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Semiring on Roughsets

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Abstract

In this paper, we initiate to deal the concepts of ideals on rough sets. Also we introduce the notions Rough pivot set and semiring on the set of all rough sets for the given information system together with the operations praba Δ and praba ∇. We illustrate these concepts through examples.

Keywords: Roughset, Pivot Set, Semigroup, Monoid, Regular Semigroup, Semiring, Ideals

1. Introduction

The concept of rough set theory was introduced by Z. Pawalak[17] in 1982. This formal tool was implemented to process incomplete information in the information systems. Rough set theory is an extension of set theory and it is defined by a pair of sets called lower and upper approximations. In the content of data analysis, this concept will be used to discover fundamental patterns in data, remove redundancies and generate decision rules. Also rough set theory will be applied in several fields like computational intelligence such as machine learning, intelligent systems, pattern recognition, knowledge discovery, expert systems and others[2, 5, 8, 15, 21]. B. Praba and R. Mohan[18] discussed the concept of rough lattice. In this paper the authors considered an information system I = (U, A). A partial ordering relation was defined on T = {RS (X) | X $\subseteq$ U}. The least upper bound and greatest lower bound were established. They have also defined the operation Praba Δ and Praba ∇. B. Praba, V. M. Chandrasekaran and A. Manimaran[19] discussed the commutative regular monoid on rough sets under the operation Praba Δ in 2013. In this paper the authors dealt the rough ideals on (T, Δ). A. Manimaran, B. Praba and V. M. Chandrasekaran[14] studied the notion of regular rough ∇ monoid under Praba ∇ in 2014. N. Kuroki and P. P. Wang[13] discussed some properties of lower and upper approximations with respect to the normal subgroup. R. Biswas and S. Nanda[1]introduced the notion of rough groups and rough subgroups. M. Kondo[12] introduced the concepts on the structure of generalized roughsets in 2006. Changzhong Wang and Degang Chen[4] discussed about a short note on some properties of rough groups and the authors have studied the image and inverse image of rough approximations of a subgroup with respect to a homomorphism between two groups in 2010. Yonghong Liu[23] dealt the concepts of special lattice of rough algebras in 2011. Ronnason Chinram[20] introduced the concept of rough prime ideals and rough fuzzy prime ideals in gamma semigroups in 2009. Also the authors T. B. Iwinski[10] Z. Bonikowaski[3] have studied algebraic properties of rough sets. Then the concept of rough fuzzysets and fuzzy rough sets was introduced by D. Dubois, H. Parade[7] and Nick C. Fiala[16] discussed about semigroup, monoid and group models of groupoid identities in his paper. M. K. Sen and M. R. Adhikari[22] dealt the concepts on k-ideals of semirings. K. V. Krishna and N. Chatterjee[11] discussed the representation of near semirings and approximation of their categories. In his paper the authors have studied, S-semigroups are essentially the representations of near semirings to proceed to establish categorical representation of near semirings. Also B. Davaz[6] introduced the idea about roughness in rings in his paper and the author have studied the relationship between roughsets, ring theory and some properties of the lower and upper approximations in rings. Zadeh[24] introduced the concept of fuzzy sets in his paper.

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In the recent and past, rough set theory has triggered many researchers all around the world. The concept of rough set theory is the approximation space such as lower and upper approximations of a set determined by attributes. The pair of lower and upper approximation is called rough set also in rough set theory data can be represented in the form of an information system. An information system is a pair I = (U, A) where U is a non empty finite set of objects, called universal set and A is a nonempty set of fuzzy attributes defined by $\mu_a : U \rightarrow [0, 1]$, $a \in A$, is a fuzzy set. Indiscernibility is a core concept of rough set theory and it is defined as an equivalence between objects. Objects in the information system about which we have the same knowledge forms an equivalence relation.

Formally any set $P \in A$, there is an associated equivalence relation called P – Indiscernibility relation defined as follows,

$$IND(P)=\{(x, y) \in U^2 | \forall a \in P, \mu_a(x) = \mu_a(y)\}$$

The partition induced by IND(P) consists of equivalence classes defined by

$$[x]_p = \{y \in U | (x, y) \in IND(P)\}.$$

For any $X \subseteq U$, define the lower approximation space $\underline{P}(x) = \{x \in U | [x]_p \subseteq X\}$

Also, define the upper approximation space $P(\overline{X}) = \{x \in U | [x]_p \cap X, \varphi\}$

For every subset $X \in U$, there is an associated rough set RS (X) = $(\underline{P}(\overline{X}), P(x))$.

