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Assessment of the Impact of Orthogonal Directional Pattern Generating Algorithms on the Accuracy of Wireless Direction Finding by BS-MUSIC and BS-Capon Methods

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Abstract

The paper considers the MUSIC and Capon methods for determination of radio radiation coordinates, possessing a superresolution property, which assumes that two signal sources spaced less than at the Rayleigh limit can be represented as individual peaks on the spatial direction finding relief graph. A computational burden problem arises in such spectrum estimation algorithms; it can be solved by the methods of building so-called orthogonal directional patterns, reducing computational complexity and antenna array dimension. A multibeam array pattern (spatial channel) is formed for this purpose, and then the signals are pre-processed by means of these spatial channels focused in a certain region. The paper gives numerical estimations of accuracy for determination of radio radiation source coordinates by the classical MUSIC and Capon methods, and also after preliminary transformation by orthogonal directional patterns (BS-MUSIC and BS-Capon) versus signal/noise ratio, the number of beams and averaging time of the correlation matrix. A great number of methods of mutually orthogonal spatial channel formation are presented; among them the most well-known discrete Fourier transform, spherical sequences and Taylor series expansion. It is worth noting that to date no comparative studies of the accuracy characteristics of such methods (BS-MUSIC and BS-Capon) has been performed. It is found that the accuracy of the BS-MUSIC and BS-Capon methods increases with the increase in the number of spatial channels from four to five. In this case the accuracy of the method for channel generating by spherical discrete sequences is the highest in the majority of the considered interference cases.

Keywords: Digital Antenna Array, Orthogonal Directional Patterns, Superresolution, Wireless Direction Finding

1. Introduction

Development of high-accuracy radio data transmission systems with rapid response time, large capacity and increased transmission capacity designed to work in high interference signaling environment in a limited frequency range is currently relevant.

One of the resources to increase the capacity of information data transmission systems is the use of degrees of freedom associated with adaptive beam steering. This often requires a priori information about the location of subscribers and interference sources. The accuracy of this information influences the correctness of setting antenna system directional pattern zeroes and maximums, and therefore, the signal/noise ratio for the signals received by subscriber and base station. Nowadays, angular superresolution methods based on the correlation processing of antenna array output signals are gaining ground. Among these methods it is required to distinguish, first of all, MUSIC and Capon algorithms that are suitable for any configuration of antenna array, providing high resolution of signals and being the most promising ones in the superresolution[3,18,25].

Adaptive formation of directional pattern (DP) of

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antenna arrays (AA) enables to increase considerably subscriber's signal-to-interference-plus-noise ratio (SINR). One of the pattern forming techniques is based on the estimation of angular coordinates of radio radiation sources (RRS). Such algorithms require the input signals from all antenna elements (AE) to be available in the digital form. In major applied problems the number of primary modules for the received signals processing and of analog-digital converters (ADC) can reach high values. Antenna arrays consisting of several dozen elements are not uncommon[10]. The order of the computational complexity of spectral analysis of the arrays consisting of *N* antenna elements makes O(N^3). After applying the beam-space processing, the CPU time is significantly reduced[19]. Therefore, the ways to reduce the observation vector dimension with minimal loss are of great interest (Huang et al., 2010).

2 Basic Assumptions

Assume that *M* radio signals are incoming to AA from random directions $\{\theta_m\}_{m=1}^{M}$ and are described by expression[10].

$s_m(t) = b_m(t)\exp(j2\pi f_m t)$,

where $b_m(t)$ – the amplitude of *m*-th signal, f_m – carrier frequency of *m*-th signal, *t* – time. For the linear AA the signal with azimuth θ_m on the *n*-th antenna element acquires phase shift $-2\pi D\lambda^{-1} n \sin(\theta_m)$, and it becomes possible to obtain linear array direction vector[7].

$$\vec{\mathbf{a}}(\theta_m) = [1 \quad \exp\{j[-2\pi D\lambda^{-1}\sin(\theta_m)]\} \quad \dots \quad \exp\{j[-2\pi D\lambda^{-1}(N-1)\sin(\theta_m)]\}]^T,$$

where n=1...N, $k_m = 2\pi/\lambda_m$, λ_m – wavelength of the *m*-th signal, D – interelement spacing. For AA complex vector of antenna element output signals is described by the expression[10]:

$$\vec{\mathbf{x}}(t) = \mathbf{A}\cdot\vec{\mathbf{s}}(t) + \vec{\mathbf{n}}(t),$$

where $\vec{\mathbf{x}}(t)^T$ – *N*-dimensional vector describing output signals of each , antenna element AA, $\vec{\mathbf{s}}(t)$ – *M*-dimensional vector of signals; $\vec{\mathbf{n}}(t)$ – noise vector; A – N M direction vector matrix. Then spatial correlation matrix can be written[7]:

$$\mathbf{R}_{xx} = E[\vec{\mathbf{x}}(t)\vec{\mathbf{x}}^H(t)] = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + \sigma^2\mathbf{I} = \mathbf{E}_s\Lambda_s\mathbf{E}_s^H + \mathbf{E}_n\Lambda_n\mathbf{E}_n^H,$$

where $\mathbf{R_s} = E[\vec{s}(t)\vec{s}^H(t)]$, $\lambda_1 > \lambda_2 > ... > \lambda_M > \lambda_{M+1} = ... \lambda_N = \sigma^2$ and and $\vec{e}_1, \vec{e}_2, ..., \vec{e}_N$ –eigenvalues and eigenvectors of **R** matrix, respectively, $\mathbf{E}_s$, $\mathbf{E}_n$ – signal and noise subspace matrices, Λ_S, Λ_n– diagonal matrices of signal and noise subspace eigenvalues.

3. Preprocessing Methods

In many cases the number of antenna elements *N* has high value, and then N_{BS} directional patterns are generated for preliminary processing of signals received by each AE. After that further estimation and transformation are performed (Figure 1)[7].

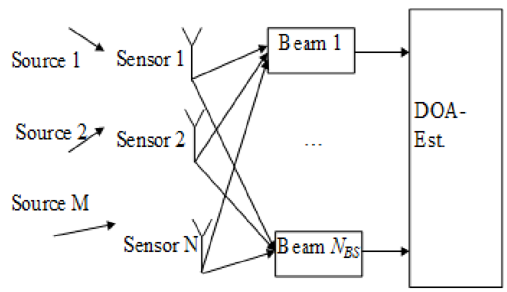


Figure 1. Tested specimen.

The scheme of Fig. 1 can be presented in the form of matrix linear transformations as follows follows[3,18,23] :

$$\vec{\mathbf{z}}(k) = \mathbf{T}_{BS}^H\vec{\mathbf{x}}(k),$$

where $\vec{\mathbf{z}}(k)$ - R-dimensional vector,

$\mathbf{T}_{BS} = \left[\vec{\mathbf{t}}(\psi_1) \quad \vec{\mathbf{t}}(\psi_2) \quad \dots \quad \vec{\mathbf{t}}(\psi_R)\right]$ - NBS×N transformation matrix consisting of NBS weight vectors which help to form the set of NBS directional patterns focused in the directions ψ_i, i=1,2,..., N_{BS} and $M < N_{BS} << N$. The transformation is usually performed in the analog part, reducing considerably the required quantity of the ADCs, and has a number of advantages[11,23]:

- Computational complexity is reduced and, therefore statistical stability of spectrum response estimation is achieved.
- It becomes possible to suppress powerful RRSs prior to apply signal parameter estimation methods within the sector.
- Noise in the c overage sector gets white with unknown dispersion.

In addition, $\mathbf{T}_{BS}$ matrix columns are orthogonal relative to each other, i.e.

$$\mathbf{T}_{BS}^H\mathbf{T}_{BS} = \mathbf{I} \tag{1}$$

In some applications the condition (1) is not satisfied, and then in order to reduce T_{no} to orthonormalized matrix it is necessary to perform preliminary transformation:

$$\mathbf{T}_{bs} = \mathbf{T}_{no}\left[\mathbf{T}_{no}^H\mathbf{T}_{no}\right]^{-0.5} \tag{2}$$

Orthogonality of the directional patterns means that each beam is focused on its radio radiation source, and nulls are formed in the directions of the remaining RRSs. Thus, in case of only one signal only one element of $\vec{\mathbf{z}}(k)$ vector contains information about this signal[16]. In case if $\mathbf{T}_{BS}$ matrix columns have been selected incorrectly, performance loss of spectral analysis methods will occur. Antenna array observation vector[3,18]:

$$\vec{\mathbf{z}}(t) = \mathbf{T}_{BS}^{H}\mathbf{A}(\theta)\vec{\mathbf{s}}(t) + \mathbf{T}_{BS}^{H}\vec{\mathbf{n}}(t)$$

Spatial covariation matrix with regard to $\mathbf{T}_{BS}$ is transformed to[1,11]:

$$\mathbf{R}_{zz} = E[\vec{\mathbf{z}}(t)\vec{\mathbf{z}}^{H}(t)] = \mathbf{T}_{BS}^{H}\mathbf{R}_{xx}\mathbf{T}_{BS} == \mathbf{T}_{BS}^{H}\mathbf{ASA}^{H}\mathbf{T}_{BS} + \sigma^{2}\mathbf{T}_{BS}^{H}\mathbf{T}_{BS} \tag{3}$$

