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Design of Multivariable Systems Controlled by Novelty Based Techniques

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Abstract

This paper proposes to build up a rotary inverted pendulum; its state space model was derived using Euler-Lagrange equation. This model was highly nonlinear. Stabilization and Self erecting a rotary inverted pendulum from sliding position and assessment of the pendulum in a straight up position was achieved by designing a control techniques like minimum order model, dead bead controller and Linear quadratic Regulator (LQR) using MATLAB domain. This concept was used in JCB, GRAIN and entertainment instrument in park.

Keywords: Quadratic Optimal Control, Real Time Control, Self Erecting

1. Introduction

The form of rotary inverted pendulum imitative the mechanical form by using Euler-Lagrange equation in state space form[1]. The Rotary Inverted Pendulum is a typical control problem that is explored often as a project in control courses due to its easily developed dynamics combined with its complexity of control design. The rotary inverted pendulum was controlled by techniques in real time earlier[2,3]. The pendulum was stabilized by FLC in simulink environment[4]. A rotary inverted pendulum was stabilized using Sliding mode control, Minimum Time Swing Up[5,6]. Although, the problem can be solved using conventional control techniques like PD/PID controllers[7], and soft computing techniques[8,9,10], the pendulum was controlled by intelligent techniques[11]. One can use other powerful techniques involving state space analysis also. The system is composed of a pendulum fond of to the end of a rotary arm controlled by a motor. The control input is in the form of voltage input to the motor. Here our objective is to stabilize the pendulum.

2. Experimental Setup and Modelling

This system contains a DC motor, an arm, controller and a pendulum as shown in Figure 1. The controller makes the pendulum stand at upright position on the rotary arm by moving the arm support of the base. The motor provides control to rotate the arm.

Here,

- θ – motor angle.
- A – pendulum angle.
- $\dot{\theta}$ – Motor velocity.
- $\dot{\alpha}$ – Pendulum velocity.

The principle of Lagrange and uses the Euler-Lagrange equation to calculate the non-linear equation of motion.

Let the potential energy be,

$$V_t = M_p g y_p$$

Resulting in,

$$V_t = M_p g(h - 1_p \cos(\alpha(t)))$$

The total kinetic energy includes the turning kinetic energy of the arm and pendulum and the translational kinetic energy of the pendulum COG.

$$T_t = T_{rot,arm} + T_{rot,pend} + T_{trans,pend}$$

The rotational kinetic energy of the arm is

$$T_{rot,arm} = \frac{1}{2} J_{eq} \left(\frac{d}{dt} \theta(t) \right)^2$$

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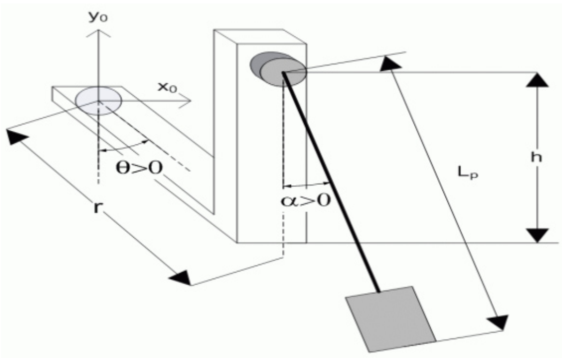

Figure 1. Experimental setup.

Rotational kinetic energy of the pendulum is,

$$T_{rot,pend} = \frac{1}{2} J_p \left(\frac{d}{dt} \theta(t) \right)^2$$

$$T_{trans,pend} = \frac{1}{2} M_p \left(\left(\frac{d}{dt} x_p(t) \right)^2 + \left(\frac{d}{dt} y_p(t) \right)^2 \right)$$

The Lagrangian of a system is,

$$L = T_t - V_t$$

Where,

Tt is the total kinetic energy of the system and Vt is the potential energy of the system.

On substituting $q_1 = \theta$ and $q_2 = \alpha$ in kinetic and potential energy equation we get,

Lq is quadratic structure given as,

$$d_{11}(q_1)\left(\frac{d^2}{dt^2} q_1(t) \right) + 2d_{12}(q_1, q_2)\left(\frac{d}{dt} q_2(t) \right)\left(\frac{d}{dt} q_1(t) \right)$$
$$+ d_{22}(q_2)\left(\frac{d^2}{dt^2}(q_2(t)) \right) - V_t(q) = \tau_{output}$$