The paper is organised as follows.

In section 2, we give the necessary definitions pertaining to rough set theory and semiring theory.

In section 3, we use the binary operation praba Δ and praba ∇[18] on T and prove that (T, Δ, ∇) is a semiring called as the rough semiring.

Section 4, deals that the concepts of rough ideals in rough semiring and we illustrate all these concepts with examples.

Section 5, gives the conclusion.

2. Preliminaries

In this section we present some preliminaries on rough sets and algebraic structures.

2.1 Rough Sets

Let I = (U, A) be an information system and for any subset X of U and $(\underline{P}(x), \overline{P}(X))$ are the lower and upper approximations respectively as defined in previous section.

Definition 2.1 (rough set).

A rough set corresponding to X, where X is an arbitrary subset of U in the approximation space P, we mean the ordered pair RS (X) = $(\underline{P}(x), \overline{P}(X))$.

Example 2.1.

[18]Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and $A = \{a_1, a_2, a_3, a_4\}$ where each a_i(i = 1 to 4) is a fuzzy set whose membership values are shown in Table 1.

Table 1. Information table 1

A/U	a_1	a_2	a_3	a_4
x_1	0	0.1	0.3	0.2
x_2	1	0.6	0.7	0.3
x_3	0	0.1	0.3	0.2
x_4	1	0.6	0.7	0.3
x_5	0.8	0.5	0.2	0.4
x_6	1	0.6	0.7	0.3

Let $X = \{x_1, x_3, x_5, x_6\}$ and P = A. Then the equivalence classes induced by IND(P) are given below.

$$X_1 = [x_1]_p = \{x_1, x_3\} \quad (1)$$

$$X_2 = [x_2]_p = \{x_2, x_4, x_6\} \quad (2)$$

$$X_3 = [x_5]_p = \{x_5\} \quad (3)$$

Hence, $\underline{P}(x) = \{x_1, x_3, x_5\}$ and $P(\overline{X}) = \{x_1, x_2, x_3, x_4, x_5, x_6\}$.

Therefore RS (X) = $(\{x_1, x_3, x_5\}, \{x_1, x_2, x_3, x_4, x_5, x_6\})$.

Note that the upper approximation space consists of those objects that are possibly members of the target set X.

Remarks 2.1.

[18]If $X \subseteq U$, then $X \subseteq \cup^{i=1} X_{ai}$ where none, one or more of the equivalence classes are contained in X. Here X_{ai}, i = 1, 2, ... rare the equivalence classes induced by Ind(P).

Definition 2.2.

[18]If $X \subseteq U$, then the number of equivalence classes (Induced by Ind(P)) contained in X is called as the Ind. weight of X. It is denoted by IW(X).

Definition 2.3.

[18]Let $X, Y \subseteq U$. The Praba Δ is defined as

$$X \Delta Y = X \cup Y, \text{ if } IW(X \cup Y) = IW(X) + IW(Y) - IW(X \cap Y).$$

If $IW(X \cup Y) > IW(X) + IW(Y) - IW(X \cap Y)$, then identify the equivalence class obtained by the union of X and Y. Then delete the elements of that class belonging to Y. Call the new set as Y. Now, obtain $X\Delta Y$. Repeat this process until $IW(X \cup Y) = IW(X) + IW(Y) - IW(X \cap Y)$.

Definition 2.4.

[18]If $X, Y \subseteq U$ then an element $x \in U$ is called a Pivot element, if $[x]_p \nsubseteq X \cap Y$, but $[x]_p \cap X \neq \varphi$ and $[x]_p \cap Y \neq \varphi$.

Definition 2.5.

[18]If $X, Y \subseteq U$ then the set of Pivot elements of X and Y is called the Pivot set of X and Y and it is denoted by $P_{X \cap Y}$.

Definition 2.6.

[18]Praba ∇ of X and Y is denoted by $X\nabla Y$ and it is defined as

$$X\nabla Y = \{x \mid [x]_p \subseteq X \cap Y\} \cup P_{X \cap Y} \text{ where } X, Y \subseteq U.$$

Note that each Pivot element in $P_{X \cap Y}$ is the representative of that particular class.

