

What Period of Time is better to Use for Moving Average to Predict Stock Price in Tehran Stock Exchange?

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Abstract

Accurately predicting stock prices has been long one of the desires of investors in the financial markets. As such, various methods and rationales have been proposed for predicting corporate stock prices. One of the most widely used analytical methods to predict time series is technical analysis in which analysts assess indicators and models to find appropriate strategies and gain profits. Currently, a large number of technical indicators are available but it is not possible to use all of them. Therefore, selecting appropriate indicators for predicting requires great skills. One of such indicators is the simple moving average that can be defined for different time periods. The aim of this study is to find out the most appropriate moving average by assessing the simple moving averages for periods of 1 to 200 days using a feature selection algorithm called Correlation-Based Filter. To this end, data from petrochemical companies list in Tehran Stock Exchange for the period 2009-2015 are analyzed. The results show that five 5, 25, 48, 50 and 89 day simple moving averages are more appropriate to predict stock prices in the Tehran Stock Exchange. To test this hypothesis, we utilize Multilayer Perceptron Artificial Neural Network as a predictor method, and on the one hand, stock prices are predicted using 5 selected indicators, and on the other hand, stock prices are predicted using all 200 indicators. Our results also suggest that using the five selective simple moving averages are more accurate for predicting stock prices.

Keywords: Artificial Neural Network, Correlation-Based Filter, Simple Moving Averages, Technical Analysis

1. Introduction

Predicting stock prices has always been on the main challenges faced by all investors in all markets throughout the world. The reason is the investors' belief in the more accurate the prediction, the higher the profit. Another reason is that the main goal of financial market players is always to gain profit. In the investment process, gaining profit is not possible without sound analysis and having familiarity with stock and market conditions. Therefore, any investor is expected to buy or sell stocks after assessing the available stocks. Two schools of thoughts governing the literature on the stock market are fundamental and technical analyses⁶.

Experts of the technical analysis who are called technicalists or chartists believe that all data needed in the stock market are hidden in the past trends. Therefore, it

is possible to gain more profits in the market by focusing on stock prices and turnover in the past and present and predicting the stock prices and turnover. Such investment is made by studying stock price behaviors and movements in the past and predicting their prices and trends and the possibility of the repetition of similar patterns in the future as according to technicalists, all current economic and industrial events and investors' expectations ultimately affect stock prices. Therefore, the historical stock price trends and charts must be assessed carefully. In fact, the analyst only needs a price trend presented in the form of a chart. Therefore, technicalists are also called chartists. Accordingly, one of the important assumptions held by technicalists is the repeated market trend patterns. As a result, it is possible to predict future price trends from the past price trends based on a set of principles.

In the recent years, a large number of indicators were

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introduced by scholars in the field of technical analysis. One of the most popular indicators is simple moving average. The simple moving averages often provide investors with many signals to buy and sell stocks and they are seen as one of the most useful models for technical analysis in the Iranian capital market⁸.

An important point about moving averages is the selection of their time period. Different moving averages with different time periods are used in studies. For instance, Samadi S, et al⁹. used 10 and 20 day moving averages to predict stock indicators in Tehran Stock Exchange. Further, Chashami SAN, et al.¹⁰ used 30, 60 and 90 day moving averages in their research. In another study, Sajjadi SH, et al.¹¹ employed 3, 7 and 10 day moving averages. Other similar studies conducted in Iran and other countries found moving averages with different time periods as useful moving averages. The lack of consensus among the previous researchers and also the market players about selection optimal time periods for moving average motivated us to study to determine optimal time periods for predicting stock prices using simple moving average indicator and feature selection.

In this study, one of the most well-known feature selection algorithms, i.e. Correlation-Based Filter (CBF) will be used to determine the optimal time periods to be used for analyses performed by simple moving average indicators. In addition, to assess the model performance; corporate stock prices will be predicted using the selected indicators and Artificial Neural Network (ANN). Finally, the prediction accuracy will be compared for all 1-200 day time periods.