Where, $d_{11}(q_1) = J_{eq} + M_p r^2 \cos(q_1)^2$

$$d_{12}(q) = \frac{1}{2} M_p r l_p \cos(q_1)\cos(q_2)$$

$$d_{22}(q_2) = J_p + M_p l_p^2$$

$$V_t(q_2) = M_p g(h - l_p \cos(q_2))$$

$$\tau_{output} = \frac{\left(K_t \left(V_m - K_m \left(\frac{d}{dt} \right)(\theta(t)) \right) \right)}{R_m}$$

The state model of time invariant linear continuous time dynamic system is

$$X(t) = AX(t) + Bu(t)$$

$$y(t) = CX(t) + Du(t)$$

The states the rotary inverted pendulum angles are

$$x_1 = \theta, x_2 = \alpha, x_3 = \frac{\partial}{\partial t}\theta, x_4 = \frac{\partial}{\partial t}\alpha$$

A is real 4*4 matrix, B is 4*1 matrix

This rotary inverted pendulum is a single input and four outputs system. Where the outputs are pendulum angle, motor angle, pendulum velocity and motor velocity. The parameters of the rotary inverted pendulum are given below in Table 1.

The obtained state space matrix is,

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{M_p^2 g l_p^2 r}{\Delta t} & -\frac{K_t K_m (J_p + M_p l_p^2)}{\Delta t} & 0 \\ 0 & \frac{-l_p M_p g (M_p r^2 + J_{eq})}{\Delta t} & \frac{l_{pM_pK_trK_m}}{\Delta t} & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ 0 \\ \frac{K_t (J_p + M_p l_p^2)}{\Delta t} \\ \frac{l_{pM_pK_tr}}{\Delta t} \end{pmatrix}$$

Table 1. Parameters of rotary inverted pendulum

Parameters	Values
Mp (kg)	0.027
Lp (m)	0.153
r (m)	0.0826
g (m/s2)	9.81
Jp (kg.m2)	0.00017
Jeq (kg.m2)	0.00018
Rm (ohm)	8.7

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Where $\Delta t = J_p M_p r^2 + J_p J_{eq} + M_p l_p^2 J_{eq}$

$$\frac{dx}{dt} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{M_p^2 g l_p^2 r}{\Delta t} & -\frac{K_t K_m (J_p + M_p l_p^2)}{\Delta t} & 0 \\ 0 & \frac{-l_p M_p g (M_p r^2 + J_{eq})}{\Delta t} & \frac{l_{pM_p K_t r K_m}}{\Delta t} & 0 \end{pmatrix}$$

$$X(t) + \begin{pmatrix} 0 \\ 0 \\ \frac{K_t (J_p + M_p l_p^2)}{\Delta t} \\ \frac{l_{pM_p K_t r}}{\Delta t} \end{pmatrix} U(t)$$

$$Y(t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} X(t) + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} U(t)$$

Obtained transfer function,

$$\frac{num1}{den} = \frac{-25.131s^3 + 17.4949s^2 + 5.2197s - 883.0695}{s^4 + 0.58328s^3 - 83.9278s^2 - 29.4415s}$$

$$\frac{num2}{den} = \frac{-11.5471s^3 + 7.443s^2 - 5.9082s}{s^4 + 0.58328s^3 - 83.9278s^2 - 29.4415s}$$

$$\frac{num3}{den} = \frac{17.4949s^3 - 59.082s^2 - 883.0695s}{s^4 + 0.58328s^3 - 83.9278s^2 - 29.4415s}$$

$$\frac{num4}{den} = \frac{7.44s^3 - 3.5328s^2 - 2.810084s}{s^4 + 0.58328s^3 - 83.9278s^2 - 29.4415s}$$

3. Controller Design

Stabilization of pendulum by minimum order model, deadbeat controller technique and LQR:

3.1 Minimum Order Model

The minimum order can be designed by first portioning the state vector x(k) in/to two parts as follows:

$$X(k) = \begin{vmatrix} X_a(k) \\ X_b(k) \end{vmatrix}$$

Where

Xa (k) is that portion of the state vector.

Xb (k) is the unmeasurable portion of the state vector.

The partitioned state equation becomes as follows

$$\begin{vmatrix} X_a(k+1) \\ X_b(k+1) \end{vmatrix} = \begin{bmatrix} A_{aa} & A_{ab} \\ A_{ba} & A_{bb} \end{bmatrix} \begin{bmatrix} X_a(k) \\ X_b(k) \end{bmatrix} + \begin{bmatrix} B_a \\ B_b \end{bmatrix} (k) \quad (1)$$

$$X_a(k+1) = A_{aa} X_a(k) + A_{bb} X_b(k) + B_a U(k) \quad (2)$$

This equation relates measurable and unmeasurable quantities of the state.