Theorem 2.1.

[19]Let $I = (U, A)$ be an information system where U be the universal (finite) set and A be the set of attributes and T be the set of all roughsets then (T, Δ) is a commutative monoid of idempotents.

Theorem 2.2.

[19](T, Δ) is a regular rough monoid of idempotents.

Theorem 2.3.

[14]Let $I = (U, A)$ be an information system where U be the universal (finite) set and A be the set of attributes and T be the set of all roughsets then (T, ∇) is a monoid of idempotents and it is called Rough monoid of i dempotents.

Theorem 2.4.

[14](T, ∇) is a commutative rough ∇ monoid of idempotents.

Theorem 2.5.

[14](T, ∇) is a commutative regular rough ∇ monoid of idempotents.

2.2 Algebraic Structures

Definition 2.7. (Groupoid).

[16, 9]A groupoid consists of a non-empty set equipped with a binary operation $*$, and it is denoted by $(S, *)$.

Definition 2.8. (Semigroup).

[16, 9]A semigroup $(S, *)$ is a groupoid that is associative $(x * y) * z = x * (y * z)$ forall $x, y, z \in S$).

Definition 2.9. (Monoid).

[16, 9]A semigroup $(S, *)$ is said to be a monoid if it contains an identity element $e \in S$ such that $e * x = x * e = x$ for all $x \in S$.

Definition 2.10. (Commutative monoid).

[16, 9]A monoid $(S, *)$ which satisfies commutative axiom namely $x * y = y * x$ for all $x, y \in S$, is known as Commutative monoid.

Definition 2.11. (Idempotent).

[16, 9]An element x in a groupoid $(S, *)$ is said to be idempotent, if $x * x = x$.

Definition 2.12. (Regular semigroup).

[16, 9]A semigroup $(S, *)$ is said to be regular, if there exists an element $y \in S$ such that $x = x * y * x$ for all $x \in S$.

Definition 2.13. (Semiring).

A Semiring is a system consisting of a nonempty set S together with two binary operations on S called addition and multipliation (denoted in the usual manner) such that

(i) S together with addition is a semigroup
(ii) S together with multiplication is a semigroup
(iii) $a(b + c) = ab + ac$ and $(a + b)c = ac + bc$ for all $a, b, c \in S$

Definition 2.14. (Additively Commutative).

A semiring S is said to be additivey commutative if $a + b = b + a$ for all $a, b \in S$.

Definition 2.15. (Right (Left) ideal).

[9]A nonempty subset I of a semigroup $(S, *)$ is a right (left) ideal, if it satisfies $I * S \subseteq I$ $(S * I \subseteq I)$.

Definition 2.16. (Ideal).

A nonempty subset I of a semigroup (S, $*$) is said to be an ideal, if it is both right and left ideal.

Definition 2.17. (Ideal of a semiring).

A left or right ideal of a semiring S is a non empty subset I of S such that

(a) $a + b \in I$ for all $a, b \in I$ and
(b) $ra \in I$ and $ar \in I$ for all $a \in I$ and $r \in S$

In the following section, the set of pivot rough sets is defined on the set of all rough sets and its algebraic structure is studied.

3. Semiring on Rough Sets

Throughout this section, we consider an information system I = (U, A). Now for any $X \subseteq U$ RS (X) = ($\underline{P}(X)$, $\overline{P}(X)$) and let T = {RS (X)|$X \subseteq U$} be the set of all rough sets on I.

Definition 3.1 (Pivot Rough Set).

Let I = (U, A) be an information system. $U = \bigcup_{i=1}^{r} X_i$ be the union of equivalence classes induced by Ind P and T = {RS (X) | $X \subseteq U$}. Choose a representative x_i from each equivalence class X_i. Let $B = \{x_i | x_i \in X_i$ and $|X_i| > 1\}$ be the pivot set of the information system I consisting of the representative of the equivalence classes X_i whose cardinality is greater than 1.

Let J = {RS (X) | $X \in P(B)$}. This subset J of T is called as the set of Pivot Rough sets on U.

Theorem 3.1.

Let I = (U, A) be an information system where U be the universal (finite) set and A be the set of attributes and T be the set of all roughsets then (T, Δ, ∇) is a Semiring.

Proof.