2. Theoretical Framework

This section addresses the literature including simple moving average, correlation-based filter, and multilayer perceptron artificial neural network. Then, the empirical studies conducted in the field are reviewed.

2.1 Simple Moving Average

Technical analysis emerged by papers presented by Charles Dow and William Hamilton. This type of analysis deals with the assessment of securities in terms of demand and supply. Chartists predict past stock prices and traded turnover, and future price trends using different types of charts. Prediction in the technical analysis means to determine the appropriate time to enter or leave the market so that the stock purchase and sale in the Tehran

Stock Exchange by using predicting signals of technical analysis is significantly higher than the stock purchase and sale in the Tehran Stock Exchange without using such signals. Moving averages are among the most important tools of technical analysis⁸.

Moving average refers to the average stock price in a given time period. To calculate moving average, the time period must be determined. In fact, 10, 100, 200 or n -day moving average shows the time period when the moving average is calculated. Simple moving average of each time series is calculated as the sum of time-series data in n days divided by n . As an example, to calculate 50 day simple for a given stock, the sum of the daily final stock price over 50 days is calculated and then is divided by 50. Moving averages are regarded as signals for buying and selling stocks. For examples, when the stock prices rise above the moving average technicalists often buy them and when the price reaches below moving average, they sell them. Some investors believe that higher profit is gained by shorter-term moving averages, while researchers maintain that the longer-term moving average have more valid indicators⁸. An n day moving average for period t is calculated as follows and to calculate a 4 day moving average of stock prices in 8th day, the final prices in 8th, 7th, 6th and 5th days are used.

$$MA_t(n) = \frac{1}{n} \sum_{i=0}^{n-1} P_{t-i} \quad (1)$$

2.2 Correlation-Based Filter

In correlation-based filter, the quality of a set of variables is measured in terms of the individual predictive power of each tool plus the redundancy among them. The target function is expressed as followed using Pearson correlation coefficient:

$$f(k) = \frac{k \bar{r}_{cf}}{\sqrt{k + k(k-1) \bar{r}_{ff}}} \quad (2)$$

Where, k is the number of selected feature in a feature subset, $\bar{r}_{cf}$ is the correlation mean between the features and the relevant class, and $\bar{r}_{ff}$ shows the correlation mean between a given feature and other features. The numerator in the above equations shows the predictive power of a set of features while the denominator shows the redundancy between these features. Irrelevant features are less correlated with their own class, thus reducing the value of the numerator in the above equation.

In addition, excluded variables are different from other variables as there is a strong correlation between

them and other variables, thus increasing the numerical value of the numerator in the above equation. As a result, function $f(k)$ assumes a lower value for less relevant features. The inclusion of a feature depends mainly on the ability of that feature for class prediction in the sample which is not predictable by other features¹.

The value of $f(k)$ can be calculated in two ways: First, it is assumed that no variable is selected in the model and all individual variables are progressively are considered as selected features and then the number of feature increases to the extent that $f(k)$ follows an increasing trend. In contrast, the second approach is the opposite as all variables are simultaneously included in the model and they are excluded individually to the extent that $f(k)$ maintains its increasing trend.

2.3 Artificial Neural Network

ANNs are among the most commonly used prediction methods in finance and can be used for predicting each financial time series including the prediction of stock prices. An ANN refers to an input-output system in which the data processing is performed by hidden layers. ANN function is similar to that of an advanced regression in which regression coefficients are estimated with the difference that the term *learning* or *training* is used in the ANN literature for estimating the network weights with the aim of minimizing the sum of error squares as the difference between real output values and the output processed by the ANN. To do so, the data are divided into two training and testing sets. The training set is used for the network learning and the testing set is employed for prediction purposes⁷.

Like computer networks, ANNs can be organized in several different ways. Most of the processing elements perform their calculations simultaneously when processing data. This parallel processing is similar to the brain function and different from serial processing. Currently, ANNs are used for many different purposes. Some examples of ANNs are illustrated in Table 1:

In feed-forward networks, each unit (except for input layer units) receives their inputs directly from the previous layers and sends their outputs directly to the next layer (except for output layer units). Each unit acts only as input for the next layer without any feedback between the units. Theoretically, this network has the simplest analytical structure among other neural networks because of the possibility of providing directly network outputs from

inputs and weights. Recurrent networks act similarly to feed-forward networks with the difference that in recurrent networks there is at least one recurrent signal from a neuron to the same neuron or neurons at the same layer or neurons from the previous layers¹³.

