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Hesitancy Fuzzy Graphs

T. Pathinathan[1*], J. Jon Arockiaraj[2] and J. Jesintha Rosline[3]

[1]Department of Mathematics, Loyola College, Chennai - 600034, Tamil Nadu, India; nathanpathi@hotmail.com
[2]Department of Mathematics, St. Joseph's College of Arts and Science, Cuddalore - 607001, Tamil Nadu, India; jonarockiaraj@gmail.com
[3]Department of Mathematics, SRM University, Chennai - 603203, Tamil Nadu, India; jesi.simple@gmail.com

Abstract

Objectives: To remove the hesitation that appears in choosing membership degree of an element from some possible values. **Methods:** Through Intuitionistic Fuzzy Graph and Intuitionistic Double Layered Fuzzy Graph, we developed a new graph to represent the hesitancy degree. **Findings:** Further we have proved some theoretical concepts and principle properties which have the roots from Double Layered Fuzzy Graph and Intuitionistic Double layered Fuzzy Graph. **Conclusion:** In this paper,a new fuzzy graph called Hesitancy Fuzzy Graphs (HFGs) is introduced.

Keywords: Hesitancy Fuzzy Graphs (HFGs), Intuitionistic Double Layered Fuzzy Graphs (IDLFGs)

1. Introduction

Fuzzy set theory was introduced by[1]. Most of the real world problems are extremely complex and contain vague information. In order to measure the lack of certainty, further development to Fuzzy sets was introduced by[2] and he named it as Hesitant Fuzzy Sets (HFSs). HFSs are motivated to handle the common difficulty that appears in fixing the membership degree of an element from some possible values. This situation is rather common in decision making problems too while an expert is asked to assign different degrees of membership to a set of elements {x, y, z,…} in a set A. Often problems arise due to uncertain issues and situations hence one is faced with hesitant moments. The researcher had to find ways and means to take the problems and arrive at a solution. Therefore researchers have taken up the study and application of HFS. HFSs have been extended[3,4] from different perspectives such as, both quantitative and qualitative.

The aim of this paper is to construct a new graph called Hesitancy Fuzzy Graphs (HFGs) and also to discuss some basic concepts, notations, remarks, proofs related with Hesitancy Fuzzy Graphs (HFGs).

The paper is organised as follows. Section 2 focuses on the concept of Fuzzy Graphs (FGs) and some basic definitions on Fuzzy graphs. Section 3 introduces the new graph called Hesitancy Fuzzy Graphs (HFGs), and deals with some basic notations and theoretical validation of Hesitancy Fuzzy Graphs (HFGs) followed by conclusion in section 4.

2. Preliminaries

Fuzzy graphs were introduced by[5]. The following provides its definition[5,6], which will be needed throughout the paper:

Definition 1

Let V be a non empty set. A fuzzy graph is a pair of functions $G : (\sigma, \mu)$ where σ is a fuzzy subset of V, μ is a symmetric fuzzy relation on σ . i.e., $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ such that $\mu(u, v) \le \sigma(u) \wedge \sigma(v)$ for all u, v in V. The underlying crisp graph of the fuzzy graph $G : (\sigma, \mu)$ is denoted as $G^* : (\sigma^*, \mu)$ where σ^* is referred to as the nonempty set V of nodes and $\mu = E \subseteq V \times V$.

** Author for correspondence*

The crisp graph (V, E) is a special case of the fuzzy graph G with each vertex and edge of (V,E) having degree of membership**1.**

Definition 2

A fuzzy graph G : (σ, μ) is said to be a strong fuzzy graph if $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ for all (x, y) in μ^* .

Definition 3

A fuzzy graph G : (σ, μ) is said to be a complete fuzzy graph if $\mu(x, y) = \sigma(x) \wedge \sigma(y)$ for all x, y in σ^* .

Definition 4

Let G : (σ, μ) be a fuzzy graph. The complement of G is defined as $\overline{G}:(\overline{\sigma},\overline{\mu})$ where
$\overline{\sigma} = \sigma$ and $\overline{\mu}(u,v) = \sigma(u) \wedge \sigma(v) - \mu(u,v)$ for every u, v $\in \sigma$.

3. Hesitancy Fuzzy Graph

In this section, we define a new fuzzy graph called Hesitancy Fuzzy Graph and various properties have been studied.