From theorem 2.1 and theorem 2.3, we know that (T, Δ) and (T, ∇) are the Rough Semigroup respectively.

Claim: Both operations are realted by distributive laws For RS (X), RS (Y) and RS (Z) ∈ T

(i) $RS(X\Delta(Y\nabla Z)) = RS((X\Delta Y)\nabla(X\Delta Z))$
(ii) $RS(X\nabla(Y\Delta Z)) = RS((X\nabla Y)\Delta(X\nabla Z))$

Proof.

(i) $RS(X\Delta(Y\nabla Z)) = RS((X\Delta Y)\nabla(X\Delta Z))$

Claim: (i) $\underline{P}(X\Delta(Y\nabla Z)) = \underline{P}((X\Delta Y)\nabla(X\Delta Z))$

(ii) $\overline{P}(X\Delta(Y\nabla Z) = \overline{P}((X\Delta Y)\nabla(X\Delta Z))$

Proof for claim (i):

Let $x \in \underline{P}(X\Delta(Y\nabla Z))$

$$\Rightarrow [x]_p \subseteq X\Delta(Y\nabla Z)$$

$$\Rightarrow [x]_p \subseteq X \text{ or } [x]_p \subseteq Y\nabla Z$$

$$\Rightarrow [x]_p \subseteq X \text{ or } [x]_p \subseteq ((Y \cap Z) \cup P_{Y\cap Z})$$

$$\Rightarrow [x]_p \subseteq ((X\Delta Y)\nabla(X\Delta Z))$$

$$\Rightarrow x \in P((X\Delta Y)\nabla(X\Delta Z))$$

$\therefore P(X\Delta(Y\nabla Z)) \subseteq \underline{P}((X\Delta Y)\nabla(X\Delta Z))$

similarly, we prove that

$$\underline{P}((X\Delta Y)\nabla(X\Delta Z)) \subseteq \underline{P}(X\Delta(Y\nabla Z))$$

$\therefore \underline{P}(X\Delta(Y\nabla Z)) = \underline{P}((X\Delta Y)\nabla(X\Delta Z))$

Proof for claim (ii):

Let $x \in \overline{P}(X\Delta(Y\nabla Z))$

$$\Rightarrow [x]_p \cap (X\Delta(Y\nabla Z)) \neq \varphi$$

$$\Rightarrow [x]_p \cap X \neq \varphi \text{ or } [x]_p \cap (Y\nabla Z) \neq \varphi$$

$$\Rightarrow [x]_p \cap X \neq \varphi \text{ or } [x]_p \cap (Y \cap Z) \cup P_{Y\cap Z} \neq \varphi$$

Subcase (i):

$$[x]_p \cap X \neq \varphi \text{ or } [x]_p \cap (Y \cap Z) \neq \varphi$$

$$\Rightarrow [x]_p \cap (X\Delta Y) \neq \phi \text{ or } [x]_p \cap (X\Delta Z) \neq \phi$$

$$\Rightarrow [x]_p \cap ((X\Delta Y)\nabla(X\Delta Z)) \neq \phi$$

$$\Rightarrow x \in P((\overline{X}\Delta Y)\nabla(X\Delta Z))$$

$\therefore P(\overline{X}\Delta(Y\nabla Z)) \subseteq P((\overline{X}\Delta Y)\nabla(X\Delta Z))$

Subcase (ii):

$$[x]_p \cap X \neq \phi \text{ or } [x]_p \cap P_{Y\cap Z} \neq \phi$$

$$\Rightarrow [x]_p \cap X \neq \phi \text{ or } [x]_p \cap Y \neq \phi \text{ and } [x]_p \cap Z \neq \phi$$

$$\Rightarrow [x]_p \cap ((X\Delta Y)\nabla(X\Delta Z)) \neq \phi$$

$$\Rightarrow x \in P((\overline{X}\Delta Y)\nabla(X\Delta Z))$$

$\therefore P(\overline{X}\Delta(Y\nabla Z)) \subseteq P((\overline{X}\Delta Y)\nabla(X\Delta Z))$

similarly, we prove that

$$\overline{P}((X\Delta Y)\nabla(X\Delta Z)) \subseteq \overline{P}(X\Delta(Y\nabla Z))$$

$\therefore\ P(\overline{X}\Delta(Y\nabla Z) = P((\overline{X}\Delta Y)\nabla(X\Delta Z))$

$\therefore$ RS $(X\Delta(Y\nabla Z))$ = RS $((X\Delta Y)\nabla(X\Delta Z))$

Proof.

