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Design of Control Systems with the Increased Potential for Aircraft Model

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Abstract

Background/Objectives: In this paper proposes a new approach to the study robust stability of system in the class of 3-parameter structurally steady maps. **Methods/Statistical Analysis:** The research is based on a novelty approach derived from the geometric interpretation of Lyapunov theorem. Lyapunov functions is synthesized in the form of vector functions, and its antigradient sets in the specified component of the velocity vector of system in the form of velocity tensor. The region of system stationary states stability is obtained in the form of simple inequalities for uncertain parameters of the control object. **Findings:** The proposed aspect to choice control laws for linearized system in a class of three-parameter structurally stable maps from catastrophe theory allows to the construct an automatic control system with a marginal increased potential of robust stability. Synthesized control system maintains stability property in an extremely wide range of object uncertain parameters and given system parameters. Also, the results of experimental laboratory research in the Simulink Matlab show, that the proposed system have higher quality. **Application/Improvements:** An example showing the effectiveness of the method in aircraft control systems.

Keywords: Three-Parameter Structurally Stable Maps, Lyapunov's Function, Robust Stability, Control Aircraft, Angular Motion

1. Introduction

Today robust analysis for linear and nonlinear systems appears to be one of the active fields of studies. Models with parametric uncertainty carry out important function as in the theory so in practical usage of robust control. They are described by the mathematical model containing parameters that are not accurately known, but the values are in the range of given intervals. Such kind of uncertainty can take place in the control of real processes, for instance, as a consequence of modeling effort, imprecise measuring (worn-out parts, weight change of the aircraft, temperature, fuel quality) or the impact of definite extrinsic conditions. The techniques of robust stability analysis raised and developed a high interest in previous studies. A vast amount of works was dedicated to the problem of robust stability study control systems. Nevertheless, many of them are specialized in concrete uncertainty structure systems.

This work provides a method designed at the application of a universal approach in the analysis of robust stability for aircraft model. A significant task is to solve the problem of analysis of control systems and synthesis of control laws. All this provides for the best protection from high imprecision of object characteristics. The problem in question is robust manageability of system with parametric or non-parametric imprecisions[1,2]. Taking that the system is manageable, a sufficient condition is offered to sustain the characteristics of object (parameters of control systems) when system imprecisions are introduced. The most important approach in the study of robust stability is to specify constraints for changes in control system characteristics that sustain stability. For the aim of studying the system dynamics and their control, we contemplated models of observing input and output signals of the object and the representing its behavior in the state space as most proper.

In this work, we offer another approach to the composition of Lyapunov's vector functions. The constituents of a vector of an anti-gradient of vector functions out of geometrical interpretations of theorems of Lyapunov's second method are set by speed vector constituents (the right member of equation of a state)[3-5].

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The research of system robust stability is carried out by way of constructing a character-negative functions equal to the scalar product of the gradient vector to the velocity vector and is used theorem of Morse catastrophe theory[6,7].

The stability condition can be obtained from the positive definiteness of Lyapunov's function in the shape of systems of in equations for imprecise parameters of control objects and set parameters of the regulator. Currently, the construction of systems of equations with a sufficiently vast scope of robust stability under conditions of great imprecision parameters of the control object and the drift of their characteristics within wide ranges are also urgent problems[8-12]. A certain interest represents to construct system of automatic control in a category of accidents an elliptic umbilic in the conditions of big imprecision in parameters of the control objects. Also, this mathematical models accompanying difficult behavior of system, namely the having set of consistently steady decisions called by systems of the equation with the enhanced capacity of robust stability.

The content of this work is arranged in the following way: in section #2 we propose robust stability discussion by considering our problem and research the main equations of the model and their wide form with control method. Section #3 is dedicated to explaining the stability of model. We obtained the Lyapunov function, geometric interpretation, gradient vector constituents and system superstability condition. The principal points (region stability) of this study are given in section #3. In section #4, we pass to the proof method of the method offered. We demonstrate how the offered approach applies to the sample and construct a task of the control aircraft by the pitch. In addition we give a case study with a practical instance. And eventually, the principal conclusions of this study are given in Section #5.