Then eigenvector and eigenvalue decomposition is as follows[23]:

$$\mathbf{R}_{zz} = \mathbf{E}_{BS-s}\Lambda_{BS-s}\mathbf{E}_{BS-s}^{H} + \mathbf{E}_{BS-n}\Lambda_{BS-n}\mathbf{E}_{BS-n}^{H}, \tag{4}$$

where $E_{BS\text{-}s}$ – $N_{Bs}\times M$ matrix of signal subspace vectors, $E_{BS\text{-}n}$ – $N_{Bs}\times(N_{Bs}\text{-}M)$ matrix of noise subspace vectors consisting of eigenvectors, corresponding to the least eigenvalues (N_{Bs}-M), $\Lambda_{BS\text{-}s}$, $\Lambda_{BS\text{-}n}$ - diagonal matrices of eigenvalues. $\mathbf{T}_{BS}$ linear transformation matrix reflects vector space of total dimension to the less dimensional subspace. Design of such matrix shall follow certain criteria.

3.1. Discrete Fourier Transformation (DFT)

The most widespread $\mathbf{T}_{BS}$ transformation matrix has columns consisting of directional pattern generating coefficients with maximums spaced apart to 2π/N. Then $\mathbf{T}_{BS}^{H}$ matrix lines are equal to:

$$\mathbf{T}_{BS}^{H} = \frac{1}{N}e^{j\left(\frac{N-1}{2}\right)m\frac{2\pi}{N}}\begin{bmatrix}1 & e^{jm\frac{2\pi}{N}} & \dots & e^{j(N-1)m\frac{2\pi}{N}}\end{bmatrix} \tag{5}$$

where m – value determining the beam number in the coverage sector.

Transformations to reduce the level of side lobes, such as Hamming windows, can be used together with DFT method.

3.2 Beams of Taylor series.

In case of two signal sources the beam matrix looks as follows:

$$\mathbf{T}_{no} = \begin{bmatrix}\vec{\mathbf{a}}(\psi_1) & \vec{\mathbf{a}}(\psi_2) & \vec{\mathbf{a}}(\psi_{mid})\end{bmatrix} \tag{6}$$

where ( ) - direction vector corresponding to the true angular coordinates of RRSs ψ_1 and ψ_2, $\psi_{mid} = \frac{(\psi_1 + \psi_2)}{2}$ - is a midpoint. However, it is impossible to build $\mathbf{T}$ according to[6], since true values of ψ_1 and ψ_2 are not known for certain and they must be estimated.

Nevertheless, the matrix (6) can be approximated with the help of Taylor series expansion of $\vec{\mathbf{a}}$ vectors around the region containing useful signals, ψ_{mid}:

$$\vec{\mathbf{a}}(\psi_i) = \vec{\mathbf{a}}(\psi_{mid}) + \vec{\mathbf{a}}^{(1)}(\psi_{mid})(\psi_i - \psi_{mid}) + \vec{\mathbf{a}}^{(2)}(\psi_{mid})\frac{(\psi_i - \psi_{mid})^2}{2!} + \dots,$$

$$\vec{\mathbf{a}}^{(1)}(\psi_{mid}) = \frac{\partial^k \vec{\mathbf{a}}^{(1)}(\psi_{mid})}{\partial^k \psi_{mid}}$$

where $i = 1,2,\dots m$ and k – order of the derived direction vector relative to the angle ψ_{mid}. Then

$$\mathbf{T}_{no} = \begin{bmatrix}\vec{\mathbf{a}}(\psi_{mid}) & \vec{\mathbf{a}}^{(1)}(\psi_{mid}) & \vec{\mathbf{a}}^{(2)}(\psi_{mid}) & \dots & \vec{\mathbf{a}}^{(m)}(\psi_{mid})\end{bmatrix} \tag{7}$$

Having obtained the matrix (7), one must make use of transformation[2] to reduce $\mathbf{T}_{no}$ to the orthonormalized form of $\mathbf{T}_{BS}$.

3.3 Discrete convex spherical sequences

We define vectors $\vec{\mathbf{t}}_i, i = 1,2\dots M$ as columns of $\mathbf{T}_{BS}$ orthogonal directional pattern matrix. Let ratio of the i-th beam energy in the scanning region $[-\psi_0, \psi_0]$ to the i-th DP energy in general be defined by the relationship:

$$\alpha_i = \frac{\int_{-\psi_0}^{\psi_0}\left|\vec{\mathbf{t}}_i\vec{\mathbf{a}}(\psi)\right|^2 d\psi}{\int_{-\pi}^{\pi}\left|\vec{\mathbf{t}}_i\vec{\mathbf{a}}(\psi)\right|^2 d\psi}, i = 1,2\dots M$$