(ii) RS $(X\nabla(Y\Delta Z))$ = RS $((X\nabla Y)\Delta(X\nabla Z))$

Claim: (i) $\underline{P}(X\nabla(Y\Delta Z)) = \underline{P}((X\nabla Y)\Delta(X\nabla Z))$

(ii) $\overline{P}(X\nabla(Y\Delta Z) = \overline{P}((X\nabla Y)\Delta(X\nabla Z))$

Proof for claim (i):

Let $x \in \underline{P}(X\nabla(Y\Delta Z))$

$$\Rightarrow [x]_p \subseteq X\nabla(Y\Delta Z)$$

$$\Rightarrow [x]_p \subseteq (X \cap (Y\Delta Z) \cup P_{X\cap(Y\Delta Z)})$$

$$\Rightarrow [x]_p \subseteq X \cap Y \text{ or } [x]_p \subseteq X \cap Z$$

$$\Rightarrow [x]_p \subseteq (X\nabla Y)\Delta(X\nabla Z)$$

$$\Rightarrow x \in \underline{P}((X\nabla Y)\Delta(X\nabla Z))$$

$\therefore\ \underline{P}(X\nabla(Y\Delta Z)) \subseteq \underline{P}((X\nabla Y)\Delta(X\nabla Z))$

similarly, we prove that

$$\underline{P}((X\nabla Y)\Delta(X\nabla Z)) \subseteq \underline{P}(X\nabla(Y\Delta Z))$$

$\therefore\ \underline{P}(X\nabla(Y\Delta Z)) = \underline{P}((X\nabla Y)\Delta(X\nabla Z))$

Proof for claim (ii):

Let$x \in \overline{P}(X\nabla(Y\Delta Z))$

$$\Rightarrow [x]_p \cap (X\nabla(Y\Delta Z)) \neq \varphi$$

$$\Rightarrow [x]_p \cap (X \cap (Y\Delta Z) \cup P_{X\cap(Y\Delta Z)}) \neq \varphi$$

$$\Rightarrow [x]_p \cap X \neq \phi \text{ and } [x]_p \cap Y\Delta Z \neq \phi \text{ or } [x]_p \cap P_{X\cap(Y\Delta Z)} \neq \phi$$

Subcase (i):

$$[x]_p \cap X \neq \phi \text{ and } [x]_p \cap (Y\Delta Z) \neq \phi$$

$$\Rightarrow [x]_p \cap X\nabla Y \neq \phi \text{ or } [x]_p \cap (X\nabla Z) \neq \phi$$

$$\Rightarrow [x]_p \cap (X\nabla Y)\Delta(X\nabla Z) \neq \phi$$

$$\Rightarrow x \in P((\overline{X}\nabla Y)\Delta(X\nabla Z))$$

$\therefore\ P(\overline{X}\nabla(Y\Delta Z)) \subseteq P((\overline{X}\nabla Y)\Delta(X\nabla Z))$

Subcase (ii):

$$[x]_p \cap X \neq \phi \text{ and } [x]_p \cap P_{X\cap(Y\Delta Z)} \neq \phi$$

$$\Rightarrow [x]_p \cap X \neq \phi \text{ and } [x]_p \cap Y\Delta Z \neq \phi$$

$$\Rightarrow [x]_p \cap X\nabla Y \neq \phi \text{ or } [x]_p \cap (X\nabla Z) \neq \phi$$

$$\Rightarrow [x]_p \cap (X\nabla Y)\Delta(X\nabla Z) \neq \phi$$

$$\Rightarrow x \in P((\overline{X}\nabla Y)\Delta(X\nabla Z))$$

$\therefore\ P(\overline{X}\nabla(Y\Delta Z)) \subseteq P((\overline{X}\nabla Y)\Delta(X\nabla Z)$

similarly, we prove that

$$\overline{P}((X\nabla Y)\Delta(X\nabla Z)) \subseteq \overline{P}(X\nabla(Y\Delta Z))$$

$\therefore$ RS $(X\nabla(Y\Delta Z))$ = RS $((X\nabla Y)\Delta(X\nabla Z))$

$\therefore$ (T, Δ, ∇) is a semiring.

