

Underwater Passive Target Tracking in Constrained Environment

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Abstract

In underwater scenario, observer manoeuvre is required to find out target motion parameters in bearings only passive target tracking. Sometimes, due to tactical constraints, observer is not able to carry out manoeuvre. In this paper, it is shown that target motion parameters can be obtained in such situation, if the knowledge of any one of the target motion parameters is available. There are other practical problems like spurious bearings are generated by sonar, etc. In addition, auto tracking fails often and some bearings will be missed. Pseudo Linear Kalman Filter is made flexible to address these practical problems.

Keywords: Constrained Environment, Direction of Arrival Estimation, Kalman Filtering, Monte Carlo Simulation, Passive Target Tracking, Target Detection

1. Introduction

A familiar approach for underwater passive target tracking is by using bearings only measurement. Generally, the sonar bearings are corrupt or noisy. These are transmitted by the radiating target and monitored by the observer. There is a preliminary assumption that the course of the target motion is invariable. The obtained measurements are processed by the observer and target motion parameters are found out. Here, making the whole process is essentially non-linear because the measurements are non-linear. The measurements related to bearing are obtained from a single sensor. The identification of the location and velocity is difficult until observer executes a proper manoeuvre as shown in Figure 1. However, to acquire target parameters, the techniques¹⁻⁵ are available in the above situation.

If any one of the target kinematics is known in addition to the bearing measurements, the remaining target motion parameters can be estimated easily. These are called constraint solutions. Depending upon the type of information available, the solution obtained is called range, course or speed constraint solution. This range,

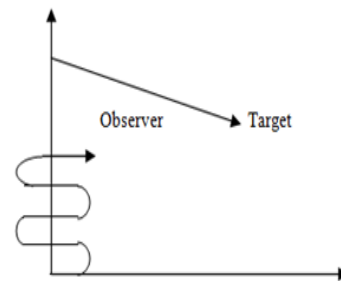


Figure 1. Observer in S-manoeuvre on LOS.

course or speed inputs can be obtained approximately, from different sources. For example, the speed of the target can be calculated approximately using the sound pattern of the received bearing measurement. The bearing measurements can be plotted and the course of the target can be found out approximately. In this paper, the Pseudo Linear Kalman Filter (PLKF) is suitably modified to find out constraint solutions. The other practical problem underwater is that the auto tracking may not be continuously possible making the availability of the measurements discontinuous. Sometimes, highly noisy spurious bearing measurements will also be generated by sonar. To overcome such situations, in this

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paper, the algorithm is made flexible such that it takes care of missed or highly noisy bearings. The missed bearings will be replaced by the bearings estimated by an algorithm that works in parallel with the main algorithm. The calculation of measurement error variance is on-line and if this variance exceeds the threshold (or expected) variance, then these are treated as spurious bearings and will be replaced by the estimated ones. As the process requires external input to find out target motion parameters, closed loop estimators are cannot be used here¹⁻⁶. Pseudo Linear Estimator (PLE) using continuous bearings⁷ is adapted so that the algorithm can accept range/course/speed of the target as external input. As this estimator is similar to KF, it is also called as PLKF. In case of PLE in batch processing mode¹ against recursive mode, it is derived from the first principles after modelling observer and target dynamics. In order to suit real time applications, it is later converted into recursive mode.

The impact of all the bearing measurements in covariance matrix is preserved in the form of recursive SUMS (notation to denote a function of bearing estimate). The updating of the bearing measurement to the SUMS is to be carried out by performing calculation related to current measurement^{7,8}. Here, 20 SUMS are required and the time taken for 20 iterations is to be stored and updated. Therefore, solution is upgraded whenever the measurement is received. While upgrading the interim time, the solution will be extrapolated at every second. This estimate is utilised to figure out target. Monte-Carlo simulation is performed and tested against a few strategic scenarios. One of the tactical scenario is presented for illustration in this paper. It is watched that the outcomes are acceptable.