The numerator α_i looks like:

$$\alpha_{iN} = \vec{\mathbf{t}}_i\mathbf{A}_{DPSS}\vec{\mathbf{t}}_i^{H},$$

where $\mathbf{A}_{DPSS} = \int_{-\psi_0}^{\psi_0}\vec{\mathbf{a}}(\psi)\vec{\mathbf{a}}^{H}(\psi)d\psi$. For the linear AA the

mn-th element of $\mathbf{A}_{DPSS}$ matrix is respectively determined as:

$$\mathbf{A}_{DPSS}^{mn} = \frac{2\psi_0\sin(m-n)}{(m-n)}, m \neq n$$

$$\mathbf{A}_{DPSS}^{mn} = 2\psi_0, m = n$$

The denominator of the α_i matrix looks like:

$$\alpha_{iD} = 2\pi\vec{\mathbf{t}}_i^{H}\vec{\mathbf{t}}_i$$

Then

$$\alpha_i = \frac{\vec{\mathbf{t}}_i \mathbf{A}_{DPSS} \vec{\mathbf{t}}_i^H}{2\pi \vec{\mathbf{t}}_i^H \vec{\mathbf{t}}}, i = 1,2 \ldots N_{se}$$

It is necessary to maximize α_i, i=1,2,..., N_{se} providing mutual orthogonality $\vec{\mathbf{t}}_i$, which is equivalent to the search of $\mathbf{A}_{DPSS}$ matrix eigenvectors, corresponding to the highest eigenvalues M. Thus, we have equality[6]

$$2\pi\lambda \vec{\mathbf{t}}_i = \mathbf{A}_{DPSS} \vec{\mathbf{t}}_i.$$

What is more?

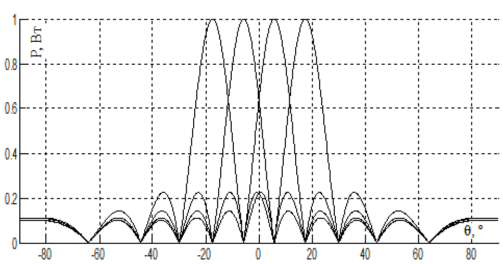
$$\sum_{n=1}^{N} \frac{\psi_0 \sin(m-n)}{(m-n)} t_n = \pi \lambda t_m, m = 1,2,\ldots,K,N$$

For each of the highest eigenvalues M, we obtain a sequence that defines vector-column $\vec{\mathbf{t}}_i$. The number of significant eigenvalues, and, therefore, of DPs is determined as:

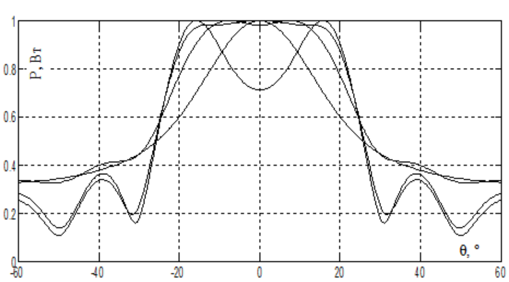
$$N_{se} = \frac{\psi_0}{\pi} N + 1\cdot$$

Then the coverage sector value ψ_0 for this method determines the quantity of orthogonal beams N_{se}, as eigenvectors of $\mathbf{A}_{DPSS}$ matrix, which in fact form T_{bs} columns.

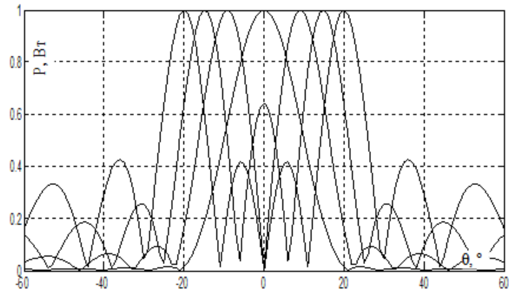
For illustrative purposes Figure 2 shows 4 orthogonal DPs for window coverage (-20°; 20°), obtained according to DFT method (Figure 2a), by Taylor series expansion (Figure 2b) and discrete spherical sequences (Figure 2c) for linear antenna array from 10 half-wavelength spaced AEs.



(a)



(b)



(c)

Figure 2. Set of 4 orthogonal DP for (a) DFT; (b) Taylor series (c) spherical sequences.

DP for Taylor beams (Figure 2b) have been obtained by building matrix (8) to the central angle $\psi_{mid} = 0^0$. Then T_{no} is expanded to the orthogonal form by means of Equation (3). In Figure 1(c) DPs are taken as 4 eigenvectors of $\mathbf{A}_{DPSS}$ matrix. It is seen from the Figure 2 that the highest level of side lobes is given by the Taylor beam method.