$\therefore$ This semiring is called as a rough semiring.

4. Rough Ideals in Semiring

In this section, we discuss about the ideals of Rough semiring (T, Δ, ∇)

Definition 4.1 (Rough ideal).

Consider the Rough semiring (T, Δ, ∇). A left or right Rough ideal of a Rough semiring is a non empty subset J of T such that

(a) RS $(X)\Delta$RS $(Y) \in J$ for all RS (X), RS $(Y) \in J$ and
(b) RS $(X)\nabla$RS $(Y) \in J$ and RS $(Y)\nabla$RS $(X) \in J$ for all RS $(Y) \in J$ and RS $(X) \in T$

Theorem 4.1.

The Pivot rough set is an ideal of the semiring (T, Δ, ∇)

Proof.

(i) Let RS (X), RS $(Y) \in J$, then RS $(X) = (\varphi, V)$, RS $(Y) = (\varphi, W)$ where V and W are the union of equivalence classes containing the pivot elements in X.

$\therefore$ RS $(X\Delta Y) = (\varphi, V \cup W)$

(ii) Let RS $(Y) \in J$ and RS $(X) \in T$ such that RS $(X\nabla Y) \in J$

case (1) :

$$X\nabla Y = \{x \mid [x]_p \subseteq X \cap Y\} \cup P_{X\cap Y}$$

$$\Rightarrow \text{If } X \cap Y = \varphi \text{ then } X\nabla Y = P_{X\cap Y} \in P(B)$$

$\therefore$ RS $(X\nabla Y) \in J$

case (2) :

If $X \cap Y$, φ then $X\nabla Y = P_{X\cap Y} \in P(B)$

$\therefore$ RS $(X\nabla Y) \in J$

$\therefore$ J is the ideal in T

Example 4.1. (Example)

Let us consider the information system I = (U, A) where $U = \{x_1, x_2, x_3, x_4\}$ and $A = \{a_1, a_2, a_3, a_4\}$ where each $a_i (i = 1 \text{ to } 4)$ is a fuzzy set of attributes whose membership values are shown in Table 2.

Table 2. Information table 2

A/U	a_1	a_2	a_3	a_4
x_1	0.2	0.3	1	0
x_2	0.8	0.4	0.1	0.9
x_3	0.2	0.3	1	0
x_4	0.8	0.4	0.1	0.9

Let $X = \{x_1, x_2, x_3, x_4\} \subseteq U$ then the equivalence classes induced by IND(P) are given below

$$X_1 = [x_1]_p = \{x_1, x_3\} \quad (4)$$

$$X_2 = [x_2]_p = \{x_2, x_4\} \quad (5)$$

and let $T = \{RS(X) \mid X \subseteq U\}$ be the set of all rough sets such that,

$$T = \{RS(\varphi), RS(U), RS(X_1), RS(X_2), RS(\{x_1\}), RS(\{x_2\}), RS(X_1 \cup \{x_1\}), RS(\{x_1\} \cup X_2), RS(\{x_1\} \cup \{x_2\})\} \quad (6)$$

(a) It can be verified that (T, Δ) is a commutative Rough monoid of idempotents
(b) Similarly we can verify (T, ∇) is a commutative Rough ∇ monoid
(c) (Distributive law) for $RS(X_1), RS(X_2), RS(\{x_1\}) \in T$,

To Prove:

(i) $RS(X_1 \Delta (X_2 \nabla \{x_1\})) = RS((X_1 \Delta X_2) \nabla (X_1 \Delta \{x_1\}))$
(ii) $RS(X_1 \nabla (X_2 \Delta \{x_1\})) = RS((X_1 \nabla X_2) \Delta (X_1 \nabla \{x_1\}))$

Case(i):

$$RS(X_1 \Delta (X_2 \nabla \{x_1\})) = RS(X_1) \Delta (RS(X_2) \nabla RS(\{x_1\})) = RS(X_1) \Delta RS(\varphi) = RS(X_1)$$

$$RS((X_1 \Delta X_2) \nabla (X_1 \Delta \{x_1\})) = (RS(X_1) \Delta RS(X_2)) \nabla (RS(X_1) \Delta RS(\{x_1\})) = RS(U) \nabla RS(X_1) = RS(X_1)$$

thus $RS(X_1 \Delta (X_2 \nabla \{x_1\})) = RS((X_1 \Delta X_2) \nabla (X_1 \Delta \{x_1\}))$