The full-order observer can be given by

$$X_a(k+1) = (G - K_e C) X(k) + BU(k) + K_e Y(k) \quad (3)$$

Making following substitutions in equation (3)

$$X(k) = X_b(k),\ A = A_{bb},\ Bu(k) = A_{ba} X_a(k) + B_b u(k),$$

$$Y(k) = X_a(k+1) - A_{aa} X_a(k) - B_a u(k),\ C = A_{ab}$$

$$X_b(k+1) = (A_{bb} - K_e A_{ab}) X_b(k) + A_{ba} X_a(k) + B_b u(k)$$
$$+ K_e [X_a(k+1) - A_{aa} X_a(k) - B_a u(k)] \quad (4)$$

Since, motor and pendulum angles are our output,

$$Y(k) = X_a(k) \quad (5)$$

Sub eqn (5) in eqn (4)

$$\eta(k+1) = (A_{bb} - K_e A_{ab})(k) - [(A_{bb} - K_e A_{ab})$$
$$+ (A_{ba} - K_e A_{aa}) Y(k)] + (B_b - K_e B_a) u(k)$$

This equation is minimum order observer.

The error equation can be written as

$$e(k+1) = (A_{bb} - k_e A_{ab}) e(k)$$

Ackermanns formula is

$$k = \varnothing(A_{bb})\begin{bmatrix} A_{ab} & A_{bb} \\ A_{ab} & A_{bb} \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\varnothing(A_{bb}) = A_{bb}^2 + a_1 A_{bb} + a_2 I$$

The characteristics polynomial equation of minimum order observer is given by

$$|zI - A_{bb} + k_e A_{ab}| = 0$$

3.2 Deadbeat Controller

For the discrete time state space model, the state equation becomes u(k) = –kx(k),

State equation,

$$X(k+1) = (A_d - B_d k)X(k)$$

If the Eigen values of matrix (Ad-BdK) lies inside the unit circle, then the system is stable.

It follows that, by choosing all Eigen values of (Ad-BdK) to be zero, it is possible to get the deadbeat response, or since, our system is completely controllable, we can choose the desired Eigen values to be zero.

This implies that,

$$(z-\mu_1)(z-\mu_2)(z-\mu_3)(z-\mu_4) = z^4 + a_1 z^3 + a_2 z^2 + a_3 z + a_4 = z^4$$

Which implies,

$$a_1 = 0, a_2 = 0, a_3 = 0, a_4 = 0$$

Let the original system characteristic equation is given by

$$|ZI - A_d| = z^4 + a_1 z^3 + a_2 z^2 + a_3 z + a_4$$

To achieve specified poles we define transformation matrix as follows:

$$T = MW$$

$$M = \left[B_d, \quad A_d B_d, \quad A_d^2 B_d, \quad A_d^2 B_d\right]$$

which is rank 4.

$$W = \begin{pmatrix} a_3 & a_2 & a_1 & 1 \\ a_2 & a_1 & 1 & 0 \\ a_1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Using x(k) = Tx(k), we get following state space matrices

(Note: Any coordinate transformation of the state vector, yields the same Markov parameters of the system)

$$\widehat{A_d} = T^{-1} A_d T = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_4 & -a_3 & -a_2 & -a_1 \end{pmatrix}$$

$$\widehat{B_d} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\widehat{K} = KT = \left[\delta_4 \quad \delta_3 \quad \delta_2 \quad \delta_1\right]$$

The characteristic equation becomes as follows

$$|ZI - A_d + B_d k| = 0$$

Comparing equation

Hence, for dead beat response

$$k = KT^{-1} = \left[-a_1, \quad -a_2, \quad -a_3, \quad -a_4\right]T^{-1}$$

The obtained results for minimum order model,

4. Result and Discussion

In Figure 1 shown the three dimensional diagram of inverted pendulum. The step response of motor pendulum angle was obtained using minimum order observer model in figure 3. Figure 4 shows the error vectors between observed and actual states. The voltage input was given to the motor and that response was obtained using dead beat algorithm (Figure 5). Similarly, motor and pendulum angle response was obtained using dead beat algorithm (Figure 6). Finally, arm velocity and pendulum velocity was obtained using LQR (Figure 7).

5. Application

There are some application already been used in real life using pendulum principle such as some part of rides at amusement park, crane system and pendulum clock. Modelling and control of rotary inverted pendulum system is very useful to improve the application.