2. Theoretical Approach of SISO Model with Control Law

Let us consider the problem of making of control systems with an enhanced capability of robust stability in a class 3-parameter structurally steady maps (catastrophe elliptic umbilic)[13,14] for objects with single input and single outputs. The control system is shown by this equation: Let us consider the control system with SISO, shown by the equation of state $x(t) = Ax(t) + Bu(t)$, (1), where $x(t)\in R^n$ - the state vector control object; $u(t)\in R^1$- Scalar function of control actions; $A\in R^{nxn}$ - Matrix object control with imprecise parameters dimension nxn, $B \in R^{mx1}$ - the dimension of the matrix control. Matrix A and b have the following form

$$A=\begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \dots & -a_1 \end{pmatrix} \quad b=\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_n \end{pmatrix} \tag{1}$$

The control law u (t) is given in the form of 3-parameters structurally steady maps (catastrophe elliptic umbilic):

$$\begin{aligned} u(t) &= -x_2^3+3x_2x_1^2-k_{12}\left(x_1^2+x_2^2\right)+k_2x_2+k_1x_1-x_4^3 \\ &+3x_4x_3^2-k_{34}\left(x_4^2+x_3^2\right)+k_4x_4+k_3x_3-,\dots,-x_n^3 \\ &+3x_nx_{n-1}^2-k_{n-1,n}\left(x_n^2+x_{n-1}^2\right)+k_nx_n+k_{n-1}x_{n-1}, \end{aligned} \tag{2}$$

The system (1) is written in expanded form

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dots = \dots \\ \dot{x}_{n-1} = x_n \\ \dot{x}_n = b_n\left[3x_2x_1^2-x_2^3-k_{12}\left(x_1^2+x_2^2\right)+\left(k_1-a_n\right)x_1\right. \\ +\left(k_2-a_{n-1}\right)x_2+3x_4x_3^2-x_4^3-k_{34}\left(x_3^2+x_4^2\right) \\ +\left(k_3-a_{n-2}\right)x_3+\left(k_4-a_{n-3}\right)x_4+,\dots,+3x_nx_{n-1}^2- \\ \left.-x_n^3-k_{n-1,n}\left(x_n^2+x_{n-1}^2\right)+\left(k_{n-1}-a_2\right)x_{n-1}+\left(k_n-a_1\right)x_n\right] \end{cases} \tag{3}$$

Stationary states of the system are determined by solution of the equation:

$$\begin{cases} x_{2s}=0, x_{3s}=0,\dots,x_{n-1,s}=0, x_{ns}=0 \\ 3x_{2s}x_{1s}^2-x_{2s}^3-k_{12}\left(x_{1s}^2+x_{2s}^2\right)+\left(k_1-a_n\right)x_{1s}+ \\ +\left(k_2-a_{n-1}\right)x_{2s}+3x_{4s}x_{3s}^2-x_{4s}^3-k_{34}\left(x_{3s}^2+x_{4s}^2\right)+ \\ +\left(k_3-a_{n-2}\right)x_{3s}+\left(k_4-a_{n-3}\right)x_{4s}+,\dots,+3x_{ns}x_{n-1,s}^2- \\ -x_{ns}^3-k_{n-1,n}\left(x_{ns}^2+x_{n-1,s}^2\right)+ \\ +\left(k_{n-1}-a_2\right)x_{n-1,s}+\left(k_n-a_1\right)x_{ns}=0, \end{cases} \tag{4}$$