Table 1. Different types of ANNs

Networks	Examples
Recurrent networks	Adaptive resonance theory
	Cohnen networks
	Hapfield networks
	Competitive networks
Feed-forward networks	Single layer perceptron
	Multilayer perceptron
	Radial basis networks

The most widely used network is the multilayer perceptron model⁷ which is a type of recurrent network and is generally made of one or more hidden layers and one output layer as shown in Figure 1.

Data are processed in ANNs by the following components:

- Inputs: Each input is related to a unique feature. The values of the features comprise the network inputs.
- Outputs: A network output is a response to a problem we seek to solve. In addition, a layer whose output is the final network output is called the output layer and other layers are terms middle layers.
- Weights: Weights are the main components of ANNs which show the relative power of mathematical value of initial input data and various interactions transferring the data from one layer to the other. The impact of inputs on outputs is determined by weights. In other words, weights show the relative importance of each processing elements. Weights are of high significance as the network learning occurs are a result of their repeated adjustment¹².

3. Related Works

Moshiri S, Moravat H, et al.¹⁴ used linear and nonlinear models to predict the total stock return in the Tehran Stock Exchange using daily and weekly data over 1998-2033 through various methods such as ARIMA, ARFIMA, GARCH, and ANN. A comparison of the above models showed that ANN is more powerful and accurate for daily and weekly predictions.

Sinaee H, et al.¹⁵ predicted stock prices using multilayer

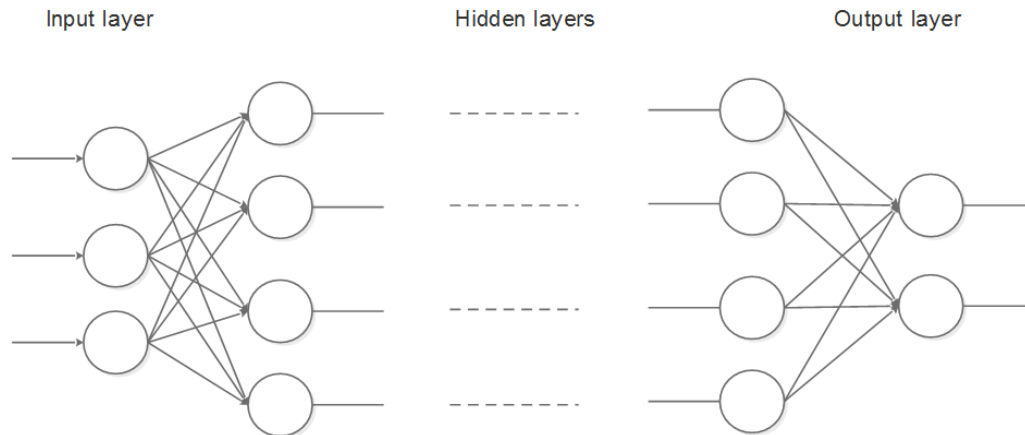


Figure 1. Multilayer perceptron structure.

ANN and ARIMA. They used different lags of stock prices, oil price per barrel, dollar prices, and inflation as input variables. Finally, the best designed network contained ten inputs including two lags of economic factors and two lags for the total stock index. Finally, errors from the two methods were compared and the results showed the superiority of ANN in terms of prediction.

Tehrani R, et al.¹⁶ assessed conditional and unconditional fluctuation models in terms of their power to predict cash return and stock prices in the Tehran Stock Exchange based on Root Mean Square Error (RMSR). The results indicated that the 250 day moving average and CGARCH are more superior to other models in terms of RMSR. The results of mixed models also suggested that unconditional models generally outperformed conditional models. In addition, the results of the Diebold-Mariano (DM) Test indicated that there is no significant difference between the 250-day moving average and CGARCH in terms of predictive power.