4 Estimation of RRS Angular Coordinates

4.1 Capon and BS-Capon Method

The problem is set up as follows: it is necessary to find $\vec{\mathbf{w}}$ weight vector minimizing average AA output power provided that for a certain angle of arrival θ the array transmission coefficient is fixed and equals, for example, to unity:

$$\min_w \left\langle \left| \vec{\mathbf{w}}^H \vec{\mathbf{x}} \right|^2 \right\rangle = \min P(\vec{\mathbf{w}}) \text{ при } \vec{\mathbf{w}}^H \vec{\mathbf{a}}(\theta) = 1$$

To solve this problem Lagrangian functional shall be composed as:

$$\Phi(\vec{\mathbf{w}}) = \left\langle \left| \vec{\mathbf{w}}^H \vec{\mathbf{x}} \right|^2 \right\rangle - \chi\left(\vec{\mathbf{w}}^H \vec{\mathbf{a}}(\theta) - 1 \right),$$

where χ - undetermined Lagrange multiplier. In this case $\left\langle \left| \vec{\mathbf{w}}^H \vec{\mathbf{x}} \right|^2 \right\rangle = \vec{\mathbf{w}}^H \mathbf{R} \vec{\mathbf{w}}$. We equate the gradient of this functional to zero and obtain an equation minimizing average output power according to Capon criterion4:

$$\vec{\mathbf{w}} = \frac{\mathbf{R}^{-1} \vec{\mathbf{a}}(\theta)}{\vec{\mathbf{a}}^H(\theta) \mathbf{R}^{-1} \vec{\mathbf{a}}(\theta)}$$

Thus, for Capon method a decision function[4]:

$$P_{Capon}(\theta) = \frac{1}{\vec{\mathbf{a}}^H(\theta)\mathbf{R}^{-1}\vec{\mathbf{a}}(\theta)}$$

This method is able to solve correlated signals. However, its accuracy in general in lower than that of eigenstructure methods, such as MUSIC.

Assuming vector $\vec{\mathbf{a}}$ and R matrix columns undergo mapping to one and the same subspace formed by T_{bs} matrix columns will result in the fact that Capon method modified to be used with orthogonal DP (so-called BeamSpace Capon or BS-Capon) reduces to the computation of function:

$$P_{BS-Capon}(\theta) = \frac{1}{\vec{\mathbf{a}}^H(\theta)\mathbf{T}_{Bs}\mathbf{R}_{zz}^{-1}\mathbf{T}_{Bs}^H\vec{\mathbf{a}}(\theta)}$$

4.2 MUSIC and BS-MUSIC Method

To implement the MUSIC method the orthogonality of noise subspace and signal vectors[17]:

$$\mathbf{E}_n^H\vec{\mathbf{a}}(\theta_m) = 0, \theta_m \in \{\theta_1, \theta_2, ..., \theta_M\} \tag{8}$$

And then signal coordinates will correspond to the maxima of the function[17]:

$$P_{MUSIC}(\theta) = \frac{1}{\vec{\mathbf{a}}^H(\theta)\mathbf{E}_n\mathbf{E}_n^H\vec{\mathbf{a}}(\theta)}.$$

The MUSIC method possesses infinitely high resolution in the absence of noise and amplitude and phase mismatch of AA channels. This method is unstable if the signals are correlated[13,14,15].

According to Equations (3) and (4) it can be concluded that:

$$\Re\{\mathbf{E}_{BS-s}\} = \Re\{\mathbf{T}^H\mathbf{A}(\theta)\},$$

where $\Re\{\mathbf{M}\}$ means space formed by the columns of random matrix **M**. Then $\Re\{\mathbf{E}_{BS-s}\}$ may be used as signal subspace that is orthogonal to the noise subspace formed by $\Re\{\mathbf{E}_{BS-n}\}$, i.e.

$$\Re\{\mathbf{E}_{BS-s}\} \perp \Re\{\mathbf{E}_{BS-n}\} \Rightarrow \Re\{\mathbf{T}^H\mathbf{A}(\theta)\} \perp \Re\{\mathbf{E}_{BS-n}\}$$

Therefore, similarly to[8] it is possible to write expressions for RRS coordinates to be determined after transformation by the orthogonal DPs[20,21]:

$$P_{BS-MUSIC}(\theta) = \frac{1}{\vec{\mathbf{a}}^H(\theta)\mathbf{T}_{Bs}\mathbf{E}_{BS-n}\mathbf{E}_{BS-n}^H\mathbf{T}_{Bs}^H\vec{\mathbf{a}}(\theta)}, \tag{9}$$

where T_{Bs} is selected with the preset number of beams focused in a definite region of space and formed by one of the above methods. The Expression (9) is called BEAMSPACE MUSIC (BS-MUSIC).