From (4) can be stationary state determined by the trivial solution of (4):

$$x_{1s}=0, x_{2s}=0,\dots,x_{n-1,s}=0, x_{ns}=0, \tag{5}$$

Some other stationary states will be determined by solving the equations

$$-k_{i,i+1}x_{is}+k_i-a_{n-i+1}=0, x_{j1}=0\, at\, i\neq j, i=1,\dots,n \tag{6}$$

$$or -x_{i+1,s}{}^2-k_{i,i+1}x_{i+1,s}+k_{i=1}-a_{n-i+2}=0, x_{js}=0 \quad at\, i+1\neq j, i=1,\dots,n \tag{7}$$

Equation (6) has a solution

$$x_{is}=\frac{k_i-a_{n-i+1}}{k_{i,i+1}}, x_{js}=0\ at\, i\neq j, i=1,\dots,n \tag{8}$$

Equation (7) for negative values $k_{i,i+1}{}^2+4(k_i-a_{n-i+2})<0, i=1,\dots,n$ have minimal solutions that cannot match what - either physically possible situation. For $k_{i,i+1}{}^2+4(k_i-a_{n-i+2})>0, i=1,\dots,n$ equation (7) admit the following decisions:

$$x^2_{i+1,s}=\frac{-k_{i,i+1}-\sqrt{k^2_{i,i+1}+4(k_i-\mathrm{a}_{\text{n-i+2}})}}{2}, x_{js}=0\ for\ j\neq i+1; i=1,\dots,n \tag{9}$$

$$x^3_{i+1,s}=\frac{-k_{i,i+1}+\sqrt{k^2_{i,i+1}+4(k_i-\mathrm{a}_{\text{n-i+2}})}}{2}, x_{js}=0 \quad for\ j\neq i+1; i=1,\dots,n \tag{10}$$

The stability of the stationary states of (5), (8), (9) and (10) of (3) will investigate on the basis of the approach of Lyapunov function method[3,4].

3. Region of Stability

3.1 The Stability of the Steady State (5)

3.1. Let us research stability of the state (5). To this end, we denote the components of the antigradient - Lyapunov function through:

$$\frac{\partial V_1(x)}{\partial x_1}=0, \frac{\partial V_1(x)}{\partial x_2}=0, \frac{\partial V_1(x)}{\partial x_3}=0,\dots,\frac{\partial V_1(x)}{\partial x_n}=0$$

$$\frac{\partial V_2(x)}{\partial x_1}=0, \frac{\partial V_2(x)}{\partial x_2}=0, \frac{\partial V_2(x)}{\partial x_3}=0,\dots,\frac{\partial V_2(x)}{\partial x_n}=0$$

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$$\frac{\partial V_{n-1}(x)}{\partial x_1}=0, \frac{\partial V_{n-1}(x)}{\partial x_2}=0, \frac{\partial V_{n-1}(x)}{\partial x_3}=0,\dots,\frac{\partial V_{n-1}(x)}{\partial x_n}=0$$

$$\frac{\partial V_n(x)}{\partial x_1}=b_nk_{12}x_1^2-b_n(k_1-a_n)x_1-3b_nx_2x_1^2$$

$$\frac{\partial V_n(x)}{\partial x_2}=b_nx_2{}^3+b_nk_{12}x_2{}^2-b_n(k_2-a_{n-1})x_2$$

$$\frac{\partial V_n(x)}{\partial x_2}=b_nx_2{}^3+b_nk_{12}x_2{}^2-b_n(k_2-a_{n-1})x_2$$

$$\frac{\partial V_n(x)}{\partial x_3}=b_nk_{34}x_3^2-3b_nx_4x_3{}^2-b_n(k_3-a_{n-2})x_3$$

$$\frac{\partial V_n(x)}{\partial x_4}=b_nk_{34}x_4{}^2+b_nx_4{}^3-b_n(k_4-a_{n-3})x_4$$

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$$\frac{\partial V_n(x)}{\partial x_{n-1}}=b_nk_{n-1,n}x_{n-1}{}^2-3b_nx_nx_{n-1}{}^2-b_n(k_{n-1}-a_2)x_{n-1}$$

$$\frac{\partial V_n(x)}{\partial x_n}=b_nx_n{}^3+b_nk_{n-1,n}x_n{}^2-b_n(k_n-a_1)x_n$$

