_2	φ^*
DMU_1	2	2	2	2	2	1
DMU_2	4	4	4	4	4	1
DMU_3	4	6	4	2	6	1
DMU_4	6	6	2	2	6	2
DMU_5	3.5	5	1	1	5	3.5

(cooper's definition¹), it is clear that DMU_3 and DMU_5 have congestion, which this method can't determine their congestion.

With respect to this arise problem in Noura et al.'s method, Hosseinzadeh et al.¹⁶ have modified it. They proposed the following model (15) for any DMU_o .

$$\begin{aligned} \text{Max } z &= \sum_{r=1}^s s_r^+ \\ \text{s.t. } \sum_{j=1}^n \lambda_j x_{ij} &\leq x_{io} \quad i = 1, \dots, m \\ \sum_{j=1}^n \lambda_j y_{rj} - s_r^+ &= y_{ro} \quad r = 1, \dots, s \\ \sum_{j=1}^n \lambda_j &= 1 \\ s_r^+, \lambda_j &\geq 0 \quad r = 1, \dots, s, \quad j = 1, \dots, n \end{aligned} \quad (15)$$

Suppose, (λ^*, z^*) are the optimal solution of this model. Note that, z^* for DMU_j is shown by z_j^* . As Noura et al. they defined E as follows:

$$E = \{j : z_j^* = 0\} \quad (16)$$

Highest value of each input is evaluated for all components in set of E and then, the following revised definition is suggested to identifying congestion.

DEFINITION 2.

Congestion occurs if and only if, in every optimal solution of (15) for DMU_o , the following condition is satisfied:

$z^* > 0$, and there is at least one $x_{io} > x_i$, $i = 1, \dots, m$.

Now, applying this modified method for above example, we get:

$$E = \{DMU_1, DMU_2\}$$

and $x^* = (4, 4)$ so, DMU_3, DMU_4, DMU_5 have congestion by this method. Therefore, arisen problem in Noura et al.'s method was revised.

3. The Proposed Method

One of the famous basic DEA model is "Multiplier form of BCC with output oriented"¹². The efficiency for DMU_o

is evaluated by this model that each $DMUs$ have m inputs and s outputs:

$$\begin{aligned} \text{Min } & \frac{v^t x_o + v_o}{u^t y_o} \\ \text{s.t. } & \frac{v^t x_j + v_o}{u^t y_j} \geq 1 \quad j = 1, \dots, n \\ & u^t \geq 1_s \varepsilon \\ & v^t \geq 1_m \varepsilon \end{aligned} \quad (17)$$

where the decision variables are the weight vectors $u^t = (u_1, \dots, u_s)$, $v^t = (v_1, \dots, v_m)$ and $x_j^t = (x_{1j}, \dots, x_{mj})$, $y_j^t = (y_{1j}, \dots, y_{sj})$ are the input and output vectors for $DMU_j (j = 1, \dots, n)$. If the optimal value of above model is equal to one DMU_o is efficient, otherwise it is inefficient. The equivalent linear programming of above model will be substituted as follows:

$$\begin{aligned} \text{Min } & v^t x_o + v_o \\ \text{s.t. } & u^t y_o = 1 \\ & v^t x_j + v_o - u^t y_j \geq 0 \quad j = 1, \dots, n, \\ & u^t \geq 1_s \varepsilon \\ & v^t \geq 1_m \varepsilon \end{aligned} \quad (18)$$

As we can see in feasible region of model 18, $v^t x_j + v_o - u^t y_j \geq 0 (j = 1, \dots, n)$. In this model the DMU is more preferable when it has the smaller value. Hence when we minimize the $v^t x_j + v_o - u^t y_j \geq 0 (j = 1, \dots, n)$, we can reach our purpose. Therefore instead to solve above model, one may minimize the $v^t x_j + v_o - u^t y_j \geq 0 (j = 1, \dots, n)$, with respect to same region. So based on Noura et al. methodology¹², we proposed the following Multi Objective Linear Programming (MOLP) to clarifying CSW.

$$\begin{aligned} \text{Min } & v^t x_j + v_o - u^t y_j \quad j = 1, \dots, n \\ \text{s.t. } & v^t x_j + v_o - u^t y_j \geq 0 \quad j = 1, \dots, n, \\ & u^t \geq 1_s \varepsilon \\ & v^t \geq 1_m \varepsilon \end{aligned} \quad (19)$$

The CSW with equal weights is applied to solve the above MOLP as follows:

$$\begin{aligned}
 & \text{Min } \sum_{j=1}^n (v^t x_j + v_o - u^t y_j) \\
 & \text{s.t. } v^t x_j + v_o - u^t y_j \geq 0 \quad j = 1, \dots, n \\
 & \quad u^t \geq 1_s \varepsilon \\
 & \quad v^t \geq 1_m \varepsilon
 \end{aligned} \quad (20)$$

Suppose $(u^*, v^*)^t$ will be the optimal solution of the model (20), then it is considered as CSW and for ranking and comparing DMUs, we use the efficiency score of DMU_j ($j = 1, \dots, n$) as $\phi_j^* = \frac{v^{*t} x_j + v_o^*}{u^{*t} y_j}$. When ϕ_j^* is equal to one we can conclude that the DMU under evaluation (DMU_j) is efficient.