Chashami SAN, et al.¹⁰ explored the efficiency of moving average in predicting stock prices using technical analysis. To this end, the stock prices in the Tehran Stock Exchange from 2004 to 2008 were predicted using simple moving average, weighted moving average, and exponential moving average for 30, 60 and 90 time intervals using Excel. Then, the predicted prices were compared with real prices. In addition, the prediction methods were assessed using validation indices such as the Mean Absolute Error (MAE) and the tracing indicator. The results indicated that of the three methods, the componential moving average has a higher validity and reliability for stock price prediction.

Heibati F, et al.¹⁷ explored the relationship between the technical and fundamental stock pricing approaches in the Tehran Stock Exchange. The technical approach was estimated using dual moving average, exponential moving average, relative power, cash flow, and moving average, while the fundamental approach was assessed the asset pricing model. The results of testing the research hypothesis for a five year period from 2004 to 2008 suggested that there was a positive significant relationship between the calculated stock returns (using five technical indicators) and the actual market returns and also between the returns from the asset pricing model and the actual market returns. It was also noted that there was a positive significant correlation between the expected returns from dual moving average, relative power, cash flow and the returns from the asset pricing model. However, there was no significant relationship between returns from exponential moving average and moving average and returns from the asset pricing model.

Khorram M.¹⁸ studied the stock movement direction in the Tehran Stock Exchange using 22 technical prediction indicators based on the data over 2006 to 2013 (1688 transaction days). He used various feature selection algorithm to determine more appropriate indicators to predict stock exchange index in the Tehran Stock Exchange. The results showed that when using the Correlation-Based Feature selection filter, ROC 5, %R 14, and % K 14 indicators were more accurate over the other indicators in terms of stock return prediction in the Tehran Stock Exchange.

Kim KJ, et al.² introduced a model for predicting stock price changes using SVMs. He used eleven technical

indicators as the model inputs. The model outputs were divided into 0 and 1 categories in which 0 showing the future indicator is less than the current indicator while 1 showing the future indicator is greater than the current indicator. It was found that the prediction accuracy was 57% for SVMs and 54% for ANN.

Yu et al.⁴ used the mixed model of SVMs and genetic algorithm to predict the direction of the stock market changes. The reason for using the genetic algorithm was the more effective selection of variables to reduce the model complexity and increase the model speed. The researchers used 18 technical variables as the input variables and the values of S&P 500 from 1 January 2000 to 31 December 2004 were selected as output variables. The results indicated that the mixed model of SVMs and genetic algorithm is more accurate and faster than other models for predicting stock market changes⁴.

Researchers in⁵ predicted the stock price changes using SVMs and the data from daily stock prices in Shanghai Stock Market from 2003 to 2005. Then, the price indicators were classified into training and testing indicators. The researchers also estimated and normalized 13 technical analysis indicators and used recommendations made by almost 400 capital market analysts and their predictions as input variables. The results showed that SVMs have a high predictive ability and when SVMs are combined with the analysts' recommendations better results are obtained compared with the time only when SVMs are used.

Huang CJ, et al.³ used feature selection methods and artificial intelligence methods to find more appropriate input variables for predicting market change trends. They employed data envelope analysis for selecting the most appropriate set of model features including 23 technical indicators. Then, a voting scheme which consisted of common classification algorithms was used for model classification. The researcher predicted the direction of daily stock changes in Korean and Taiwanese Stock Exchange. To this end, indicators which increased two days later compared with the day before were shown by "1" and those which showed a decrease compared with the previous day were shown by "-1". The dataset included 365 transaction days of which 294 were used to train the model and 71 to test the model. In other words, almost 80% of the data were used for training the model and 80% for testing the model. The results indicated that when feature selection algorithms are used in the model, it has a stronger predictive power.