Full derivative of Lyapunov's vector-function is equal to:

$$\begin{aligned}\frac{dV}{dt}=\frac{\partial V}{\partial x}*\frac{dX}{dt}&=-x_2^2-x_3^2-,\dots,-x_n^2-\\ &-b_n^2\left[k_{12}x_1^2-3x_2x_1^2-(k_1-a_n)x_1\right]^2\\ &-b_n^2\left[x_2^3+k_{12}x_2^2-(k_2-a_{n-1})x_2\right]^2-\\ &-b_n^2\left[k_{34}x_3^2-3x_4x_3^2-(k_3-a_{n-2})x_3\right]^2\\ &-b_n^2\left[k_{34}x_4^2+x_4^3-(k_4-a_{n-3})x_4\right]^2-,\dots,-\\ &-b_n^2\left[k_{n-1,n}x_{n-1}^2-3x_nx_{n-1}^2-(k_{n-1}-a_2)x_{n-1}\right]^2\\ &-b_n^2\left[x_n^3+k_{n-1,n}x_n^2-(k_n-a_1)x_n\right]^2\end{aligned} \tag{11}$$

From (11) we have that the total time derivative of the vector - Lyapunov function will sign-negative function, therefore, a sufficient condition for the asymptotic stability of the system (3) with respect to the steady state (5) is satisfied.

By components of the vector, Lyapunov's function is building components of Lyapunov's vector - function in the form

$$V_i(x)=(V_{i1}(x),V_{i2}(x),\dots,V_{in}(x),): V_{11}(x)=0, V_{12}(x) =-\frac{1}{2}x_2{}^2, V_{13}(x)=0,\dots,V_{1n}(x)=0$$

$$V_{21}(x)=0, V_{22}(x)=0, V_{23}(x)=-\frac{1}{2}x_3{}^2,\dots,V_{2n}(x)=0$$

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$$V_{n-1,1}(x)=0, V_{n-1,2}(x)=0, V_{n-1,3}(x)=0,\dots,V_{n-1,n}(x)=-\frac{1}{2}x_n{}^2$$

$$V_{n1}(x)=\frac{1}{3}b_n k_{12} x_1{}^3 - b_n x_2 x_1{}^3 - \frac{1}{2} b_n (k_1 - a_n) x_1{}^2$$
$$V_{n1}(x)=\frac{1}{3}b_n k_{12} x_1{}^3 - b_n x_2 x_1{}^3 - \frac{1}{2} b_n (k_1 - a_n) x_1{}^2$$
$$V_{n2}(x)=\frac{1}{4}b_n x_2{}^4 + \frac{1}{3} b_n k_{12} x_2{}^3 - \frac{1}{2} b_n (k_2 - a_{n-1}) x_2{}^2$$
$$V_{n3}(x)=\frac{1}{3}b_n k_{34} x_3{}^3 - b_n x_4 x_3{}^3 - \frac{1}{2} b_n (k_3 - a_{n-2}) x_3{}^2$$
$$V_{n4}(x)=\frac{1}{3}b_n k_{34} x_4{}^3 + \frac{1}{4} b_n x_4{}^4 - \frac{1}{2} b_n (k_4 - a_{n-3}) x_4{}^2$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$V_{n,n-1}(x)=\frac{1}{3}b_n k_{n-1,n} x_{n-1}{}^3 - b_n x_n x_{n-1}{}^3 - \frac{1}{2} b_n (k_{n-1} - a_2) x_{n-1}{}^2$$
$$V_{n,n}(x)=\frac{1}{4}b_n x_n{}^4 + \frac{1}{3} b_n k_{n-1,n} x_n{}^3 - \frac{1}{2} b_n (k_n - a_1) x_n{}^2$$