Now, As Noura et al¹⁰ E is defined as follows:

$$E = \{j : \phi_j^* = 1\}$$

And x_i^* is defined the highest value of each input for all components in set of E, the following revised definition is suggested to identifying congestion.

DEFINITION 3.

Congestion in DMU_o eventually occurs if for the optimal solution of ϕ_o^* for DMU_o , the following condition is satisfied:

$\phi_o^* > 1$, and there is at least one $x_{io} > x_i^*$, $i = 1, \dots, m$.

The amount of congestion in the i th input of DMU_o is shown by $s_i^{c'}$ as follow:

$$s_i^{c'} = x_{io} - x_i^*$$

The sum of all $s_i^{c'}$ is the amount of congestion in DMU_o .

Congestion does not present when $x_{io} \leq x_i^*$ or $s_i^{c'} = 0$ for all $i = 1, \dots, m$.

THEOREM 2.

Amount of congestion in proposed method is equal amount of congestion in Cooper method; but vice versa is not true.

PROOF.

Reference 8.

THEOREM 3.

If $\forall i, i = 1, \dots, m$, we have $x_{ij} - x_i^* \leq 0$, then there exists no congestion in DMU_j .

PROOF.

Reference 8.

Now, we are using modified method for example 1. For this example, we have with respect to optimal solution of model 20:

$$E = \{DMU_1, DMU_2\}$$

So $x^* = (4, 4)$ and therefore, DMU_3, DMU_4, DMU_5 that have congestion by the proposed method.

Our proposed method is similar to the method of Noura et al⁸ with the advantage of this method is to solve one linear programming for all DMUs by the same condition instead of n linear programming. Therefore, this method greatly reduces the computational. Moreover, it provides an easy understandable knowledge of congestion.

4. Numerical Examples

In this section, we apply our proposed methodology to selected examples that have been previous analyzed using alternative approaches.

4.1 Example 2

Consider eight DMUs, A, B, C, D, E, F, G and H, with one input and one output. It is shown in Figure 1. This example was solved by Cooper et al¹¹ using Model (10), the results of which are provided in Table 2.

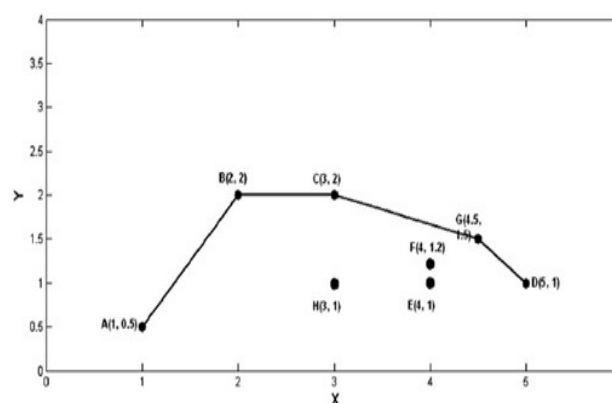


Figure 1. Numerical example 2, source: Cooper et al¹¹.

Table 2. Results from Cooper et al's approach

DMU	A	B	C	D	E	F	G	H
ϕ^*	1	1	1	2	2	1.667	1.667	2
s^{+*}	0	0	0	0	0	0	0	0
s^{-c*}	0	0	0	2	1	1	1.5	0

We now apply our proposed method to solve the same problem. Considering the efficiency of the DMUs, we have $E = \{B\}$, where $x^* = 2$. As it is well-known, the necessary condition for congestion is inefficiency. Thus, congestion is calculated as follows using the proposed method.

$$\phi_A^* > 1 \quad x_A = 1 \quad x_A \leq x_B^* \quad \text{so there is not congestion}$$

$$\phi_C^* > 1 \quad x_C = 3 \quad s_1^{c'} = x_C - x_B = 3 - 2 = 1 \quad \text{Congestion occur}$$

$$\phi_D^* > 1 \quad x_D = 5 \quad s_1^{c'} = 5 - 2 = 3 \quad \text{Congestion occur}$$

$$\phi_E^* > 1 \quad x_E = 4 \quad s_1^{c'} = 4 - 2 = 2 \quad \text{Congestion occur}$$

$$\phi_F^* > 1 \quad x_F = 4 \quad s_1^{c'} = 4 - 2 = 2 \quad \text{Congestion occur}$$

$$\phi_G^* > 1 \quad x_G = 4.5 \quad s_1^{c'} = 4.5 - 2 = 2.5 \quad \text{Congestion occur}$$

$$\phi_H^* > 1 \quad x_H = 3 \quad s_1^{c'} = 3 - 2 = 1 \quad \text{Congestion occur}$$

Comparing of the proposed method and Noura et al. and Cooper et al. are given in Table 3.