Definition 1

A Hesitancy Fuzzy Graph is of the form G = (V,E), where

- V = $\{v_1, v_2, v_3 \ldots v_n\}$ such that μ_1:V→[0,1],γ_1: V→[0,1] and β_1: V→[0,1] denote the degree of membership, non-membership and hesitancy of the element $v_i \in$ V respectively and

$\mu_1(v_i) + \gamma_1(v_i) + \beta_1(v_i) = 1$
for every $v_i \in$ V, where $\beta_1(v_i) = 1-[\mu_1(v_i) + \gamma_1(v_i)]$ and (1)

- E ⊆ V×V where μ_2: V×V →[0,1], γ_2: V×V →[0,1] and β_2: V×V →[0,1] are such that,

$\mu_2(v_i,v_j) \le \min[\mu_1(v_i), \mu_1(v_j)]$ (2)
$\gamma_2(v_i,v_j) \le \max[\gamma_1(v_i), \gamma_1(v_j)]$ (3)
$\beta_2(v_i,v_j) \le \min[\beta_1(v_i), \beta_1(v_j)]$ (4)
and $0 \le \mu_2(v_i,v_j) + \gamma_2(v_i,v_j) + \beta_2(v_i,v_j) \le 1$ for every $(v_i,v_j) \in$ E. (5)
(see Example 1, Figure 1).

Example 1

In this example, we obtained the new graph called Hesitancy Fuzzy Graph using the above mentioned (Definition 1) properties.
Consider G = (V,E), where V = {a,b,c,d}

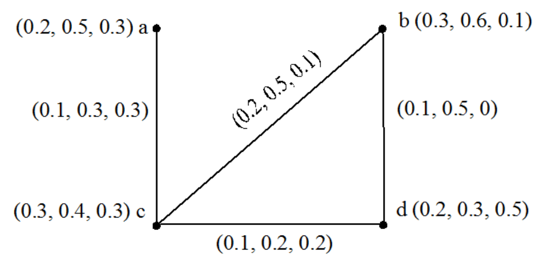

Figure 1. Hesitancy Fuzzy Graph

Example 2

Consider a HFG, G = (V,E) with V = {a,b,c,d}

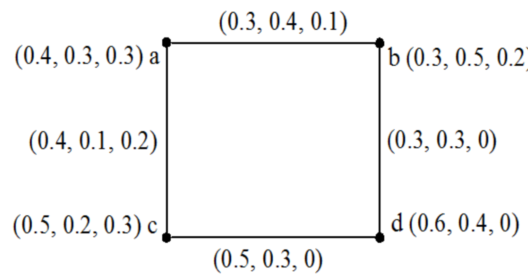

Figure 2. Hesitancy Fuzzy Graph.

Example 3

Consider a HFG, G = (V,E) with V = {a,b,c}

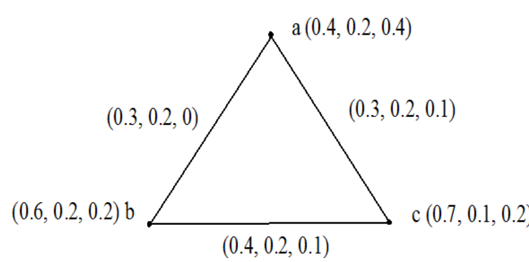

Figure 3. Hesitancy Fuzzy Graph.

Example 4

Consider a HFG, G= (V,E) with V={a,b,c,d}

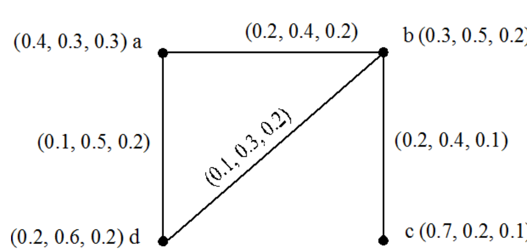

Figure 4. Hesitancy Fuzzy Graph.

3.1 Notations

Here $<v_i, \mu_{1i}, \gamma_{1i}, \beta_{1i}>$ denotes the vertex, degree of membership, non-membership and hesitancy of the vertex v_i.

Also $<e_{ij}, \mu_{2ij}, \gamma_{2ij}, \beta_{2ij}>$ denotes the edge, degree of membership, non-membership and hesitancy of the edge relation $e_{ij} = (v_i, v_j)$ on V.

3.2 Remark

- 1. If $\beta_{1i} = 0$ for every i, then the Hesitancy Fuzzy Graph (HFG) becomes Intuitionistic Fuzzy Graph (IFG). Then we can call the IFG as perfect IFG.
- 2. If one of the inequalities (1) or (2) or (3) or (4) or (5) is not satisfied, then G is not an HFG.

Definition 2

A HFG G = (V,E) is said to be a μ-strong HFG if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$, for all $(v_i, v_j) \in E$.

Example 5

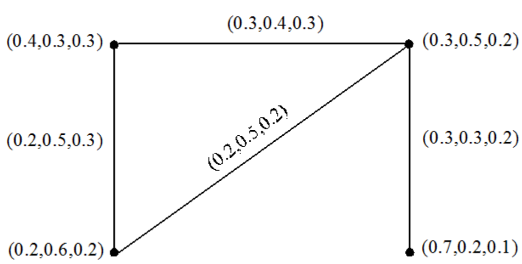

Figure 5. μ-strong Hesitancy Fuzzy Graph.