Case(ii):

$$RS(X_1 \nabla (X_2 \Delta \{x_1\})) = RS(X_1) \nabla (RS(X_2) \Delta RS(\{x_1\})) = RS(X_1) \nabla RS(X_2 \cup \{x_1\}) = RS(\{x_1\})$$

$$RS((X_1 \nabla X_2) \Delta (X_1 \nabla \{x_1\})) = (RS(X_1) \nabla RS(X_2)) \Delta (RS(X_1) \nabla RS(\{x_1\})) = RS(\varphi) \Delta RS(\{x_1\} = RS(\{x_1\})$$

thus $RS(X_1 \nabla (X_2 \Delta \{x_1\})) = RS((X_1 \nabla X_2) \Delta (X_1 \nabla \{x_1\}))$

from case(i) and case(ii), Distributive law holds. The verification of the distributive property for all the other elements can be done simillarly (T, Δ, ∇) is a Semiring.

(d) (Ideals in semiring)

For the information system given in table 2, let $X = \{x_1, x_2, x_3, x_4\} \subseteq U$ and $Y = \{x_1, x_2\} \subseteq U$ where

$$U = \{x_1, x_2, x_3, x_4\}$$

case(i) If $B = \{x_1, x_2\}$ then $P(B) = \{\varphi, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$

$J = \{(\varphi,\varphi), (\varphi, X_1), (\varphi, X_2), (\varphi, X_1 \cup X_2)\} = \{RS(\varphi), RS\{x_1\}, RS\{x_2\}, RS\{x_1, x_2\}\}$

from the cayley's table under prabaΔ

case (ii)

from the cayley's table under praba∇, it is obvious that for all $RS(X) \in T$ and $RS(Y) \in J$ such that $RS(X) \nabla RS(Y) \in J$ thus J is said to be a rough ideal of T.

Table 3. Cayley's table under praba Δ

Δ	RS (φ)	RS $\{x_1\}$	RS $\{x_2\}$	RS $\{x_1, x_2\}$
RS (φ)	RS (φ)	RS $\{x_1\}$	RS $\{x_2\}$	RS $\{x_1, x_2\}$
RS $\{x_1\}$	RS $\{x_1\}$	RS $\{x_1\}$	RS $\{x_1, x_2\}$	RS $\{x_1, x_2\}$
RS $\{x_2\}$	RS $\{x_2\}$	RS $\{x_1, x_2\}$	RS $\{x_2\}$	RS $\{x_1, x_2\}$
RS $\{x_1, x_2\}$	RS $\{x_1, x_2\}$	RS $\{x_1, x_2\}$	RS $\{x_1, x_2\}$	RS $\{x_1, x_2\}$

Table 4. Cayley's table under praba ∇

∇	RS (φ)	RS (U)	RS (X_1)	RS ({x_2})	RS ({x_1})	RS ({x_2})	RS (X_1 U {x_2})	RS ({x_1} U X_2)	RS ({x_1}, {x_2})
RS (φ)	RS (φ)	RS (φ)	RS (φ)	RS (φ)	RS (φ)	RS (φ)	RS (φ)	RS (φ)	RS (φ)
RS ({x_1})	RS (φ)	RS ({x_1})	RS ({x_1})	RS (φ)	RS ({x_1})	RS (φ)	RS ({x_1})	RS ({x_1})	RS ({x_1})
RS ({x_2})	RS (φ)	RS ({x_2})	RS (φ)	RS ({x_2})	RS (φ)	RS ({x_2})	RS ({x_2})	RS ({x_2})	RS ({x_2})
RS ({x_1}, {x_2})	RS (φ)	RS ({x_1}, {x_2})	RS ({x_1})	RS ({x_2})	RS ({x_1})	RS ({x_2})	RS ({x_1}, {x_2})	RS ({x_1}, {x_2})	RS ({x_1}, {x_2})

5. Conclusion

In this paper, we dealt the two operations praba Δ and praba ∇ on the set of all rough sets T for a given information system I = (U, A) and we proved that (T, Δ, ∇) is a semiring on the set of all rough sets T also we extended the concepts to ideal of semiring on T. The future study is to investigate and explore these ideals.

6. References

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