Lyapunov's function in the scalar form can be written as:

$$\begin{aligned}
V(x) &= \frac{1}{3}b_n k_{12} x_1^3 - b_n x_2 x_1^3 - \frac{1}{2} b_n (k_1 - a_n) x_1^2 \\
&+ \frac{1}{4} b_n x_2^4 + \frac{1}{3} b_n k_{12} x_2^3 - \\
&- \frac{1}{2} b_n \left(k_2 - a_{n-1} + \frac{1}{b_n} \right) x_2^2 + \frac{1}{3} b_n k_{34} x_3^3 - b_n x_4 x_3^3 \\
&- \frac{1}{2} b_n \left(k_3 - a_{n-2} + \frac{1}{b_n} \right) \times - \\
&\times x_3^2 + \frac{1}{3} b_n k_{34} x_4^3 + \frac{1}{4} x_4^4 - \frac{1}{2} b_n \left(k_4 - a_{n-3} + \frac{1}{b_n} \right) x_4^2 + \\
&,\dots,+ \frac{1}{3} b_n k_{n-1,n} x_{n-1}^3 - \\
&- b_n x_n x_{n-1}^3 - \frac{1}{2} b_n \left(k_{n-1} - a_2 + \frac{1}{b_n} \right) x_{n-1}^2 + \\
&+ \frac{1}{4} b_n x_n^4 + \frac{1}{3} b_n k_{n-1,n} x_n^3 - \\
&- \frac{1}{2} b_n \left(k_n - a_1 + \frac{1}{b_n} \right) x_n^2,
\end{aligned}$$

Terms of positive or negative definite functions (12) is not obvious, so we use Lemma Morse of catastrophe theory.

By Lemma Morse, Lyapunov's function (12) locally in a neighborhood of the stationary state can be represented as a quadratic form

$$\begin{aligned}
V(x) &= -b_n (k_1 - a_n) x_1^2 - b_n \left(k_2 - a_{n-1} + \frac{1}{b_n} \right) x_2^2 - b_n \times \\
&\left(k_3 - a_{n-2} + \frac{1}{b_n} \right) x_3^2 - b_n \left(k_4 - a_{n-3} + \frac{1}{b_n} \right) x_4^2 -,\dots,- b_n \times \\
&\left(k_{n-1} - a_2 + \frac{1}{b_n} \right) x_{n-1}^2 - b_n \left(k_n - a_1 + \frac{1}{b_n} \right) x_n^2,
\end{aligned} \tag{13}$$

A necessary condition for the stability of the steady state (5) will be determined by the system of inequalities at:

$$\begin{cases}
k_1 - a_n < 0, k_2 - a_{n-1} + \dfrac{1}{b_n} < 0, \\
k_3 - a_{n-2} + \dfrac{1}{b_n} < 0, \\
k_4 - a_{n-3} + \dfrac{1}{b_n} < 0, \dots \\
\dots, k_{n-1} - a_2 + \dfrac{1}{b_n} < 0, \\
k_n - a_1 + \dfrac{1}{b_n} < 0
\end{cases} \tag{14}$$

3.2 The Stability of the Steady State (8)

We investigate robust stability at steady state (8) on the basis of Lyapunov function method. The state equation (3) can be represented in deviations with respect to the steady state (8) and writes:

$$\begin{cases}
\dot{x}_1 = x_2 \\
\dot{x}_2 = x_3 \\
\dots = \dots \\
\dot{x}_{n-1} = x_n \\
\dot{x}_n = b_n \big[3x_2 x_1^2 - x_2^3 - k_{12}(x_1^2 + x_2^2) + 12(k_1 - a_n) x_1 x_2 \\
-(k_2 - a_{n-1}) x_2 - (k_1 - a_n) x_1 + \\
+3x_3 x_4 - x_4^3 - k_{34}(x_3^2 + x_4^2) + 12(k_3 - a_n) x_3 x_4 \\
+12(k_1 - a_n) x_3 x_4 - (k_3 - a_{n-2}) x_3 - \\
-(k_4 - a_{n-3}) x_4 +,\dots,+ 3x_n{}^2 x_{n-1}^2 - x_n^3 \\
-k_{n-1,n}(x_n^2 + x_{n-1}^2) + 12(k_n - a_1) x_{n-1} x_n - \\
-(k_{n-1} - a_2) x_{n-1} - (k_n - a_1) x_n \big]
\end{cases}$$