4.3 Assessment of RRS angular accuracy

There are known properties of the BS-MUSIC method concerning angular accuracy: root-mean-square deviation of RRS azimuth coordinate estimates for any T_{BS} matrix cannot be less than that of the classical MUSIC method[12,24]. To prove this an expression for the MUSIC method dispersion can be given[20,21]:

$$\mathrm{var}_{MUSIC}(\theta) = \frac{\sigma^2}{2N}\left[\frac{\mathbf{R}_{ss} + \sigma^2\mathbf{R}_{ss}^{-1}\left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{R}_{ss}^{-1}}{\mathbf{D}^H\left(\mathbf{I} - \mathbf{A}\left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H\right)\mathbf{D}}\right] \tag{10}$$

where $\mathbf{D} = \left[\vec{\mathbf{d}}(\theta_1), \ldots, \vec{\mathbf{d}}(\theta_M)\right], \vec{\mathbf{d}}(\theta_m) = \frac{\partial}{\partial\theta_m}\vec{\mathbf{a}}(\theta_m), m = 1\ldots M.$

The Expression (10) can be expanded for BS-MUSIC[20,21]:

$$\mathrm{var}_{BS-MUSIC}(\theta) = \frac{\sigma^2}{2N}\left[\frac{\mathbf{R}_{ss} + \sigma^2\mathbf{R}_{ss}^{-1}\left(\mathbf{A}^H\mathbf{Q}\mathbf{A}\right)^{-1}\mathbf{R}_{ss}^{-1}}{\mathbf{D}^H\mathbf{Q}\left(\mathbf{I} - \mathbf{A}\left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H\right)\mathbf{D}}\right]$$

where $\mathbf{Q} = \mathbf{T}_{BS}\mathbf{T}_{BS}^H$. In this case

$$\mathrm{var}_{MUSIC}(\theta) \le \mathrm{var}_{BS-MUSIC}(\theta).$$

In addition[20,21], it is shown in that in case of two T_{BS1} matrices (with dimension $N \times N_{BS1}$) and T_{BS2} (with dimension $N \times N_{BS2}$), where $N_{BS1} \ge N_{BS2}$, the root-mean-square deviation of RRS angular accuracy estimates obtained by means of T_{BS2} is not less than the estimates of T_{BS1}.

The expression for T_{BS} can be used to compare the characteristics of the classical MUSIC method with BS-MUSIC. In particular, the comparison with the respective Cramer-Rao bound is of interest, its analytical expression for the stochastic case looks like:

$$CRB = \frac{\sigma^2}{2N}\left[Re\left\{\left(\mathbf{D}^H\mathbf{P}_{\mathbf{A}}^{\perp}\mathbf{D}\right) \otimes \left(\mathbf{R}_{ss}\mathbf{A}^H\mathbf{R}_{xx}^{-1}\mathbf{A}\mathbf{R}_{ss}\right)\right\}\right]^{-1},$$

where $\mathbf{P}_{\mathbf{A}}^{\perp} = \mathbf{I} - \mathbf{A}\left(\mathbf{A}^H\mathbf{A}\right)^{-1}\mathbf{A}^H$ - matrix of the orthogonal projection on the noise subspace[20,21].

Lower bound of RRS coordinates estimation variance for the preliminary processing by the orthogonal DPs[2,20,21] looks like:

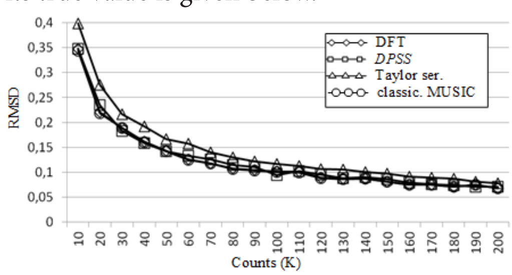
$$CRB_{BS} = \frac{\sigma^2}{2N}\left[Re\left\{\left(\mathbf{D}^H\mathbf{T}_{BS}\mathbf{P}_{BS-\mathbf{A}}^{\perp}\mathbf{T}_{BS}^H\mathbf{D}\right) \otimes \left(\mathbf{R}_{ss}\mathbf{A}^H\mathbf{T}_{BS}\mathbf{R}_{xx}^{-1}\mathbf{T}_{BS}^H\mathbf{A}\mathbf{R}_{ss}\right)\right\}\right]^{-1}$$

where $\mathbf{P}_{\mathbf{A}}^{\perp} = \mathbf{I} - \mathbf{T}_{BS}^{H}\mathbf{A}\left(\mathbf{A}^{H}\mathbf{Q}\mathbf{A}\right)^{-1}\mathbf{A}^{H}\mathbf{T}_{BS}$.

