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Design and Control of Lateral Axis of Aircraft using Sliding Mode Control Methodology

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Abstract

The automatic control system of an aircraft is very complicated in nature with lot of logics and operational constraints. The study of the detailed automatic flight control and thrust control involves complicated logics for the control functions with closed loop control systems which can be realized in the software. The work is focussed on the redesign of automatic flight control system using robust control methods in MATLAB/Simulink environment. This paper aims at a complete study, analysis, design and performance monitoring of new generation autopilot system using Sliding Mode Control method and the results of simulation of the above analysis shows an optimal performance of the autopilot system.

Keywords: Autopilot, Flight Control, FMS, SMC, Yaw Control

1. Introduction

An aircraft control system is an assemblage of mechanical and electronic equipment that let on an aircraft to sail with remarkable accuracy and fidelity. This system comprises cockpit controls, sensors, actuators (hydraulic, mechanical or electrical) and computers.

The introduction of control systems is to establish an additional role for stabilization devices to play since if a device is coupled to control devices then it could correct any departure from a stabilized condition. The earlier flight control system designs are mechanical based ones. They have a huge collection of mechanical parts such as rods, cables, pulleys, and chains are used to transfer the forces to the primary control surfaces from the flight deck controls. As technology grows, the system designs are influenced not only by the advances in aerodynamics and aircraft controllability characteristics but also by the advances taking place in other technological fields, i.e. the changeover from pneumatic operation of gyroscopes to electrical operation, the processing of control signals by electron tubes and magnetic amplifiers, the introduction of semiconductor and also the vast potential of digital processing technology. The diversity of present day automatic flight control system arises principally to suit the aerodynamic and flight handling characteristics of individual types of aircraft. The main parts of the aircraft consist of the primary and secondary systems. The primary control systems of flight are the ailerons, elevator, and rudder. These are required for the safe control of an aircraft during flight such as landing, take-off, attitude changes etc. The wing flaps, leading edge devices, spoilers, and trim systems consists the secondary control devices. By using these devices, the performance characteristics of the aircraft are improved and also the pilot will be relieved from carrying out enormous control forces[1]. Commonly, the aircraft is free to revolve about the pitch, roll and yaw axes which are perpendicular to each other. In this research work, an optimal and robust control method: the Sliding mode controller is designed and the yaw axis of the aircraft is controlled. The automatic control system using the above method is established and the performance characteristics of this controller are studied by simulating in MATLAB environment.

2. Modeling of Yaw Control System

Flight dynamics characterizes the motion of a flight vehicle in the atmosphere. As such, it can be considered

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a branch of systems dynamics in which the system studies is a flight vehicle. For an aircraft, there are two types of dynamic equations: 1. Longitudinal Equations. 2. Lateral Equations.

The pitch control is a longitudinal problem and roll & yaw control are the lateral problem and all the three axes of aircraft are controlled to stabilize the system when the aircraft performs the particular motion[2,3]. Figure 1 shows the forces, moments and velocity components in the body – fixed co-ordinate where X_B, Y_B, Z_B denoted the aerodynamic force components. φ and δr represents the orientation of aircraft in the earth axis system and rudder deflection angle respectively. L, M, N denotes the aerodynamic moment components. p, q, r represents the angular rates of roll, pitch and yaw axis and u, v, w represents the velocity components of roll, pitch and yaw axis. Figure 2 shows the angular orientation and velocities of gravity vector. During the modeling of yaw control systems, few assumptions are considered.

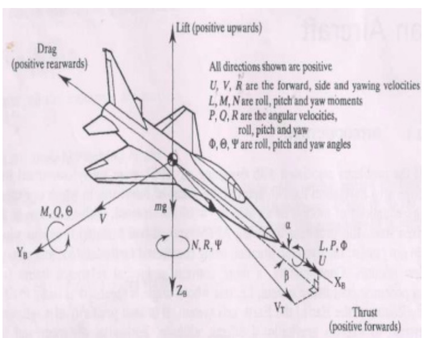

Figure 1. Aerodynamic forces, moments and velocity components.

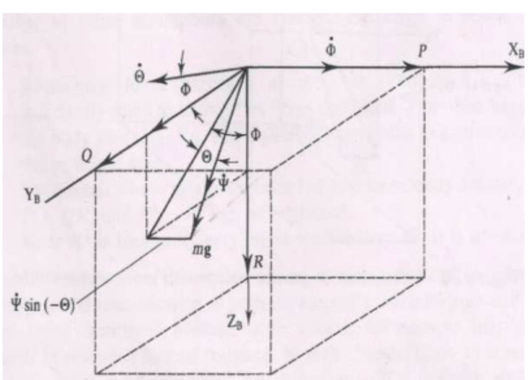

Figure 2. Angular orientation and velocities of gravity vector.