Full time derivative will be determined by:

$$\frac{dV}{dt}=\frac{\partial V}{\partial x}\frac{dx}{dt}=-\frac{1}{2}x_2^2-\frac{1}{2}x_3^2-\frac{1}{2}x_4^2-,\ldots,-\frac{1}{2}x_n^2-$$
$$-b_n^2[3x_2x_1^2-k_{12}x_1^2+6(k_1-a_n)x_1x_2-(k_1-a_n)x_1]^2-$$
$$-b_n^2[-x_2^3-k_{12}x_2^2+6(k_1-a_n)x_1x_2-(k_2-a_{n-1})x_2]^2-$$
$$-b_n^2[3x_4x_3^2-k_{34}x_2^2+6(k_3-a_{n-2})x_3x_4-(k_3-a_{n-2})x_3]^2- \quad (15)$$
$$-b_n^2[-x_4^3-k_{34}x_4^2+6(k_3-a_{n-2})x_3x_4-(k_4-a_{n-3})x_4]^2-$$
$$-,\ldots,-b_n^2[3x_nx_{n-1}^2-k_{n-1,n}x_{n-1}^2+6(k_n-a_1)x_nx_{n-1}-$$
$$-(k_{n-1}-a_2)x_{n-1}]^2-b_n^2[-x_n^3-k_{n,n-1}x_n^2+$$
$$+6(k_n-a_1)x_{n-1}x_n-(k_n-a_1)x_n]^2$$

Scalar function (15) is always negative sign, therefore, a condition for the asymptotic stability of (8) is always satisfied. Lyapunov's function in the scalar form obtained in the form:

$$V(x)=-b_nx_2x_1^3+\frac{1}{3}b_nk_{12}x_1^3-3b_n(k_1-a_n)x_2x_1^2+\frac{1}{2}b_n\times$$
$$(k_1-a_n)x_1^2+\frac{1}{4}b_nx_2^4+\frac{1}{3}b_nk_{12}x_2^3-3b_n(k_1-a_n)x_1x_2^2+$$
$$+\frac{1}{2}b_n(k_2-a_{n-1})x_2^2-,\ldots,-b_nx_nx_{n-1}^3+b_nk_{n-1,n}\times$$
$$\times x_{n-1}^3-3b_n(k_n-a_1)x_nx_{n-1}^2+\frac{1}{2}b_n(k_{n-1}-a_2)x_{n-1}^2+$$
$$+\frac{1}{4}b_nx_n^4+\frac{1}{3}b_nk_{n,n-1}x_n^3-3b_n(k_n-a_1)x_{n-1}x_n^2-$$
$$-\frac{1}{2}b_n(k_n-a_1)x_n^2-\frac{1}{2}x_2^2-\frac{1}{2}x_3^2-,\ldots,-\frac{1}{2}x_n^2 \quad (16)$$

Making use of Lemma Morse we have next quadratic form

$$V(x)\approx\frac{b_n}{2}\Big[(k_1-a_n)x_1^2+\left(k_2-a_{n-1}-\frac{1}{b_n}\right)x_2^2$$
$$+\left(k_3-a_{n-2}-\frac{1}{b_n}\right)x_3^2+\left(k_4-a_{n-3}-\frac{1}{b_n}\right)x_4^2+ \quad (17)$$
$$+,\ldots,+\left(k_n-a_1-\frac{1}{b_n}\right)\Big]x_n^2$$

A necessary condition for steady state (8) will be determined by the system of inequalities at:

$$\begin{cases} k_1-a_n>0, k_2-a_{n-1}-\frac{1}{b_n}>0, k_3-a_{n-2}-\frac{1}{b_n}>0,\\ k_4-a_{n-3}-\frac{1}{b_n}>0,\ldots\ldots,\\ k_{n-1}-a_2-\frac{1}{b_n}>0, k_n-a_1-\frac{1}{b_n}>0\end{cases} \quad (18)$$

From the system of inequalities (14) and (18) it is clear that the control system with increased form of robust provides stability system (3) for any changes uncertain parameters.