As we see in the results, Cooper et al. and Noura et al. don't show congestion in DMU_C and DMU_H while if one notice to Figure 1 it is clear that congestion in DMU_C and DMU_H are occurred instead our proposed method is shown the congestion in DMU_C and DMU_H which is compatible with data.

4.2 Example 3

Consider the six hypothetical DMUs, A, B, C, D, G and R, in Figure 2, each using two inputs, x_1 and x_2 , to produce one output, y . Figure 2 illustrates a pyramid with R at its

Table 3. Comparing of proposed method and other methods

DMU	Cooper et al. method	Noura et al. method	Proposed method
A	No congestion	No congestion	No congestion
B	No congestion	No congestion	No congestion
C	No congestion	No congestion	Congestion
D	Congestion	Congestion	Congestion
E	Congestion	Congestion	Congestion
F	Congestion	Congestion	Congestion
G	Congestion	Congestion	Congestion
H	No congestion	No congestion	Congestion

vertex, producing an input of $y = 10$. The output from all other DMUs is $y = 1$.

This problem has been solved in Cooper et al.¹¹, Congestion and efficiency of DMUs are presented in Table 4.

In order to solve the above example using our method, we will get $E = \{R\}$ and $x_R^* = (5, 5)$. Then, congestion is calculated as follows using the proposed method.

$$\phi_A^* > 1 \quad x_A = (5, 1) \quad x_A \leq x_R^* \quad \text{so there is not congestion}$$

$$\phi_B^* > 1 \quad x_B = (1, 5) \quad x_B \leq x_R^* \quad \text{so there is not congestion}$$

$$\phi_C^* > 1 \quad x_C = (5, 10) \quad s_1^{c'} = 0 \quad s_2^{c'} = 10 - 5 = 5 \quad \text{Congestion occur}$$

$$\phi_D^* > 1 \quad x_D = (10, 5) \quad s_1^{c'} = 10 - 5 = 5 \quad s_2^{c'} = 0 \quad \text{Congestion occur}$$

$$\phi_G^* > 1 \quad x_G = (7.5, 7.5) \quad s_1^{c'} = 7.5 - 5 = 2.5 \quad s_2^{c'} = 7.5 - 5 = 2.5 \quad \text{Congestion occur}$$

As can be observed, our results are identical to those obtained by Cooper et al. (Table 4).

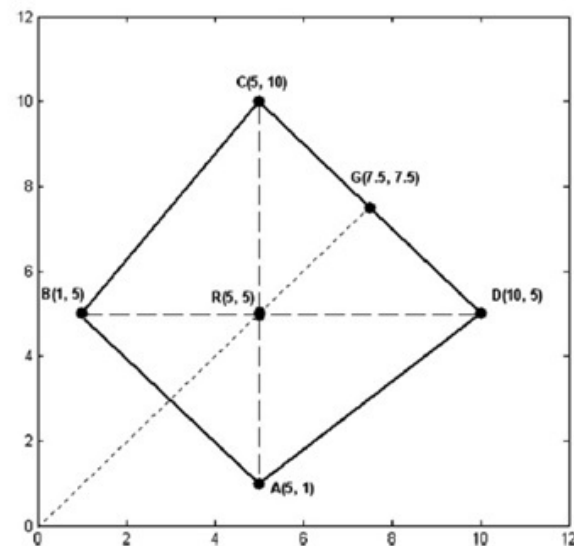


Figure 2. Numerical example 3, source: Cooper et al.⁷.

Table 4. Results from Cooper et al. approach

DMU	A	B	C	D	G	R
ϕ^*	1	1	10	10	10	1
s_1^{-c*}	0	0	0	5	2.5	0
s_2^{-c*}	0	0	5	0	2.5	0

5. Conclusion

In this paper, we revised Noura et al.'s method and then, proposed a new method for calculating congestion with applying common weights based on comparing inputs. Common weights are applied to all units of the same priority. The main advantage of the proposed method is to solve one linear programming for all DMUs. Therefore, this method greatly reduces the computational burden. Moreover, it provides an easily understandable knowledge of congestion. Finally, the proposed method was illustrated by examples.

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