- The thrust and drag cancel out each other since at constant altitude and velocity the aircraft is considered to be a steady state cruise.
- At any situation, the changes in yaw angle does not change the aircraft's speed.

The control of yaw axis is related to the lateral dynamics and in this work the aircraft's yaw angle is controlled in order to maintain the stability of the system when an yawing motion is performed by the aircraft.

With respect to Figure 1 & 2, the force and moment equations are given in equation (1) to (3).

$$Y + mg\cos\theta\sin\varphi = m[\dot{v} + ru - pw] \tag{1}$$

$$L = I_{xz}\dot{p} - I_{xz}\dot{r} + qr(I_{zz} - I_{yy}) \tag{2}$$

$$N = -I_{xz}\dot{p} - I_{zz}\dot{r} + pq(I_{yy} - I_{xx}) + I_{xz}qr \tag{3}$$

The above equations are nonlinear and simplified by considering the aircraft to comprise two components: a mean motion that represents the equilibrium or trim conditions and a dynamic motion which accounts for the perturbations about the mean motion[4]. Thus every motion variable is considered to have two components.

$$\begin{aligned} &U \triangleq U_o + \Delta u, \triangleq Q_o + \Delta q, R \triangleq R_o + \Delta r, \\ &M \triangleq M_o + \Delta m, Y \triangleq Y_o + \Delta y, P \triangleq P_o + \Delta p, \\ &L \triangleq L_o + \Delta l, V \triangleq V_o + \Delta v, \delta \triangleq \delta_o + \Delta\delta \end{aligned} \tag{4}$$

Two assumptions are made such that the flight conditions at equilibrium are considered to be symmetric and the propulsive forces are contemplated as constants.

$$v_o = q_o = u_o = r_o = \varphi_o = \psi_o = 0 \tag{5}$$

The complete linearized equations of motion are obtained as below where sideslip angle is used.

$$\left(\frac{d}{dt} - Y_v\right)\Delta v - Y_p\Delta p + (u_0 - Y_r)\Delta r - (g\cos\theta_0)\Delta\varphi = Y\delta r\cdot\Delta\delta r \tag{6}$$

$$-L_v\Delta v + \left(\frac{d}{dt} - L_p\right)\Delta p - L_r\Delta r = L\delta r\cdot\ \Delta\delta r + L\delta a\cdot\Delta\delta a \tag{7}$$

$$-N_v\Delta v + \left(\frac{d}{dt} - N_r\right)\Delta r - N_p\Delta p = N\delta r\cdot\ \Delta\delta r + N\delta a\cdot\Delta\delta a \tag{8}$$

Substituting, $\Delta v = \Delta\beta$, $Y_v = Y_\beta$, $L_v = L_\beta$, $N_v = N_\beta$ and $\Delta_\beta = \Delta_v/u_o$

$$\left(\frac{Y_\beta}{u_o}\right)\Delta\beta + \frac{Y_p}{u_o}\Delta p - \left(1 - \frac{Y_r}{u_0}\right)\Delta r + \frac{(g\cos\theta_0)}{u_0}\Delta\varphi = \frac{Y\delta r}{u_0}\cdot\Delta\delta r \tag{9}$$

$$-L_{\beta}\Delta\beta+\left(L_{p}\right)\Delta p-L_{r}\Delta r=L\delta r\cdot\ \Delta\delta r+L\delta a\cdot\Delta\delta a \quad (10)$$

$$-N_{\beta}\Delta\beta+\left(N_{p}\right)\Delta p-N_{r}\Delta r=N\delta r\cdot\ \Delta\delta r+N\delta a\cdot\Delta\delta a \quad (11)$$

Using the equations (9), (10), (11) the state space model for the roll and yaw control problem can be formulated.

[(█(Δ˙↓β@(Δ˙↓p)˙@(Δ˙↓r)˙@(Δ˙↓φ)˙))˙] = [█(■(Y↓
↓β/u↓0 &Y↓p/u↓0 & − [1 − Y↓r/u↓0]@L↓β&L↓p&L↓r@N↓
↓β&N↓p&N↓r)■(@@)■((gcosθ↓0)/u↓0 @0@0)@0 1
0 0 @)][■(Δβ@Δp@Δr@Δφ)] + [■(0 (12)

In this work, the modelling is based on the data from the general aviation aircraft NAVION. The yaw control problem has the input as rudder deflection angle and output as change in the yaw angle of aircraft. The lateral directional stability derivative parameters for Navion[5] are taken from the Table 1, which are the standard data of the general aviation aircraft Navion.