4. Case Study

As case study we consider a task of the control aircraft by the pitch. We know that aircraft has constants, apriori-imprecise parameters, which values are placed in a chosen area[15]. In addition we will note that a similar condition may happen when aircraft were flying on different modes: height, velocity and load varying slowly, in comparison with the rate of angular movement. To describe the dynamics of the aircraft angular movement we apply the following linearized questions[4]

$$\begin{cases} \dot{\alpha}(t)=\omega_z(t)+\alpha_y^{\alpha}(t)\alpha(t)+\alpha_y^{\delta\beta}\delta_\beta(t)\\ \dot{\omega}_z(t)=-\alpha_{mz}^{\alpha}\alpha(t)+\alpha_{m_z}^{\omega_z}+\alpha_{mz}^{\delta\beta}\delta_\beta(t)\\ \dot{\upsilon}(t)=\omega_z(t)\end{cases} \quad (19)$$

where $\upsilon(t)$ $\omega(t)$ – the angle and the rate of pitch, $\alpha(t)$ – the attack angle, $\delta_\beta(t)$ – the deviation angle of the rudder height; $\alpha_y^{\alpha}(t),\alpha_y^{\delta\beta},\alpha_{mz}^{\alpha},\alpha_{mz}^{\omega_z},\alpha_{mz}^{\delta\beta}$ – aircraft parameters. Their values are contingent on the factors given above and may change over a vast scope, being dependent on the height and the velocity of flight. The precise values of parameters a priori are not defined. Also, we presume that dynamics of the executive body are possible to neglect and account that control object is the deviation of rudder

$$\delta_\beta(t): x_1=\upsilon(t), x_1=\omega_z(t), x_1=\alpha(t),$$
$$a_1=\alpha_{mz}^{\alpha}, a_2=\alpha_{mz}^{\omega_z}, a_3=\alpha_{mz}^{\delta\beta}, a_4=\alpha_y^{\alpha}(t),$$
$$a_5=\alpha_y^{\delta\beta}, u=\delta_\beta(t)$$

Then, the system of the aircraft movement will present the form:

$$\begin{cases} \dfrac{dx_1}{dt} = x_2(t) \\ \dfrac{dx_2}{dt} = -a_1 x_3(t) - a_2 x_2(t) - a_3 u \\ \dfrac{dx_3}{dt} = x_2(t) + a_4 x_3(t) + a_5 u \end{cases} \quad (20)$$

For the control law, we study: $u = -x_2^3 + 3x_1^2 x_2 - k_{12}(x_1^2 + x_2^2) + k_1 x_1 + k_2 x_2$

From the previews form we define siso model:

$$\begin{cases} \dfrac{dx_1}{dt} = x_2(t) \\ \dfrac{dx_2}{dt} = x_3(t) \\ \dfrac{dx_3}{dt} = x_2(t) + a_4 x_3(t) + a_5(-x_2^3 + 3x_1^2 x_2 - k_{12}(x_1^2 + x_2^2) \\ +k_1 x_1 + k_2 x_2 - x_3^3 + 3x_2^2 x_3 - k_{23}(x_2^2 + x_3^2) + k_2 x_2 + k_3 x_3) \end{cases}$$