Technical analysis rules were examined in the Chinese Capital Market by Wang S, et al.¹⁹. They analyzed Shanghai Securities Composite Index (SSCI) over 1992-2013 and also China Securities Index 300 (CSI 300) over 2005-2013. The results of their study showed that technical indicators and rules are appropriate to predict market trades. In addition, the less efficient the market, the higher will be the predictive ability of the chartists.

4. Method

The aim of this study was to find answer to the question that for which time periods the simple moving average is more effective in predicting the stock prices in the Tehran Stock Exchange. Accordingly, the present study is regarded as an applied research in terms of its objectives. The scope of the study includes all chemical firms listed in the Tehran Stock Exchange excluding investment firms. The time period under study was a 6 year period from 2009 to 2015. Firms whose fiscal year did not end on 19 March and those which were listed after 2009 and the firms whose data for the period under study were not available were excluded for the research sample. Accordingly, the final sample included 9 firms as shown in Table 2:

Table 2. Firms in research sample

Firm's name	Exchange symbol
Fars Petrochemical Industries	Shafarass
Kermanshah Petrochemical Industries	Kermasha
La'abiran	Shalab
Carbon Iran	Shakarbon
Shazand Petrochemical Company	Sharak
Khark Petrochemical Company	Shekhark
Shiraz Petrochemical Company	Shiraz
Farabi Petrochemical Company	Shafara
Fanavaran Petrochemical Company	Shafan

The final daily stock prices of the sample firms were used as ANN inputs and 1-200 moving averages of the sample firms were selected as input variables. In addition, 80% of the input and output variables were used as the training data and the remaining 20% were used for testing the model.

In correlation-based filter, the linear correlation between the input variables and then the linear correlation between the output variables are estimated. Next, the

target function $f(k)$ will be calculated using equation (1). To estimate $f(k)$, forward selection method is used. To this end, a null set will be selected and then all moving average will be added one by one into the set in order to optimize the function $f(k)$. The process continues until the value of $f(k)$ becomes incremental. The chosen set includes moving averages which are more important for prediction by using the technical analysis. To test this assumption, 1-200 day moving averages and multilayer perceptron NN will be used to predict stock prices in the firms under study. In the next stage, the optimized simple moving averages selected through correlation-based filter will be used as inputs for ANNN and the resulting predictions will be compared.

To compare the performances of the two models, the Mean Absolute Error (MAE) for each method is calculated and the model with a lower MAE is expected to produce better results. The Mean Absolute Error (MAE) is calculated as follows:

$$MAE = \frac{1}{n} \sum_{i=1}^n (|a_i - p_i| / p_i) \quad (3)$$

Where, a_i and p_i are actual and predicted values, respectively.

In this study, feed-forward perceptron ANN with one hidden layer was used. Since estimating the number of neurons in the middle layer can strongly affect the model performance and prediction accuracy, the number of neurons in the hidden layer was 5, 10, 15 and 20 neurons. Then, the number of neurons in the network with the highest performance was selected as the neurons in the hidden layer. The transfer function for the middle layer was the sigmoid function and that for the output layer was a linear transfer function. To train the model, the common error back-propagation algorithm with 1000 training reiteration was used. It is worth mentioning that the data were extracted using Rahavard Novin Software and were analyzed by Excel and Matlab software.

5. Results and Discussions

Of 1-200 day moving averages, those which were more appropriate to predict stock prices were determined using correlation-based filter as illustrated in Table 3:

Table 3. Selected moving averages

Firm symbol	Selected simple moving averages
Shafarass	MA11 – MA25 – MA28 – MA41 – MA50
Kermasha	MA5 – MA15 – MA25 – MA48 – MA67 – MA89 – MA112
Shalab	MA19 – MA25 – MA33 – MA50 – MA61 – MA68 – MA89 – MA95 – MA121 – MA158
Shakarbon	MA3 – MA41 – MA58 – MA59 – MA60 – MA88 – MA116
Sharak	MA5 – MA25 – MA31 – MA48 – MA54 – MA78 – MA99 – MA120 – MA122 – MA134 – MA154 – MA 188
Shekhark	MA5 – MA33 – MA48 – MA50
Shiraz	MA8 – MA19 – MA37 – MA49 – MA103 – MA154
Shafara	MA12 – MA31 – MA48 – MA67 – MA70 – MA72 – MA89 – MA109 – MA134 – MA136 – MA145 – MA156 – MA160
Shafan	MA25 – MA26 – MA27 – MA28 – MA96 – MA161