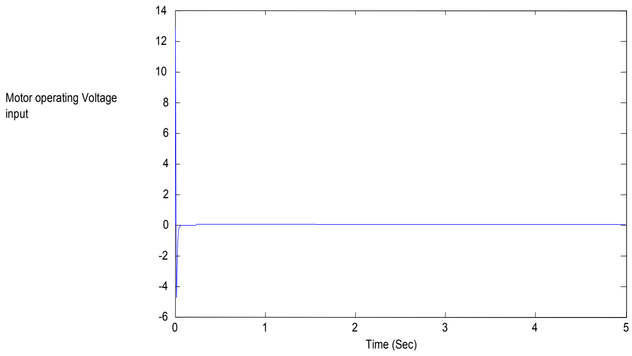

Figure 2. Minimum order model Control Effort.

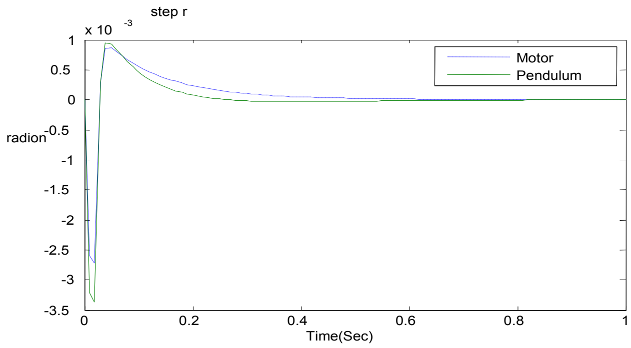

Figure 3. Step response of Motor and Pendulum angle minimum order observer.

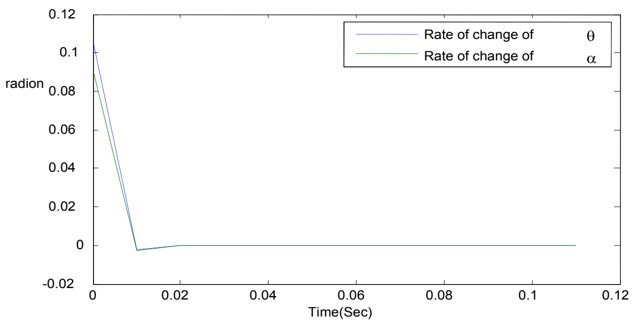

Figure 4. Error vectors between observed and actual States.

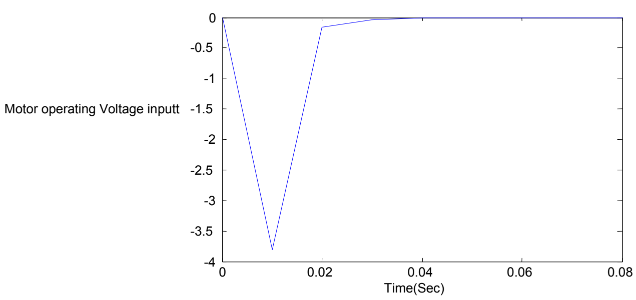

Figure 5. The obtained results for dead beat controller Control Effort.

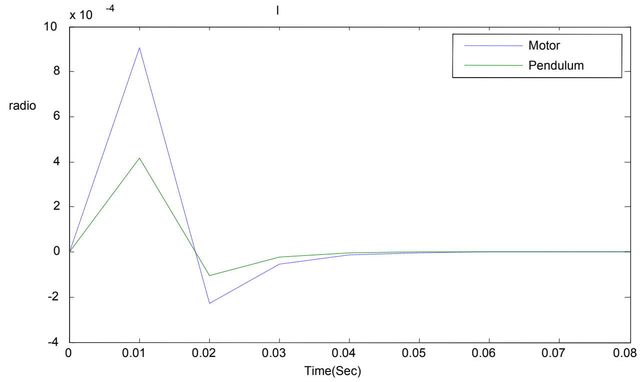

Figure 6. Step response of Motor and Pendulum range Deadbeat model.

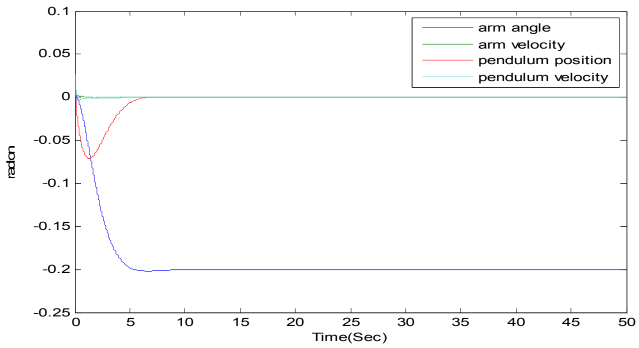

Figure 7. Output response from LQR.

6. Conclusion

A mathematical model of rotary inverted pendulum system has been achieved. Thereafter, using state space equation, the controller design is made using minimum order model LQR and deadbeat controller methods.

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