2. Mathematical Modelling

It is desired to estimate the target state vector⁶⁻⁸,

$$X_s = \begin{bmatrix} \dot{x}_t & \dot{y}_t & x_t & y_t \end{bmatrix} \quad (1)$$

Where $(\dot{x}_t, \dot{y}_t)$ and (x_t, y_t) are target velocity and position components, using noise corrupted passive bearing measurements. The state equation is given by

$$X_s(k+1) = \phi(k+1/k) X_s(k) \quad (2)$$

where $\phi(k+1/k)$ is a deterministic transition matrix.

The measured bearing is given by

$$B_m(k) = B(k) + \gamma(k) \quad (3)$$

$$\text{Where } B(k) = \tan^{-1} \left(\frac{x_t(k) - x_o(k)}{y_t(k) - y_o(k)} \right) \quad (4)$$

Where (x_o, y_o) denote observer position. $\gamma(k)$ is zero mean white Gaussian sequence Equation 4 is rewritten as

$$\frac{\sin B(k)}{\cos B(k)} = \frac{x_t(k) - x_o(k)}{y_t(k) - y_o(k)} \quad (5)$$

Substituting Equation (3) in the Equation (4) straight forward formulation yields

$$\begin{aligned} x_t(k) \cos B_m(k) - y_t(k) \sin B_m(k) + r_s(k) \gamma(k) \\ = x_o(k) \cos B_m(k) - y_o(k) \sin B_m(k) \end{aligned} \quad (6)$$

Again by simple algebraic formulation of the Equation (6) we get measurement equation

$$z(k) = H(k) X_t(k) + \gamma(k) \quad (7)$$

where the pseudo measurement $z(k)$ is given by

$$z(k) = x_o(k) \cos B_m(k) - y_o(k) \sin B_m(k) \quad (8)$$

and measurement matrix for bearing measurement is given by

$$H(k) = \begin{bmatrix} 0 & 0 & \cos B_m(k) & -\sin B_m(k) \end{bmatrix} \quad (9)$$

and $\zeta(k) = \zeta(k) r_s(k)$ where $r_s(k)$ is slant range. To find out initial estimate, $X_t(0, k)$, $z(k)$ is modified as

$$z(k) = H(k) \phi(k, 0) X_t(0, k) + \gamma'(k) \quad (10)$$

Where

$$\phi(k, 0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ ts_1 + ts_2 + \dots + ts_k & 0 & 1 & 0 \\ 0 & ts_1 + ts_2 + \dots + ts_k & 0 & 1 \end{bmatrix} \quad (11)$$

Consider the modified measurements in matrix form as

$$Z(k) = [z(1) z(2) z(3) \dots z(k)]^T \quad (12)$$

Let us use the familiar Least Square Estimator equation to find out the initial estimate of target state

$$\hat{X}_t(0, k) = A^T(k, 0) A(k, 0) Z(k) \quad (13)$$

where

$$A(k, 0) = [H(1)\phi(1, 0) \quad H(2)\phi(2, 0) \quad \dots] \quad (14)$$

Using Equation (12) we can write that

$$\phi(1, 0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ ts_1 & 0 & 1 & 0 \\ 0 & ts_1 & 0 & 1 \end{bmatrix} \quad \text{and}$$

$$\phi(2,0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ ts_1 + ts_2 & 0 & 1 & 0 \\ 0 & ts_1 + ts_2 & 0 & 1 \end{bmatrix}$$

and so on

Along these lines

$$H(1) \phi(1,0) = \begin{bmatrix} 0 & 0 & \cos B_m(1) & -\sin B_m(1) \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ ts_1 & 0 & 1 & 0 \\ 0 & ts_1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} ts_1 \cos B_m(1) & -ts_1 \sin B_m(1) & \cos B_m(1) & -\sin B_m(1) \end{bmatrix} \quad (15)$$

Similarly

$$H(k) \phi(k,0) = \begin{bmatrix} (ts_1 + ts_2 + \dots + ts_k) \cos B_m(k) \\ -(ts_1 + ts_2 + \dots + ts_k) \\ \sin B_m(k) \cos B_m(k) & -\sin B_m(k) \end{bmatrix}$$

$$\hat{X}_s(0|k) = \left[A^T(k,0) A(k,0) \right]^{-1} A^T(k,0) Z(k) \quad (16)$$

The assumption is that observer makes proper manoeuvre, and the system is observable.