Definition 3

AHFG, G = (V,E) is said to be a γ-strong HFG if $\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$, for all $(v_i, v_j) \in E$. Example 6

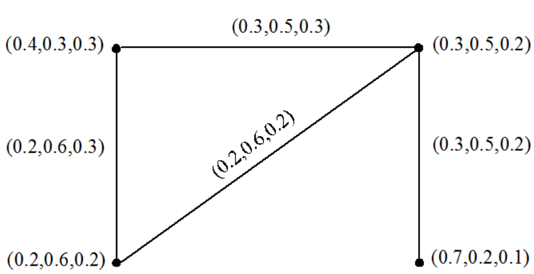

Figure 6. Υ-strong Hesitancy Fuzzy Graph.

Definition 4

A HFG, G = (V,E) is said to be a β-strong HFG if $\beta_{2ij} = \min(\beta_{1i}, \beta_{1j})$, for all $(v_i, v_j) \in E$. Example 7

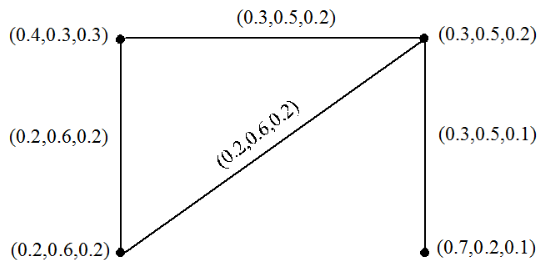

Figure 7. β-strong Hesitancy Fuzzy Graph.

Definition 5

AHFG, G = (V,E) is said to be a strong HFG if,

$\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$

$\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$

$\beta_{2ij} = \min(\beta_{1i}, \beta_{1j})$, for all $(v_i, v_j) \in E$.

Example 8

Consider HFG, G = (V,E) with vertex V = {a,b,c,d}

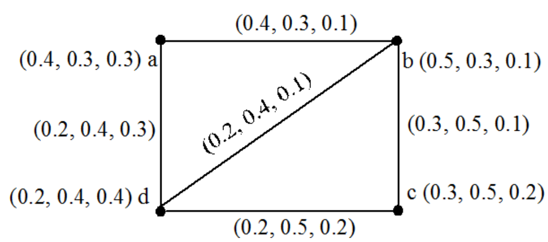

Figure 8. Strong HFG.

Definition 6

Let G = (V,E) be an HFG, then the complement of the HFG is a HFG, $\bar{G}(\bar{V}, \bar{E})$ where

$$\bar{V} = V, (i.e.,) \bar{\mu}_{1i} = \mu_{1i}; \bar{\gamma}_{1i} = \gamma_{1i}; \bar{\beta}_{1i} = \beta_{1i}$$

and

$$\bar{\mu}_{2ij} = \min(\mu_{1i}, \mu_{1j}) - \mu_{2ij},$$

$$\bar{\gamma}_{2ij} = \min(\gamma_{1i}, \gamma_{1j}) - \gamma_{2ij} \text{ and}$$

$$\bar{\beta}_{2ij} = \min(\beta_{1i}, \beta_{1j}) - \beta_{2ij} \ \forall v_i, v_j \in V$$

Example 9

Based on the above principle concepts we proved one basic theoretical proof as follows:

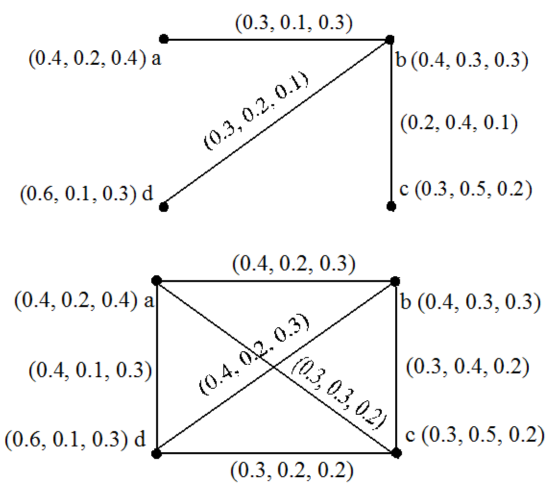

Figure 9. Complement of the Hesitancy Fuzzy Graph.

Theorem 1

If G is a strong HFG, then $\bar{G}$ is also a strong HFG.