The data required for the state space representation are substituted from the Table 1, to get the state space representation for the yaw control system as in equation (13).

$$\begin{bmatrix}\dot{\Delta}_{\beta}\\ \dot{\Delta}_{p}\\ \dot{\Delta}_{r}\\ \dot{\Delta}_{\varphi}\end{bmatrix}=\begin{bmatrix}-0.254 & 0 & -1 & 0.184\\ -15.84 & -8.349 & 2.19 & 0\\ 4.3 & -0.342 & -0.76 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta\beta\\ \Delta p\\ \Delta r\\ \Delta\varphi\end{bmatrix}+\begin{bmatrix}0.07\\ -2.67\\ -4.79\\ 0\end{bmatrix}[\Delta\delta r] \quad (13)$$

Table 1. Lateral Stability Derivatives

Lateral Derivatives	Components		
	X-Force Derivatives	Rolling moment Derivatives	Yawing moment Derivative
Pitching velocities	$Y_v = 0.254$	$L_v = -0.091$	$N_v = .025$
Sideslipangle	$Y_\beta = -44.6$	$L_\beta = -15.84$	$N_\beta = 4.3$
Rolling rate	$Y_p = 0$	$L_p = -8.349$	$N_p = -.342$
Yawing rate	$Y_r = 0$	$L_r = 2.086$	$N_r = -0.76$
Rudder Deflection	$Y_{\delta r} = 12.43$	$L_{\delta r} = -2.67$	$N_{\delta r} = -4.79$
Aileron Deflection	$Y_{\delta a} = 0$	$L_{\delta a} = -28.68$	$N_{\delta a} = -.216$

3. Design of Control Methodologies

The work is emphasized to develop a yaw control scheme for the yaw angle[6,7] of aircraft. For this, the SMC methodology was suggested and the performance of this strategy was investigated in Matlab environment[8].

3.1 Sliding Mode Control Design (SMC)

Sliding mode control method is fundamental consequence of discontinuous control. This method has long proved its significances such as, clarity of design, containment of autonomous motion as long as sliding conditions are maintained, unvarying to the characteristics of process dynamics and external disturbances, wide collection of modes of operation such as regulation, trajectory control, model following and observation. This is a non-linear method[9,11] which adapts the dynamics of the non-linear system by applying a high frequency switching control. This is also called as a variable structure control. The objective of this methodology is to change the state of the system from an initial condition x(0) to the state space origin as t → ∞. The discontinuity on the jth switching surface from the jth component U_j(j = 1, 2, … m) of the state feedback control vector U(x) is a hyperplane 'M_j' passing through the state origin.

Describing the hyper plane as

$$M_j=\{x:C_j x=0\},\ j=1,2,\ldots\ldots \mathrm{m} \quad (14)$$

Defining the system

$$\dot{x}(t)=Ax(t)+Bu(t) \quad (15)$$

From equation (15), the sliding mode satisfies the condition

$$S=Cx(t)=0,\ t\geq t_s \quad (16)$$

C = mxn matrix , t_s is the time of attainment of the sliding subspace. Differentiating eqn. (14) w.r.t. time and substituting for equation (15), we get

$$C\dot{x}(t)=CAx(t)+CBu(t)=0,\ t\geq t_s \quad (17)$$

$$CBu(t)=-CAx(t) \quad (18)$$

C is the hyperplane matrix so that $|CB|\neq 0$

The design of switching surface is predicted on the basis of investigation of the system action in sliding mode and this performance depends on the parameters of the sliding surface. So attaining the switching surface design needs empirical specification of the behaviour of the state trajectory in a sliding mode. Therefore the method of equivalent control is necessary. Equivalent control consists of a control input which when the system excited, yields the motion of the system on the sliding surface whenever the initial state is on the surface. So equation (16) can be rearranged to get equivalent control as below

$$U_{eqt} = -(CB)^{-1} CAx(t)$$

$$U_{eqt} = -Kx(t) \tag{19}$$

$$\dot{x}(t) = [A = Bk]x(t) \tag{20}$$

Equation (20) is the system equation for the closed loop system dynamics during sliding. The suitable selection of 'C' regulates the matrix 'K'. The function of the control "Ueqt' is to force the state in to the sliding subspace and maintaining that in it for all subsequent time. But the (n-m) dimensional switching surface creates 'm' constraints on the dynamics of the plant in the sliding mode. So a regular form approach is followed to change the plant dynamics. The nominal linear system can then be expressed as

$$\dot{y}_1(t) = A_{11} y_1 + A_{12} y_2$$

$$\dot{y}_2(t) = A_{21} y_1 + A_{22} y_2 + B_2 u(t) \tag{21}$$

This plays an important part in delivering the solution of the reachability problem and the sliding condition is equivalent to

$$C_1 y_1(t) + C_2 y_2(t) = 0$$

$$y_2(t) = -\frac{C_1}{C_2} y_1(t)$$

$$y_2(t) = -F y_1(t) \tag{22}$$

$$\dot{y}_1(t) = [A_{11} - FA_{12}] y_1(t) \tag{23}$$

Equation (21) is the reduced order equivalent system.