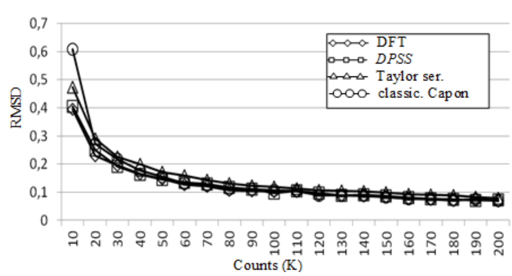
Expressions for CRB and CRB_{BS} bounds correspond to the general case and include assumption about uncorrelated signals. The MUSIC method is known to be the implementation of maximum-likelihood technique and to achieve Cramer-Rao bound in case of fairly large number of counts *N*, value of noise-to-signal ratio and provided R_{ss} is diagonal[15]. In the real situations only limited number of time snapshots *N* is available and signals have nonzero correlation coefficient, consequently estimate variance of RRS azimuth coordinates increases[5]. However, the specified factors have not been studied for BS-MUSIC and BS-Capon methods[5,12,20-22,24].

4.4 Count rate impact

Simulation of a linear equidistant AA consisting of *N*=10 AE spaced at a distance of 0.5λ was performed. Then the orthogonal DP generating technique with beam number $N_{BS}<N$ in different noise environment was applied to this AA. Coverage sector was (-20°; 20°), two RRS of equal capacity with angular coordinates -5° and 19°, respectively, were simulated. Amount of tests varied from 10 to 200. Evaluation of root-mean-square deviation (RMSD) of direction-finding bearing measurement from its true value is given below.



(a)



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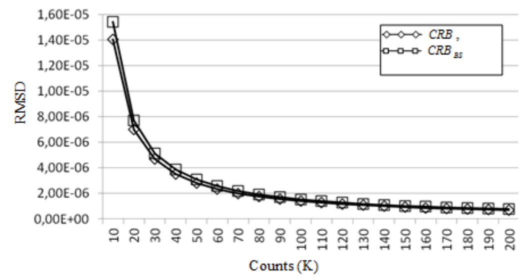

(c)

Figure 3. RMSD versus count rate (a) BS-MUSIC; (b) BS-Capon; (c) CRB.

Figure 3 shows relationship between RMSD of estimated angles of arrival of radio signals by BS-MUSIC, BS-Capon methods and averaging time K for various methods of obtaining $\mathbf{T}_{\mathbf{BS}}$ matrix. In addition, it gives the values of RMSD lower bound according to the Cramer-Rao method. Curves shown in Figure 3 allow making some conclusions. RMSD is inversely proportional to K averaging count rate and complies with the existing results[15]. Curve forms in Figure 3a) and b) coincide, however BS-Capon and BS-MUSIC will asymptotically reach Cramer-Rao bounds at a fairly high value of K which is unattainable in the real applications when it is practically impossible to diagonalize R_{ss} matrix in case of a stochastic model. RMSD of BS-Capon and BS-MUSIC methods approximate to their acceptable values at K number exceeding 100 counts.

4.5 Impact of the orthogonal DP quantity on the direction finding accuracy

Testing environment: linear equidistant AA with interelement spacing being equal to 0.5λ, number of AE is 10, quantity of orthogonal directional patterns N_{BS} amounts to 4 and 5 in order to evaluate the impact of RRS angular coordinates on the accuracy assessment, SNR ranges from -20 to 0dB. In the first case two signals with coordinates -5° and 18° are covered by the coverage sector (-20°; 20°), in the second case one more signal with azimuth 50° is added, this signal is not included in the window of interest.

Figures 4-5 demonstrate that for the cases under consideration RMSDs of BS-MUSIC and BS-Capon

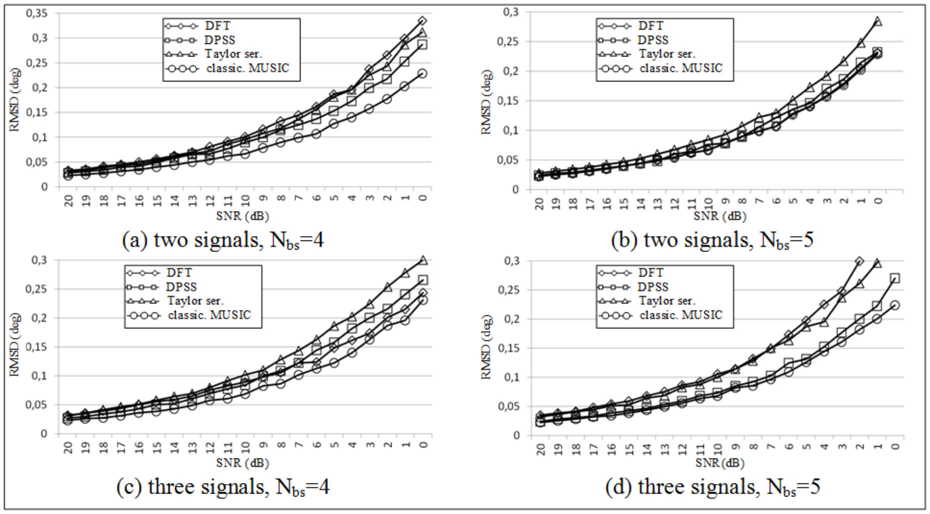

Figure 4. BS-MUSIC.