Table 4 shows the frequencies of the simple moving averages using correlation-based filter:

Table 4. Frequency of selected moving averages

Moving average	Frequency	Moving average	Frequency	Moving average	Frequency
MA3	1	MA50	3	MA109	1
MA5	3	MA54	1	MA112	1
MA8	1	MA58	1	MA116	1
MA11	1	MA59	1	MA120	1
MA12	1	MA60	1	MA121	1
MA15	1	MA61	1	MA122	1
MA19	2	MA67	2	MA134	2
MA25	5	MA68	1	MA136	1
MA26	1	MA70	1	MA145	1
MA27	1	MA72	1	MA154	2
MA28	2	MA78	1	MA156	1
MA31	2	MA88	1	MA158	1
MA33	2	MA89	3	MA160	1
MA37	1	MA95	1	MA161	1
MA41	2	MA96	1	MA188	1
MA48	4	MA99	1		
MA49	1	MA103	1		

As it can be seen in the above Table, 5, 25, 48, 50 and 89 day moving averages are more appropriated for

predicting stock prices in the Tehran Stock Exchange. To test this assumption, the stock prices for the sample firms were predicted using these five moving averages through multilayer perceptron neural network. To determine the predictive powers of the selected moving averages, the prediction results from all moving averages were compared. Interestingly, of 200 moving averages used in the period under study; only 49 moving averages were selected at least once by correlation-based filter and moving averages for 151 time periods were not even selected once. Table 5 shows the prediction of the firm stock prices using multilayer perceptron neural network:

Table 5. Predicted Stock Prices by Multilayer Perceptron Neural Network

Firm	MAEs for 1-200 day moving averages	MAEs for simple moving averages selected by correlation-based filter
Shafarass	0.2315	0.2185
Kermasha	0.1527	0.1910
Shalab	0.3174	0.2466
Shakarbon	0.1246	0.2735
Sharak	0.4163	0.2008
Shekhark	0.2134	0.1699
Shiraz	0.1527	0.1191
Shafara	0.3852	0.1189
Shafan	0.2681	0.2134

As it is shown, the neural network has more predictive power when it uses selected inputs by correlation-based filter. Besides, when the selected moving average were used the predictive power of the neural network increased considerably for 7 firms under study.

6. Conclusion

At present, technical analysis is widely used by investors as a prediction technique. Chartists always try to accurately predict the stock trends by assessing the past trends and employing more appropriate models and indicators from among thousands prediction approaches. However, an important point is how to select these indicators and models. Accordingly, the present study tried to find out optimal time periods to predict stock prices in the Tehran Stock Exchange using a technical indicator called moving

average. To this end, correlation-based filter as a feature selection algorithm was employed. The initial results indicated that 5, 25, 48, 50 and 89 day moving averages are more appropriated for predicting stock prices in the Tehran Stock Exchange. Interestingly, it was found that when moving averages are filtered, the use of moving averages with shorter time periods are more appropriate for predicting stock prices in the Tehran Stock Exchange because of all selected moving averages for the sample firms, 67% of them were of a period less than 100 days and only 33% of them had a time period of 100-200 days.

Finally, to assess the performance of 5 selected moving averages, the stock prices for the sample firms were predicted using these moving averages via ANN. It was noted that when all 1-200 day moving averages were used for prediction, ANN had the highest performance with the five selected simple moving averages.

Technical indicators are very popular and they are widely used by technicalists to predict corporate stock prices. However, selecting the best time period for these indicators is one of unresolved problems in the field of financial markets. Therefore, further research is needed to investigate the issue more profoundly. In addition, researchers can find useful indicators for other time series such as oil, exchange, and gold prices. Correlation-based filter was used in this study as a feature selection algorithm. However, there are feature selection algorithms that can be explored by future research.

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