This system has next solutions in the form:

$$x_{1s} = \frac{k_1}{k_{12}}; x_{2s} = 0; x_{3s} = 0$$

According algorithm from section 2 the full time derivative of Lyapunov function is equal to:

$$\frac{dV(x)}{dt} = -\sum_{j=1}^{i=1}\left(\sum_{j=1}^{i=1}\frac{\partial V_i(x)}{x_j}\right)\frac{dx_i}{dt} = -x_2^2 - x_3^2 -$$

$$\left(-\frac{3}{2}a_5 x_1^2 x_2 + a_5 k_{12} x_1^2 - 3a_5\frac{k_1}{k_{12}}x_1 x_2 + a_5 k_1 x_1\right)^2 - x_2^2 - a_4 x_3^2$$

Lyapunov's function in the scalar form can be written as:

$$V(x) = -\frac{1}{2}x_2^2 - \frac{1}{2}x_3^2 - \frac{1}{2}x_2^2 - \frac{1}{4}a_4 x_3^2$$

$$-\frac{1}{2}a_5 x_1^3 x_2 + \frac{1}{3}a_5 k_{12} x_1^3 - \frac{2}{3}a_5\frac{k_1}{k_{12}}x_1^2 x_2$$

$$-\frac{1}{2}a_5 k_1 x_1^2$$

Making use of Lemma Morse and locally can be in the area of the steady state function in the form of a quadratic form:

$$V(x) \approx -\frac{1}{2}a_5 k_1 x_1^2 - \frac{1}{2}(3a_5\frac{k_1}{k_{12}} + 1)^2 + a_5 k_1 x_2^2.$$

A necessary condition for the stability of the steady state will be determined by the system of inequalities at:

$$a_5 k_1 < 0; \left|3a_5\frac{k_1}{k_{12}} + a_5 k_2 + 1\right| < 0.$$

Matlab/Simulink is one of the most widely used modeling frameworks, as the bridge between applied math and information technologies. Let's give some experimental data of the described models.

Then, as an example and proof of theoretical remarks, we define the fo initial conditions and seek conditions for the system via Matlab. In three-dimensional space the system (19) is represented by the following kind of way, on figure 1. Three-dimensional picture of the system (19) show the steady focus.

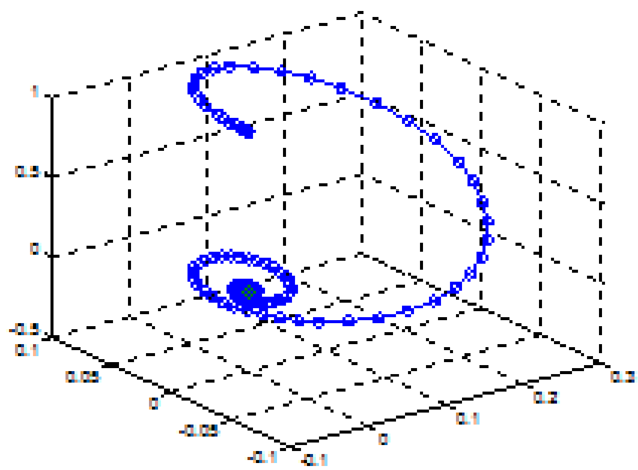

Figure 1. Three-dimensional space the system (19).

5. Conclusion

This work describes a new approach - a new theoretical technique for robust stability for a linearized system which describes forecast behavior on the dynamics of the aircraft model. Influence of nonlinear control law on model (1) has positive result[9].

In this work was substantiated an approach to building control systems with high potential for robust stability of objects with imprecise parameters with the choice of the control law in the class of three-parameter structurally stable maps. The focal point of the work was to elaborate a new conceptual framework to solve the problem of robust control for an imprecise system.

Thus, the SISO control system, built in a category of 3-parametre structurally steady maps, will be stable in beyond all wide ranges of change of imprecise parameters of the control of object. There were applied necessary and sufficient conditions for optimality and stability to develop non-linear controllers for imprecise systems. The analytical and numerical outcomes were carried out and shown.

The practical relevance of the results should motivate new research on typical applications and explain region of the applied math[16-18].

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