3. Constraint Solutions

The state vector given by

$$X_s^t = \begin{bmatrix} \dot{B}R & \dot{R} & RBr & R \end{bmatrix} \quad (17)$$

Where, $\dot{B}$ and $\dot{R}$ represent bearing rate and range rates respectively, Br represents the estimated error, R represents the relative range of the target with respect to the observer.

The state vector is obtained by shifting the coordinate system in such a way that +y axis is in line the newest bearing entered. Similar, target state vector was utilized in MPKF⁵. When the range (R_c) between observer and target is known from external sources, the entire state vector can be found out using Equation (9) by replacing R with R_c in the rotated coordinate system. Then a translation of this state vector into original rectangular coordinate system provides the position and velocity components of the target. It is observed that two valid target state vectors may also exist for the same speed - input. The speed - input can be easily converted into range - input. If two

solutions exist, the more valid will be decided by range of the day. Similarly, the course - input is converted into range - input and then range constraint algorithm is used to find out the remaining target motion parameters.

4. Spurious and Missed Bearings

The variance of the noise in the input measurements is calculated using theory of regression. If the variance of the incoming bearing measurement exceeds the threshold, this measurement is treated as spurious and is replaced by the estimated bearing, which is available from Equation (1). Sometimes bearing measurements may not be available continuously for small intervals of time due to the failures in auto tracking. The estimated bearing and bearing rates available from Equation (9) will be used for this short interval. It is found that such a procedure does not render the solution less accurate.

5. Simulation and Results

The algorithm is implemented and simulator is developed in a PC environment. All raw bearings are corrupted by additive zero mean Gaussian noise with r.m.s level of 1 degree (as shown in Table 1) and then pre-processed to find out the variance of the noise in the measurements over a period of twenty seconds. The measurements are generated at every second and the estimates are updated every 20 seconds of interval. Spurious measurements are simulated at third minute for a period lasting one minute by increasing the noise level to 1.5° rms. Missed bearings are simulated at fifth minute for a period of one minute. It is assumed that the observer knows the range at eighth minute. The actual range at second minute is 15,200 meters and the input range is 15,300 meters.

Table 1. Target Scenario

Sl. No.	Item Description	Ship Scenario 1
1.	Initial Range	18,000 meters
2.	Initial Bearing	200°
3.	Target Speed	18 knots
4.	Target Course	45°
5.	Ownship Speed	20 knots
6.	Error in Bearing (R.M.s)	5°
7.	Ownship Course	90°

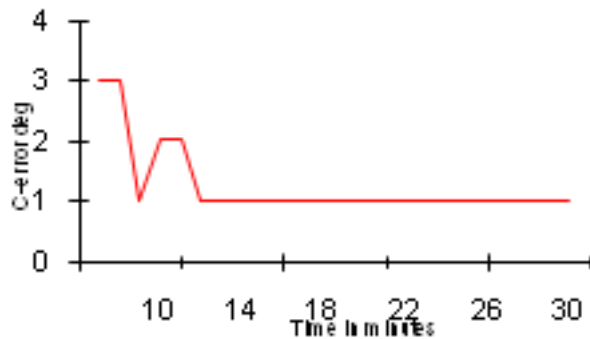


Figure 2. The estimated course in range constraint.

The parameters estimated using this range are shown in figures for a period of two minutes.

The accuracy of the estimates depends on the degree of accuracy in the inputs fed to the algorithm. The tolerances of error permitted are ten percent in range and velocity estimates while it is five degrees in course estimate. It is observed that the estimates with desired accuracies are achieved from 8.5 minute onwards. Similarly the speed and course constraint solutions are shown in figures assuming that these are known respectively at seventh minute and eighth minute. The results presented are the averaged values of the several Monte Carlo runs of the same scenario. In this extended summary, for the purpose of illustration, the course estimate in range constraint solution is shown in the Figure 2.

6. Conclusion

The estimation of target motion parameters using bearings only measurements along with range/course/speed inputs when observer is not able to carry out manoeuvre is presented in this paper. The problems of frequent

failure of auto tracking and spurious measurements are discussed and appropriate measures are taken to sort out these problems without loss of much accuracy.

7. References

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