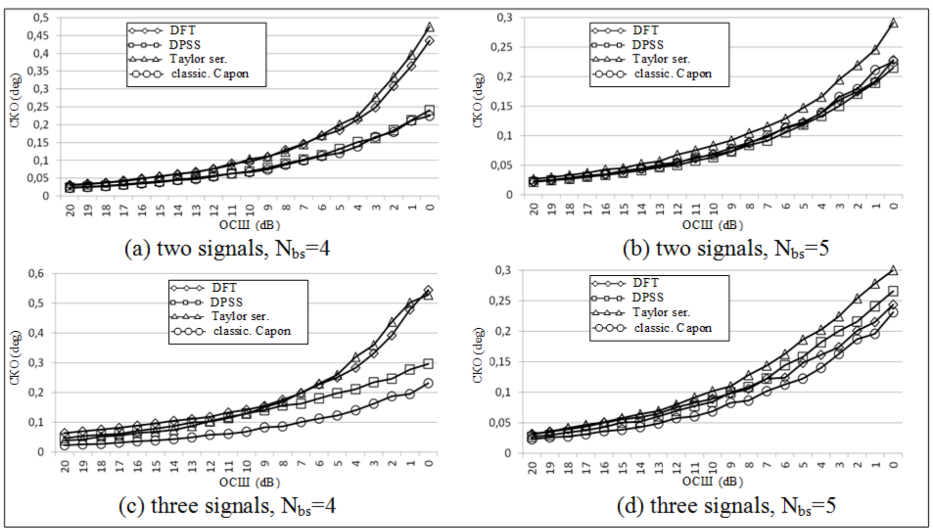

Figure 5. BS-Capon.

methods decrease with the N_{BS} increase from 4 to 5. Fore two signals covered by the sector of interest, at $N_{BS} = 5$ RMSD of the BS-MUSIC method slightly differs from MUSIC irrespective of the way of obtaining T_{BS}. For two signals, DPSS allows lowering RMSD of the BS-Capon method practically down to classical Capon values. For three signals RMSD of BS-MUSIC method at $N_{BS} = 4$ are higher than those of MUSIC. For three signals all the considered methods of obtaining $\mathbf{T}_{BS}$ together with BS-Capon have RMSD close to the values of the classical Capon method. In this regard BS-MUSIC and BS-Capon accuracy for DPSS technique is the highest for most cases, which complies with Figure 2, since side lobe level is low, high occupancy of the useful sector and at most one DP coincides with nulls of others, eliminating ambiguities in $\bar{\mathbf{z}}(t)$. Taken as a whole BS-Capon accuracy is higher than that of BS-MUSIC.

5. Conclusion

The possibility of using orthogonal directional pattern generating methods has been considered for the problem of RRS angular coordinate assessment for the linear equidistant antenna array. Application of this technique to the superresolution direction-finding problem by *MUSIC* and *BS-MUSIC* methods was analyzed using numerical simulation in conditions of different values of signal-to-noise ratio, the number of counts of the spatial correlation matrix and beams of orthogonal directional patterns. The impact of various methods of obtaining $\mathbf{T}_{BS}$ matrix on the direction-finding accuracy by means of BS-MUSIC was evaluated.

It is found that with increased number of orthogonal beams RMSD decreases. BS-MUSIC Accuracy for $\mathbf{T}_{BS}$ matrix obtained according to the DPSS method is the highest and practically coincides with MUSIC one both for the case of the signal presence only in the coverage area, and in case of the additional interference signal getting into the suppression zone $N_{BS}=5$. However, the computational complexity is reduced by 8 times, according to the criterion $O(N^3)$. In addition, with increased averaging count rate up to K=100 RMSD of BS-MUSIC method approximates to its acceptable value.

The root-mean square deviation of BS-MUSIC and BS-Capon superresolution methods increases with the increased number of signals from two to three, with one of them falling on zeroes of directional patterns. When the signals are outside the sector scanned by orthogonal DPs, their robustness and accuracy can be improved by using convex optimization algorithms[8].

6. References

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