3.1 Design of Sliding Hyperplane[9,10]

To design the hyperplane $J = \frac{1}{2}\int_{t_s}^{\infty} x^T Qx \, dt$, the quadratic performance can be minimised, where Q > 0 is positive definite symmetric matrix and t_s is the time of reaching the sliding mode. When the matrix Riccati equation is solved then the matrix 'F' will be found.

3.2 Design of Feedback Control Law

The reachability problem can be solved by selecting the sliding surface. This requires the choice of selecting a state feedback control function which will force the state 'x' in to the sliding subspace and maintain it in that subspace. This control law has two components:

- A linear control law (U_l) to stabilize the nominal linear system.
- Discontinuous component (U_n)

And the control law,

$$U(\mathrm{t}) = \mathrm{U}_l(t) + U_n(t) \tag{24}$$

Where, $U_l(t) = U_{eq}(t) = -(CB)^{-1} CAx(t) = -(\mathrm{CB})^{-1}[CA - \varphi C]x(t)$ (25)

The non-linear component is defined to be

$$U_n(t) = \rho \frac{S}{\|S\|} = \rho \frac{Cx(t)}{\|Cx(t)\|}, \rho > 0 \tag{26}$$

'C' is the symmetric positive definite matrix satisfying the Lyapunov equation, $C\varphi + \varphi^T C = -I, \varphi$ is any stable design matrix. The control law is

$$U(t) = -(CB)^{-1}[CA - \varphi C]x(t) + \rho \frac{Cx(t)}{\|Cx(t)\|} \tag{27}$$

4. Simulation and Results

An advanced Control system for controlling the yaw angle of aircraft is simulated using SMC and the results of simulation are evaluated and presented for analysis. To investigate the performance of the control strategy, the time domain specifications are analysed. The control law in SMC has the value, U(t) = [–10.93 –5.41 2.1815].

The Figure 3 shows the performance of the yaw angle control using Sliding mode control methodology.

Table 2 shows the performance characteristics of SMC controller. By analysing the performance characteristics in Table 2, it is clear that the time domain specifications are very less and are also within the limits specified earlier. This depicts that the sliding mode controller has the fastest response and gave an optimal performance in the control of the aircraft's yaw axis.

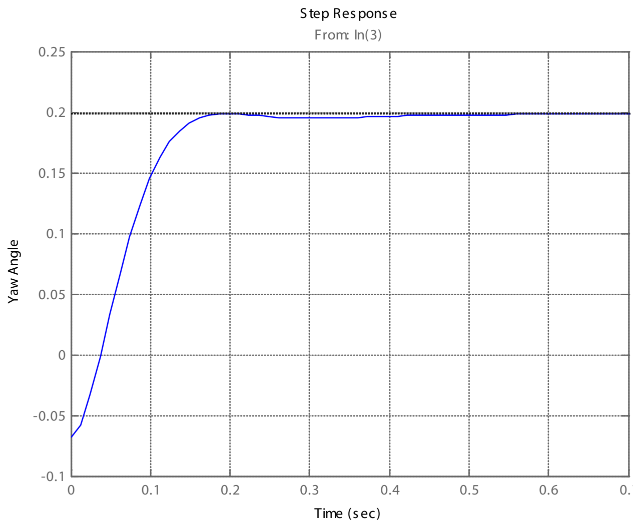

Figure 3. Response of system using SMC.

Table 2. Performance characteristics of SMC

Performance Characteristics of SMC	
Rise time (tr)	0.0999 sec
Settling time (ts)	0.155 sec
% Overshoot (%Mp)	0.0492

5. Conclusion

This research work focuses on the redesign of an autopilot system using robust method, the sliding mode controller in Matlab environment for controlling the aircraft's yaw angle .The controller was designed and the responses were analysed and verified based on investigating few design specifications on the time domain. From the response of Figure 3, it is analyzed that the robust controller gives an optimal performance in controlling the yaw axis of the aircraft efficiently thereby showing the superiority of the method.

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