Proof:

Case (i): if uv ∈ E, then

$$\bar{\mu}_2(uv) = \min(\mu_1(u), \mu_1(v)) - \mu_2(uv)$$
$$= \min(\mu_1(u), \mu_1(v)) - \min(\mu_1(u), \mu_1(v))$$
$$= 0.$$

$$\bar{\gamma}_2(uv) = \max(\gamma_1(u), \gamma_1(v)) - \gamma_2(uv)$$
$$= \max(\gamma_1(u), \gamma_1(v)) - \min(\gamma_1(u), \gamma_1(v))$$
$$= 0.$$

$$\bar{\beta}_2(uv) = \min(\beta_1(u), \beta_1(v)) - \beta_2(uv)$$
$$= \min(\beta_1(u), \beta_1(v)) - \min(\beta_1(u), \beta_1(v))$$
$$= 0.$$

$Case(ii): If\ uv \notin E, Then$

$$\bar{\mu}_2(uv) = \min(\mu_1(u), \mu_1(v)) - \mu_2(uv)$$
$$= \min(\mu_1(u), \mu_1(v))$$

$$\bar{\gamma}_2(uv) = \max(\gamma_1(u), \gamma_1(v)) - \gamma_2(uv)$$
$$= \max(\gamma_1(u), \gamma_1(v))$$

Thus if G is a strong HFG, then $\bar{G}$ is also a strong HFG.

Hence Proved.

Definition 7

AHF G, G = (V,E) is said to be a μ-complete HFG if $\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$, for all $v_i, v_j \in V$.

Example 10

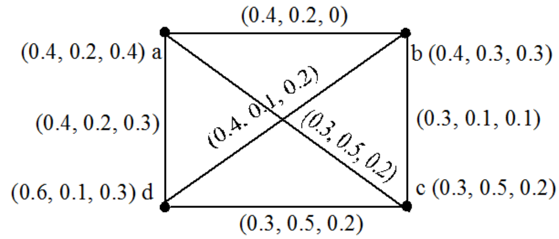

Figure 10. μ-complete Hesitancy Fuzzy Graph.

Definition 8

A HFG, G = (V,E) is said to be a γ-complete HFG if $\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$, for all $v_i, v_j \in V$.

Example 11

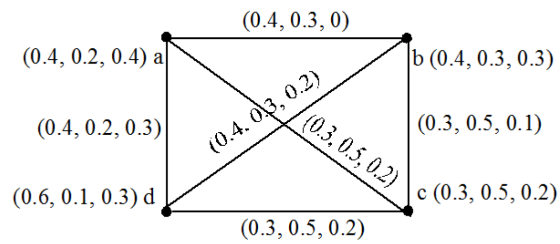

Figure 11. Υ-complete Hesitancy Fuzzy Graph.

Definition 9

A HFG, G = (V,E) is said to be a β-complete HFG if $\beta_{2ij} = \min(\beta_{1i}, \beta_{1j})$, for all $v_i, v_j \in V$. Example 12

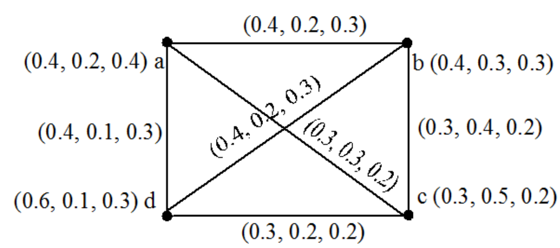

Figure 12. β-complete Hesitancy Fuzzy Graph.

Definition 10

A HFG, G = (V,E) is said to be a complete HFG if,

$\mu_{2ij} = \min(\mu_{1i}, \mu_{1j})$

$\gamma_{2ij} = \max(\gamma_{1i}, \gamma_{1j})$

$\beta_{2ij} = \min(\beta_{1i}, \beta_{1j})$ for all $v_i, v_j \in V$.

Example 13

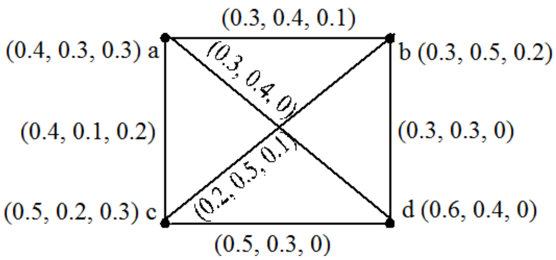

Figure 13. Complete Hesitancy Fuzzy Graph.

4. Conclusion

In this paper we have defined a new fuzzy graph called Hesitancy Fuzzy Graph (HFG) and illustrated with some examples. Also some related results have been studied and proved. These particular graph aggregate the hesitation degree raised from the human intuition in decision making process. It is an extension of known Intuitionistic Double Layered Fuzzy Graph[7]. In an upcoming article on this graph we will develop the other valid theoretical concepts with more examples.

5. References

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